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Effects of Homogeneous and Heterogeneous Reactions on the Dispersion of a Solute in MHD Newtonian Fluid in an Asymmetric Channel with Peristalsis

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Research Article

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Abstract

The dispersion of a solute in peristaltic motion of a magneto-Newtonian fluid flow through a porous medium in an asymmetric channel is studied in the presence of both homogeneous and heterogeneous chemical reaction as well. The fluid is electrically conducting by a transverse magnetic field. The channel asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitudes and phase. Applying long wavelength approximation and using Taylor's limiting condition, the effective dispersion coefficient has been found in explicit form for the two cases (homogeneous and heterogeneous chemical reactions). Moreover, the effects of various emerging parameters on the average coefficient of dispersion are discussed with the help of graphs. The results reveal that the peristaltic wave enhances dispersion of a solute but the phase difference between the two waves reduces it.

Keywords: Asymmetric channel, dispersion, chemical reaction, peristalsis.

1 Introduction

The dispersion of a solute in a solvent flowing in conduit (pipe or channel) has wide applications in chemical engineering, biomedical engineering, environmental sciences and physiological fluid dynamics. First fundamental theory on dispersion was made by Taylor [1-3], who discussed the dispersion of solute matter in the viscous, incompressible, laminar flow of a fluid in a circular pipe. He observed that, relative to a plane moving with the average speed of the flow, the solute disperses with an equivalent dispersion coefficient, which depends upon the average speed of the flow, the radius of the tube, and the molecular diffusion coefficient. In his analysis, Taylor [1] assumed that the solute does not chemically react with the fluid. However, in a variety of problems in chemical engineering, diffusion of solute takes place in the presence of irreversible first order chemical reaction. Therefore, many investigations on dispersion problem with simultaneous chemical reaction for both Newtonian and non-Newtonian have been considered [4-10]. Further, a number of authors have studied the dispersion of a solute in a porous medium

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under different conditions [11-13].

Peristalsis is a mechanism for pumping fluids by means of a contraction wave travelling along the tube. This mechanism is found in many physiological situations like urine transport from the kidney to bladder through the ureter, swallowing food through the oesophagus, movement of the chyme in the gastrointestinal tract, the movements of spermatozoa in the ductus efferentes of the male reproductive tract and the ovum in the female fallopian tube. Moreover, peristaltic mechanism is involved in transporting the lump in the lymphatic vessels, movement of the bile in the bile duct and the circulation of the blood in small blood vessels such as arterioles, venules and capillaries. In addition, the importance of such flows has also been recognized in transport of slurries, corrosive fluids, sanitary fluid and noxious fluids in the nuclear industry. Further, roller and finger pumps are widely operated under such mechanism. Some recent attempts dealing with peristaltic flow in different situations are reported in a paper of Sobh [14].

The magnetohydrodynamic (MHD) flow of a fluid in a channel with elastic, rhythmically contracting walls is of interest in connection with certain problems of the movement of conductive physiological fluids, e.g. the blood and blood pump machines. Recently, many contributions have been done to understand the MHD flow in peristaltic ducts. Some of recent papers dealing with MHD peristaltic flow are given by Abd El Naby et al. [15], Hayat et al. [16], Mekheimer and Abd elmaboud [17], Kothandapani and S. Srinivas [18], and Sobh [19].

The early studies on peristaltic transport were done in symmetric channels or tubes. Recently, physiologists observed that peristaltic motion may occur in both symmetric and asymmetric directions. After this observation, Eytan and Elad [20] have presented a mathematical model of wall-induced peristaltic fluid flow in a two-dimensional channel with wave trains having a phase difference moving independently on the upper and lower walls to simulate intra-uterine fluid motion in a sagittal cross section of the uterus. They have obtained a time independent flow solution in fixed frame by using lubrication approach. After this study, many investigations have been done to understand the mechanism of peristalsis in asymmetric channels [21-27].

Dispersion of a solute in peristaltic motion problems has not received much attention. In their recent papers, Alemayehu and Radhakrishnamacharya [28-29] have investigated the effect of peristalsis on dispersion in a micropolar fluid flowing in symmetric channel. Since peristalsis, diffusion, MHD and porosity are very important aspects in biological, chemical, environmental and bio-medical processes (Paul [30]), and since peristaltic motion may occur in both symmetric and asymmetric directions, we propose to analyze the dispersion of a solute in peristaltic flow of a Newtonian fluid in an asymmetric channel in the presence of transverse magnetic field and porous medium. The transport of nutrients in blood vessels can be considered as application to this problem, as the blood vessels have peristalsis on its walls [31]. Under long wavelength assumption and using Taylor's approach, the dispersion coefficient has been obtained in closed form for both the cases of homogeneous and heterogeneous chemical reactions. Furthermore, average effective dispersion coefficient is computed numerically and the results were discussed for various values of parameters of interest through graphics.

2 Formulations and Analysis

Consider the motion of an incompressible Newtonian fluid through a porous medium in an asymmetric channel induced by sinusoidal wave trains propagating with constant speed c along the channel walls. The fluid is subjected to uniform magnetic field B_0 , applied transversely to the flow. Let d_1+d_2 be the channel width. We select a rectangular coordinate system for the channel in such a way that *x* lies in the direction of wave propagation and *y* transverse to it. The wall surfaces are given by (Fig. 1)

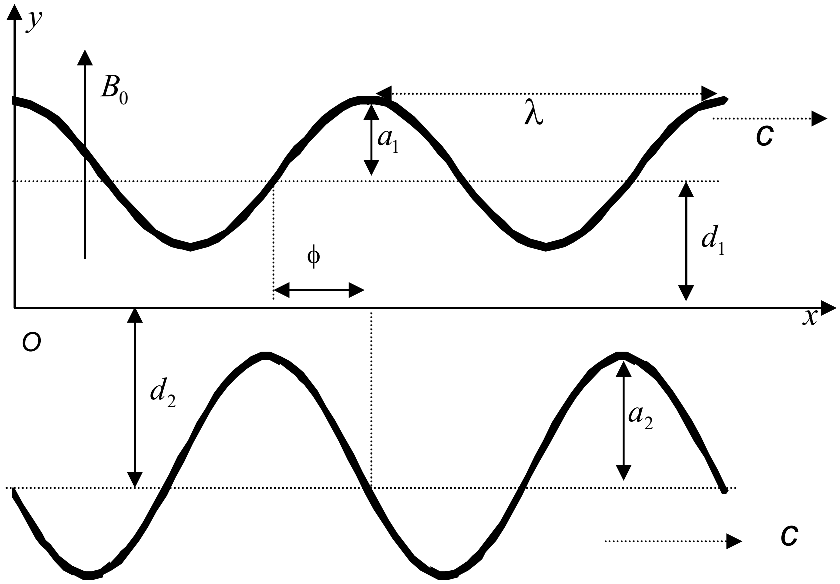

Fig. 1. Geometry of the problem

$$H_1(x,t) = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(x-ct)\right], \qquad \text{upper wall} \tag{1}$$

$$H_2(x,t) = -d_2 - a_2 \cos\left[\frac{2\pi}{\lambda}(x-ct)+\phi\right], \quad \text{lower wall} \tag{2}$$

In the above equations, a_1 and a_2 are the amplitudes of the waves, λ is the wavelength, *c* is the wave speed, and $\phi(0 \le \phi \le \pi)$ is the phase difference.

The equations governing the flow of Newtonian fluid through a porous medium in the presence of transverse magnetic field are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\mu}{k}u - \sigma B_0^2 u, \tag{4}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\mu}{\bar{k}}v, \tag{5}$$

where $u(x,y,t)$ and $v(x,y,t)$ are the velocity components in the x and y directions, ρ is the denisty, μ is the fluid viscosity, $\bar{k}$ is the permeability parameter, σ is the electrical conductivity of the fluid, B_0 is the strength of the magnetic field.

Under long wavelength approximation and low Reynolds number, the equations (3-5) become [18,19, 21]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{\bar{k}}u-\sigma B_0^2 u=0, \tag{7}$$

$$-\frac{\partial p}{\partial y}=0, \tag{8}$$

The relevant boundary conditions are

$$u=0, \quad at \quad y=H_1(x), \tag{9}$$
$$u=0, \quad at \quad y=H_2(x). \tag{10}$$

Solving Eqs. (6-8), subject to the boundary conditions (9) and (10), we obtain

$$u(x,y)=\frac{1}{\mu m_1^2}\left(\frac{\partial p}{\partial x}\right)\left[A_1^*\cosh(m_1 y)+A_2^*\sinh(m_1 y)-1\right], \tag{11}$$

Further, the mean velocity can be found as

$$\bar{u}=\frac{1}{(H_1-H_2)}\int_{H_2}^{H_1}u(y)\,dy$$

$$=\frac{1}{\mu m_1^2}\left(\frac{\partial p}{\partial x}\right)\left[\frac{A_1^*}{m_1}\left(\sinh(m_1H_1)-\sinh(m_1H_2)\right)+\frac{A_2^*}{m_1}\left(\cosh(m_1H_1)-\cosh(m_1H_2)\right)-(H_1-H_2)\right], \tag{12}$$

If we now assume that the convection is across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity can be obtained as

$$u_x = u - \bar{u} = \frac{1}{\mu m_1^2}\left(\frac{\partial p}{\partial x}\right)\left[A_1^* \cosh(m_1 y) + A_2^* \sinh(m_1 y) + A_3^*\right], \tag{13}$$

where $m_1 = \sqrt{\frac{1}{\bar{k}} + \frac{\sigma B_0^2}{\mu}}$, A_1^* , A_2^* and A_3^* are stated in the appendix.

2.1 Diffusion with Homogeneous First-Order Chemical Reaction

If we assume that the solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in peristaltic transport of a magneto-Newtonian fluid in asymmetric channel filled with porous medium, then under isothermal condition, the concentration equation of the solute is given by [10]

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - k_1 C, \tag{14}$$

where D is the molecular diffusion coefficient, assumed to be independent of C, and k_1 is the first order reaction rate constant.

For typical values of physiologically relevant parameters of this problem, it is realized that $\bar{u} \approx c$, where the solute is dispersed relative to a plane moving with the mean velocity of the fluid [28].

Using this condition and following Taylor [1-3], with the assumption that $\frac{\partial^2 C}{\partial x^2} << \frac{\partial^2 C}{\partial y^2}$, equation (14) and its boundary conditions can be written in dimensionless form as

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d_1^2}{D} C = \frac{d_1^2}{\lambda D} u_x \frac{\partial C}{\partial \xi}, \tag{15}$$

$$\frac{\partial C}{\partial \eta} = 0, \qquad for \qquad \eta = h_1, \tag{16}$$

and

$$\frac{\partial C}{\partial \eta} = 0, \qquad for \qquad \eta = h_2, \tag{17}$$

where the following non-dimensional quantities have been used

$$\theta = \frac{t}{\bar{t}}, \bar{t} = \frac{\lambda}{\bar{u}}, \eta = \frac{y}{d_1}, \xi = \frac{x - \bar{u}t}{\lambda},\ k = \frac{\bar{k}}{d_1^2}, M = \sqrt{\frac{\sigma}{\mu}} B_0\, d_1, a = \frac{a_1}{d_1}, b = \frac{a_2}{d_1}, d = \frac{d_2}{d_1},$$

$$h_1 = \frac{H_1}{d_1} = 1 + a\cos(2\pi\xi),\ h_2 = \frac{H_2}{d_2} = -d - b\cos(2\pi\xi + \phi), \tag{18}$$

Further, the relative velocity u_x in dimensionless form becomes

$$u_x = \frac{d_1^2}{\mu m^2}\left(\frac{\partial p}{\partial x}\right)\left[A_1\cosh(m\eta) + A_2\sinh(m\eta) + A_3\right], \tag{19}$$

where $m = m_1 d_1 = \left(\frac{1}{k} + M^2\right)$, A_1, A_2, and A_3 are defined in the appendix.

Assuming that $\frac{\partial C}{\partial \xi}$ is independent of η at any cross section, the solution of concentration equation (15) subject to the boundary conditions (16) & (17) is given by

$$C(\eta) = \frac{d_1^4}{\lambda D \mu m^2}\frac{\partial C}{\partial \xi}\left(\frac{\partial p}{\partial x}\right) \left[A_6\cosh(\gamma\eta) + A_7\sinh(\gamma\eta) + A_4\cosh(m\eta) + A_5\sinh(m\eta) - \frac{A_3}{\gamma^2}\right], \tag{20}$$

where $\gamma = \sqrt{\frac{k_1}{D}}\, d_1$, is the homogeneous reaction rate parameter and $A_3, \ldots, A_7$ are defined in the appendix.

Now the volume rate Q at which the solute is transported across a section of the channel of unit breadth is given by

$$Q = \int_{h_2}^{h_1} C\, u_x\, d\eta \,. \tag{21}$$

Inserting for C and u_x from (20) and (19) in (21) and carrying out the integration, we get

$$Q = -\frac{2d_1^6}{\lambda \mu D}\frac{\partial C}{\partial \xi}\left(\frac{\partial p}{\partial x}\right)^2 F(\xi, a, b, d, \phi, \gamma, M, k), \tag{22}$$

where

$$F(\xi,a,b,d,\phi,\gamma,M,k) = \frac{1}{8m^5\gamma^2(m^2-\gamma^2)}\left[(m^2-\gamma^2)\left\{-4\gamma^2 A_3(A_4B_1 + A_5B_2) - 4\gamma m A_3(A_6B_3 + A_7B_4)\right.\right.$$

$$+4A_3(A_1B_1+A_2B_2)-\gamma^2A_1(A_5B_{19}+A_4B_{20})-\gamma^2A_2(A_4B_{19}+A_5B_{20})$$
$$+2m(2A_3^2+\gamma^2A_2A_5-\gamma^2A_1A_4)(h_1-h_2)\big\}-4m\gamma^2(A_1A_6B_{15}-A_1A_7B_{16}$$
$$+A_2A_6B_{17}-A_2A_7B_{18})\big], \quad (23)$$

$B_1,\ldots,B_{20}$ are stated in the appendix.

Comparing (22) with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^*=\frac{2d_1^6}{\mu^2D}\left(\frac{\partial p}{\partial x}\right)^2F(\xi,a,b,d,\phi,\gamma,M,k). \quad (24)$$

The average effective dispersion coefficient can be found as

$$\bar{F}=\int_0^1F(\xi,a,b,d,\phi,\gamma,M,k)\,d\xi. \quad (25)$$

2.2 Diffusion with Combined Homogeneous and Heterogeneous Chemical Reactions

In this subsection, we discuss the problem of diffusion with a first order irreversible chemical reaction taking place both in the bulk of the medium (homogeneous) as well as at the walls of the channel (heterogeneous). The channel walls are assumed to be catalytic to chemical reaction. The diffusion equation (14) still holds and the differential material balance at the walls gives the boundary conditions [32]

$$\frac{\partial C}{\partial y}+f\,C=0, \qquad at \qquad y=H_1, \quad (26)$$

$$\frac{\partial C}{\partial y}-f\,C=0, \qquad at \qquad y=H_2, \quad (27)$$

Using the dimensionless variables (18) and assuming the limiting condition of Taylor [1-3], the diffusion equation remains to the non-dimensional form (15) and the boundary conditions (26) & (27) become

$$\frac{\partial C}{\partial y}+\beta\,C=0, \qquad at \qquad y=h_1, \quad (28)$$

$$\frac{\partial C}{\partial y}-\beta\,C=0, \qquad at \qquad y=h_2, \quad (29)$$

where $\beta = f\, d_1$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction at the walls.

The solution of diffusion equation (15) subject to the boundary conditions (27) & (28) is given by

$$C(\eta) = \frac{d_1^4}{\lambda D \mu m^2} \frac{\partial C}{\partial \xi} \left(\frac{\partial p}{\partial x} \right) \begin{bmatrix} A_8 \cosh(\gamma\eta) + A_9 \sinh(\gamma\eta) \\ + A_4 \cosh(m\eta) + A_5 \sinh(m\eta) - \dfrac{A_3}{\gamma^2} \end{bmatrix}, \tag{30}$$

Where A_8, A_9 are defined in the appendix.

Substituting (19) and (30) into (21) and integrating, we obtain the volume rate Q as

$$Q = -\frac{2d_1^6}{\lambda \mu D} \frac{\partial C}{\partial \xi} \left(\frac{\partial p}{\partial x} \right)^2 G(\xi, a, b, d, \phi, \gamma, \beta, M, k), \tag{31}$$

Where

$$G(\xi,a,b,d,\phi,\gamma,\beta,M,k) = \frac{1}{8m^5\gamma^2(m^2-\gamma^2)} \Big[(m^2-\gamma^2) \big\{ -4\gamma^2 A_3 (A_4 B_1 + A_5 B_2) - 4\gamma m A_3 (A_8 B_3 + A_9 B_4)$$
$$+ 4A_3(A_1B_1 + A_2B_2) - \gamma^2 A_1 (A_5 B_{19} + A_4 B_{20}) - \gamma^2 A_2 (A_4 B_{19} + A_5 B_{20})$$
$$+ 2m(2A_3^2 + \gamma^2 A_2 A_5 - \gamma^2 A_1 A_4)(h_1 - h_2) \big\} - 4m\gamma^2 (A_1 A_8 B_{15} - A_1 A_9 B_{16} + A_2 A_8 B_{17} - A_2 A_9 B_{18}) \Big], \tag{32}$$

Again, comparing (31) with Fick's law of diffusion, we find that the solute is depressed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^* = \frac{2d_1^6}{\mu^2 D} \left(\frac{\partial p}{\partial x} \right)^2 G(\xi, a, b, d, \phi, \gamma, \beta, M, k) \tag{33}$$

The average effective dispersion coefficient for this case is given by

$$\overline{G} = \int_0^1 G(\xi, a, b, d, \phi, \gamma, \beta, M, k)\, d\xi \tag{34}$$

3 Results and Discussion

It is clear that our results calculate the effective dispersion coefficient for both the two cases of homogeneous and heterogeneous chemical reactions respectively in the case of peristaltic flow in asymmetric channel. It is important to note that the case ϕ=0 corresponds to an asymmetric channel with waves out of phase. Moreover, when (ϕ=0, d=1, a=b) we obtain the results for symmetric channel (the special case). Further, equations (24) and (33) reveal that the effective

dispersion coefficient depends on the dimensionless parameters: ϕ (the phase difference of the two waves), *M* (the Hartmann number), *k* (the permeability parameter), *a*, *b*, (the amplitude ratios), the non-dimensional quantity *d*, and the homogeneous reaction parameter γ, for homogeneous reaction case, and the heterogeneous reaction rate parameter β, for heterogeneous reaction case.
In order to have an estimate of the quantitative effects of the various parameters involved in the results of the present analysis, we use the MATHEMATICA software to carry out the integrals in equations (22) and (31) numerically. The effects of emerging parameters on the average effective dispersion coefficient are illustrated graphically through Figs. (2-13).

3.1 Homogeneous Chemical Reaction

The effect of phase difference ϕ on the average effective dispersion coefficient $\overline{F}$ is shown in Fig. (2) at *a*=0.7, *b*=0.8, *d*=1.5, *M*=1, *k*=1 and (ϕ =0, π/6, π/4, π/3). It is noted that an increase in the phase difference decreases the dispersion. This means that the phase difference between the wave on the upper wall of the channel and the wave on the lower wall tends to decrease the dispersion of the solute in the flow of peristaltic transport. In other words, the dispersion of a solute in peristaltic flow through symmetric channel is greater than the dispersion in the flow through asymmetric one.

Fig. (3-5) represent the variation of the average effective dispersion coefficient $\overline{F}$ versus γ for various values of (*a*, *b*, *d*) and fixed values of other physical parameters. The graphical results reveal that $\overline{F}$ increases by increasing the non-dimensional quantities *a*, *b*, and *d*. It is deduced that peristaltic pumping enhances the dispersion. In other words, the values of the effective dispersion coefficient for peristaltic flow $(a, b \neq 0)$ are greater than its values for the flow between two parallel plates (*a*=*b*=0).

The effect of magnetic field on average effective dispersion coefficient is shown in Fig. (6) at *a*=0.7, *b*=0.8, *d*=1.5, ϕ=π/6, *k*=1 and (*M*=1.1, 1.2, 1.3, 1.4). The graph indicates that the average effective dispersion coefficient decreases by increasing the Hartmann number *M*. This result agrees with the result obtained by Alemayehu and Radhakrishnamacharya [28], for the flow of micropolar fluids in symmetric channel.

Figs. (7) is the graph of the average effective dispersion coefficient $\overline{F}$ versus γ at *a*=0.7, *b*=0.8, *d*=1.5, ϕ=π/6, *M*=1 and (*k*=0.7, 0.8, 0.9, 1). We observe that an increasing in permeability parameter *k* yields a decrease in the dispersion. In other words, the dispersion of a solute increases with the flow in porous medium.

Furthermore, as important general result from the Figs. (2-7), it is noticed that the average effective dispersion coefficient $\overline{F}$ decreases with homogeneous reaction rate parameter γ. This means that homogeneous chemical reaction tends to decrease the dispersion of the solute. This is because an increase in γ leads to increasing number of moles of solute undergoing chemical reaction, which results in the decrease of dispersion. Also, this result agrees with the previous work of Padma and Ramana [4], Gupta and Gupta [5], Ramana and Padma [6-7], and Dutta *et al.* [8].

3.2 Combined Homogeneous and Heterogeneous Chemical Reactions

Figs.(8-13) are made to see the effects of homogeneous reaction parameter γ, phase difference ϕ, amplitude ratios a and b, Magnetic parameter M, and permeability parameter k on the average effective dispersion coefficient $\overline{G}$ for the case of combined first order chemical reactions both in the bulk and at the walls. The graphical results of these six figures indicate that the average dispersion coefficient $\overline{G}$ decreases with increasing homogeneous reaction parameter, phase difference, and magnetic parameter. But $\overline{G}$ increases by increasing the amplitude ratio and the permeability parameter. Also, it is noticed from the figures that the dispersion decreases with heterogeneous reaction rate parameter β, and the decrease in the effective dispersion coefficient is sharp in a region near to the wall. This agrees with chemical point of view since the reaction which affect dispersal happens only at the surface for heterogeneous chemical reaction. This means that heterogeneous chemical reaction tends to decrease the dispersion of the solute.

4 Conclusions

The dispersion of a solute in peristaltic flow of MHD Newtonian fluid through asymmetric channel filled with porous medium is studied under long wavelength approximation and Taylor's limiting condition for both homogeneous and heterogeneous chemical reactions. The average effective dispersion coefficient is computed numerically and explained graphically in both cases. The results reveal that dispersion of a solute in peristaltic flow through symmetric channel is greater than the dispersion in the flow through asymmetric one. Furthermore, the average effective dispersion coefficient tends to decrease with homogeneous chemical reaction rate parameter γ and magnetic parameter M while it increases with increasing amplitude ratios a, b, and permeability parameter k.

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Competing Interests

Author has declared that no competing interests exist.

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Appendix

$$A_1^* = \frac{\sinh(m_1 H_1) - \sinh(m_1 H_2)}{\sinh[m_1 (H_1 - H_2)]},\ A_2^* = -\frac{\cosh(m_1 H_1) - \cosh(m_1 H_2)}{\sinh[m_1 (H_1 - H_2)]},$$

$$A_3^* = -A_1^* \frac{\sinh(m_1 H_1) - \sinh(m_1 H_2)}{m_1 (H_1 - H_2)} - A_2^* \frac{\cosh(m_1 H_1) - \cosh(m_1 H_2)}{m_1 (H_1 - H_2)},$$

$$B_1 = \sinh(mh_1) - \sinh(mh_2),\ B_2 = \cosh(mh_1) - \cosh(mh_2),\ B_3 = \sinh(\gamma h_1) - \sinh(\gamma h_2),$$

$$B_4 = \cosh(\gamma h_1) - \cosh(\gamma h_2),\ B_5 = \sinh(\gamma h_1) + \sinh(\gamma h_2),\ B_6 = \cosh(\gamma h_1) + \cosh(\gamma h_2),$$

$$B_7 = \cosh(\gamma h_1)\cosh(mh_2) - \cosh(mh_1)\cosh(\gamma h_2), B_8 = \cosh(\gamma h_1)\sinh(mh_2) - \sinh(mh_1)\cosh(\gamma h_2),$$

$$B_9 = \cosh(mh_1)\sinh(\gamma h_2) - \cosh(mh_2)\sinh(\gamma h_1), B_{10} = \sinh(mh_1)\sinh(\gamma h_2) - \sinh(\gamma h_1)\sinh(mh_2),$$

$$B_{11} = \cosh(\gamma h_1)\cosh(mh_1) - \cosh(mh_2)\cosh(\gamma h_2), B_{12} = \cosh(\gamma h_1)\sinh(mh_1) - \sinh(mh_2)\cosh(\gamma h_2),$$

$$B_{13} = \cosh(mh_1)\sinh(\gamma h_1) - \cosh(mh_2)\sinh(\gamma h_2), B_{14} = \sinh(\gamma h_1)\sinh(mh_1) - \sinh(mh_2)\sinh(\gamma h_2),$$

$$B_{15} = mB_{12} - \gamma B_{13},\ B_{16} = \gamma B_{11} - mB_{14},\ B_{17} = mB_{11} - \gamma B_{14},\ B_{18} = \gamma B_{12} - mB_{13},$$

$$B_{19} = \cosh(2mh_1) - \cosh(2mh_2),\ B_{20} = \sinh(2mh_1) - \sinh(2mh_2),$$

$$B_{21} = \cosh(\gamma h_1)\cosh(mh_2) + \cosh(mh_1)\cosh(\gamma h_2), B_{22} = \sinh(\gamma h_1)\sinh(mh_2) + \sinh(mh_1)\sinh(\gamma h_2),$$

$$B_{23} = \cosh(\gamma h_1)\sinh(mh_2) + \sinh(mh_1)\cosh(\gamma h_2), B_{24} = \sinh(\gamma h_2)\cosh(mh_1) + \cosh(mh_2)\sinh(\gamma h_1),$$

$$A_1 = \frac{B_1}{\sinh[m(h_1 - h_2)]},\ A_2 = -\frac{B_2}{\sinh[m(h_1 - h_2)]},\ A_3 = -\frac{(A_1 B_1 + A_2 B_2)}{m(h_1 - h_2)},\ A_4 = \frac{A_1}{m^2 - \gamma^2},$$

$$A_5 = \frac{A_2}{m^2 - \gamma^2},\ A_6 = \frac{m(A_5 B_7 + A_4 B_8)}{\sinh[\gamma(h_1 - h_2)]},\ A_7 = \frac{m(A_5 B_9 + A_4 B_{10})}{\sinh[\gamma(h_1 - h_2)]},$$

$$A_8 = \frac{\beta A_3(\gamma B_6 + \beta B_3) + \gamma^2 A_4(m\gamma B_8 + \beta^2 B_9 - \gamma\beta B_{21} + m\beta B_{22}) + \gamma^2 A_5(m\gamma B_7 + \beta^2 B_{10} - \gamma\beta B_{23} + m\beta B_{24})}{2\gamma^3 \beta \cosh[\gamma(h_1 - h_2)] + \gamma^2(\beta^2 + \gamma^2)\sinh[\gamma(h_1 - h_2)]},$$

$$A_9 = \frac{-\beta A_3(\gamma B_5 + \beta B_4) + \gamma^2 A_4(m\gamma B_{10} + \beta^2 B_7 + \gamma\beta B_{24} - m\beta B_{23}) + \gamma^2 A_5(m\gamma B_9 + \beta^2 B_8 + \gamma\beta B_{22} - m\beta B_{21})}{2\gamma^3 \beta \cosh[\gamma(h_1 - h_2)] + \gamma^2(\beta^2 + \gamma^2)\sinh[\gamma(h_1 - h_2)]},$$

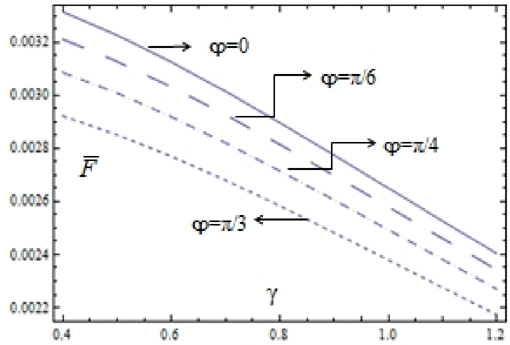

Fig. 2. Variation of $\overline{F}$ with γ for a=0.7, b=0.8, d=1.5, M=1, k=1

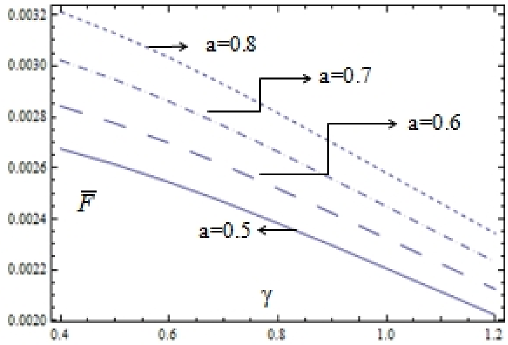

Fig. 3. Variation of $\overline{F}$ with γ for b=0.7, d=1.5, φ=π/6, k=1, M=1

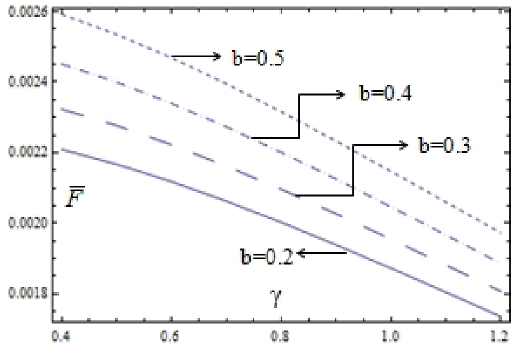

Fig. 4. Variation of $\overline{F}$ with γ for a=0.7, d=1.5, φ=π/4, k=1, M=1

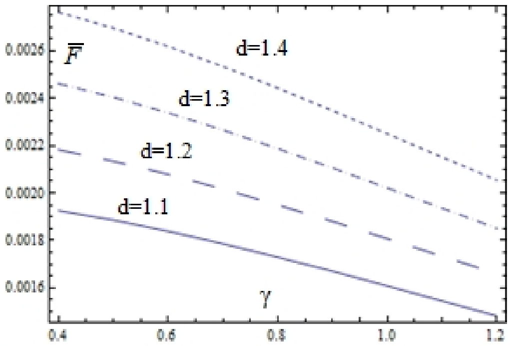

Fig. 5. Variation of $\overline{F}$ with γ for a=0.7, b=0.8, φ=π/4, k=1, M=1

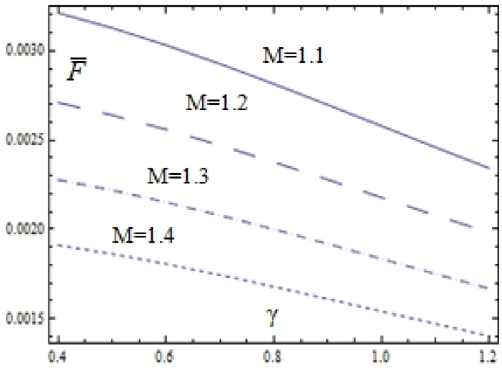

Fig. 6. Variation of $\overline{F}$ with γ for a=0.7, b=0.8, d=1.5, φ=π/6, k=1

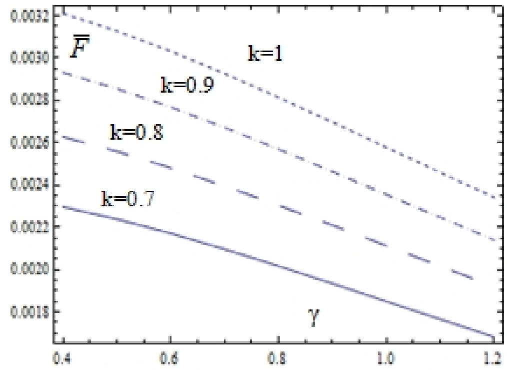

Fig. 7. Variation of $\overline{F}$ with γ for a=0.7, b=0.8, d=1.5, φ=π/6, M=1

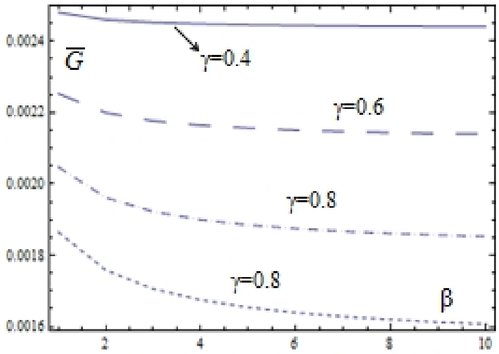

Fig. 8. Variation of $\overline{G}$ with β for k=1, a=0.5, b=0.7, d=1.5, φ=π/6, M=1

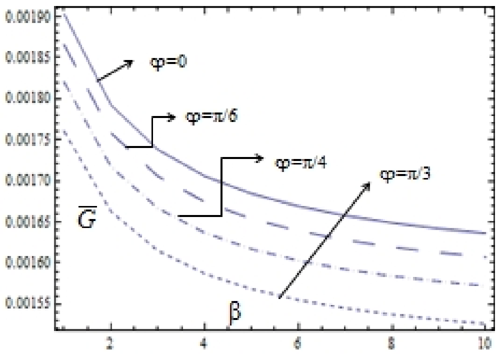

Fig. 9. Variation of $\overline{G}$ with β for γ=1 a=0.5, b=0.7, d=1.5, M=1, k=1

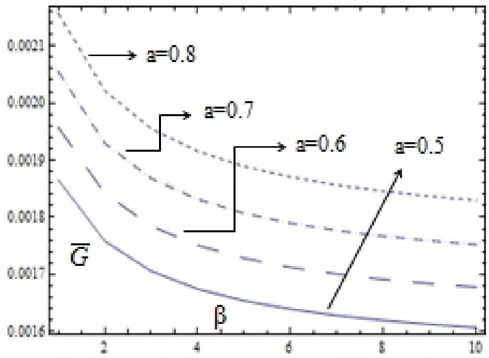

Fig. 10. Variation of $\overline{G}$ with β for γ=1, b=0.7, d=1.5, φ=π/6, k=1, M=1

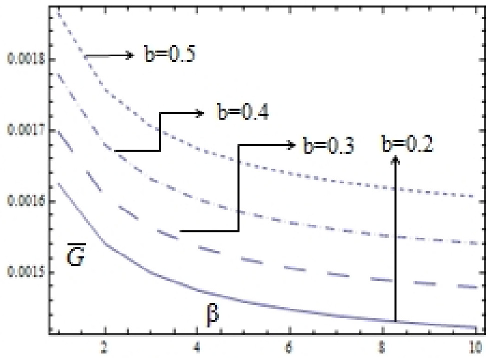

Fig. 11. Variation of $\overline{G}$ with β for γ=1, a=0.7, d=1.5, φ=π/6, k=1, M=1

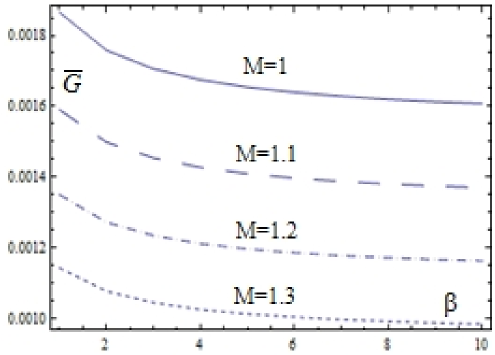

Fig.12. Variation of $\overline{G}$ with β for γ=1, a=0.5, b=0.7, d=1.5, φ=π/6, k=1

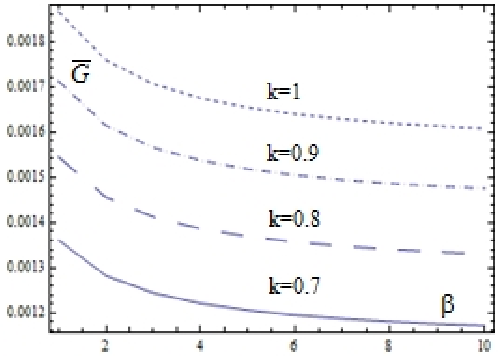

Fig.13. Variation of $\overline{G}$ with β for γ=1, a=0.5, b=0.7, d=1.5, φ=π/6, M=1

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