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The Mathematical Proof for the Beal Conjecture

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Authors' contributions

This work was carried out in collaboration between the authors. Author OSO conceptualized the study idea. The first draft of the manuscript was written by author OSO. Both authors wrote the subsequent drafts together. The authors read and approved the final manuscript.

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Abstract

The Beal conjecture is a number theory formulated in 1993 by the billionaire banker, Mr Andrew Beal. Mr Beal, very recently, declared a one-million-dollar award for the proof of this number theory. As at present, no proof of this conjecture has been generally found. In this article, we provided the proof for Beal conjecture in a crystal-clear systematic approach.

Keywords: Proof; Beal conjecture; number theory.

1 Introduction

In 1993, Andrew Beal, a Texas banker and a number theory enthusiast formulated a number theory which he called the "Beal conjecture" [1-3]. In 1997, Mr Beal publicly declared to offer a $5,000 prize for the proof. Recently, he increased the prize to a sum of $1,000,000 [1-3].

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Quite a number of mathematicians have published research articles on the proof of Beal conjecture, however they failed to provide a definitive proof for the conjecture [1,3-8].

We present such a proof in this paper.

2 The Beal Conjecture

Let A, B, C, X, Y, Z be positive integers, with X, Y, and Z greater than 2. The Beal conjecture [3] states that if:

$$A^X + B^Y = C^Z \tag{1}$$

then A, B, C must have a common prime factor.

3 Conventions

In order to present our proof as simply and clearly as possible, we shall make use in what follows of symbols and conventions presented in Table 1.

Table 1. Symbols and conventions

Symbols*	**Concepts represented**
O_x, O_y, and O_z	Odd integer: 1,3,5,7,...
E_x, E_y, and E_z	Even integer: 2,4,6,8,…

4 Cases of Addition Operations

We observed that three distinct cases can occur for addition operations between two positive integers:

1) **Case 1:**

   $O_x + O_y = E_z$

2) **Case 2:**

   $O_x + E_x = O_y$

3) **Case 3:**

   $E_x + E_y = E_z$

5 Verification

Others before us have established a highly interesting fact: any legitimate solution to the Beal conjecture equation necessarily involves three terms each of which is a "3-powerful number".

Reminder: in a 3-powerful number the exponent of each of its prime factors is at least three.

We shall use this powerful condition as a verification of the numerical examples we present hereafter.

6 The Proof

Below is the proof for the Beal conjecture, using the three cases in Section 4 as case studies.

6.1 Case 1

$$O_x + O_y = E_z$$

Let 'O_x' be A

Let 'O_y' be B

Let 'E_z' be C

$$A + B = C \qquad (2)$$

Let A = 1, let B ≠ 1, and let C ≠ 1

Equation (2) is an ordinary addition operation, and the components A, B, and C are expressible in exponential forms.

Let $B = B_1^{y1}$, where $y1 \geq 3$. Let $C = C_1^{z1}$, where $z1 \geq 1$

$$1 + B_1^{y1} = C_1^{z1} \qquad (3)$$

The authors observed that if C_1^{y1} multiplies through equation (3), a Beal conjecture can be formed.

$$1 \times C_1^{y1} + B_1^{y1} \times C_1^{y1} = C_1^{z1} \times C_1^{y1} \qquad (4)$$

By applying the laws of indices, we can have:

$$C_1^{y1} + (B_1 \times C_1)^{y1} = C_1^{z1+y1} \qquad (5)$$

C_1 is the common factor in in equation (5). Note that $C_1 > 1$, and any number >1 has a prime factor.

Comment: Beal conjecture is true for case 1

6.1.1 Numerical example

i) Derive a Beal conjecture from the equation 1 + 27 = 28.

Solution

$1 + 27 = 28$
$1 + 3^3 = 28^1$

Multiplying through by 28^3

$1 \times 28^3 + 3^3 \times 28^3 = 28^1 \times 28^3$

Applying the rules of indices, we have:

$28^3 + (3 \times 28)^3 = (28)^{1+3}$
$28^3 + 84^3 = 28^4$

2 and 7 are common prime common factors in the equation $28^3 + 84^3 = 28^4$.

ii) Derive a Beal conjecture from the equation 125 + 27 = 152

Solution

$125 + 27 = 152$
$5^3 + 3^3 = 152^1$

Multiplying through by 152^3

$152^3 \times 5^3 + 152^3 \times 3^3 = 152^1 \times 152^3$

Applying the rules of indices, we have:

$(152\times5)^3 + (152\times3)^3 = (152)^{1+3}$
$760^3 + 760^3 = 152^4$

2 , 5 and 19 are common prime common factors in the equation $760^3 + 760^3 = 152^4$.

6.2 Case 2

$O_x + E_y = O_y$

Let 'O_x' be A

Let 'E_y' be B

Let 'O_z' be C

$$A + B = C \tag{6}$$

Let A = 1, let B ≠ 1, and let C ≠ 1

Equation (6) is an ordinary addition operation, and the components A, B, and C are expressible in exponential forms.

Let $B = B_1^{y1}$, where $y1 \geq 3$. Let $C = C_1^{z1}$, where $z1 \geq 1$

$$1 + B_1^{y1} = C_1^{z1} \tag{7}$$

The authors observed that if C_1^{y1} multiplies through equation (7), a Beal conjecture can be formed.

$$1\times C_1^{y1} + B_1^{y1} \times C_1^{y1} = C_1^{z1}\times C_1^{y1} \tag{8}$$

By applying the laws of indices, we can have:

$$C_1^{y1} + (B_1 \times C_1)^{y1} = C_1^{z1+y1} \tag{9}$$

C_1 is the common factor in in equation (9). Note that $C_1 > 1$, and any number >1 has a prime factor.

Comment: Beal conjecture is true for case 2.

6.2.1 Numerical example

Derive a Beal conjecture from the equation 1 + 8 = 9

Solution

$1 + 8 = 9$
$1 + 2^3 = 3^2$

Multiplying through by 3^3

$1 \times 3^3 + 2^3 \times 3^3 = 3^2 \times 3^3$

By applying the laws of indices, we can have:

$3^3 + (2\times3)^3 = 3^{2+3}$

3 is the common prime factor in the equation $3^3 + 6^3 = 3^5$.

6.3 Case 3

$E_x + E_y = E_z$

Let 'E_x' be A
Let 'E_y' be B
Let 'E_z' be C

$$A + B = C \tag{10}$$

Equation (10) is an ordinary addition operation, and the components A, B, and C are expressible in exponential forms:

$$2^{x1} + 2^{y1} = 2^{z1} \tag{11}$$

The authors observed that if 2^{y2}, where y2 is an integer ≥ 2, multiplies through equation (11), a Beal conjecture will be formed.

$$2^{x1} \times 2^{y2} + 2^{y1}\times2^{y2} = 2^{z1}\times2^{y2} \tag{12}$$

By applying the laws of indices, we can have:

$$2^{x1+y2} + 2^{y1+y2} = 2^{z1+y2} \tag{13}$$

In equation (13), 2 is a common prime factor.

Comment: Beal conjecture is true for case 3.

6.3.1 Numerical example

Derive a Beal conjecture from the equation 2 + 2 = 4

Solution

$2^1 + 2^1 = 2^2$

Multiplying through by 2^2

$2^1 \times 2^2 + 2^1\times2^2 = 2^2\times2^2$
By applying the laws of indices, we can have:

$2^{1+2} + (2)^{1+2} = 2^{2+2}$
$2^3 + 2^3 = 2^4$

2 is the common prime factor in the equation $2^3 + 2^3 = 2^4$.

7 Conclusion

The Beal conjecture is true.

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Competing Interests

Authors have declared that no competing interests exist.

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