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Exact Explicit Traveling Solutions for a (2+1)-Dimensional Generalized KP-BBM Equation

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Research Article

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Abstract

By using the sine-cosine method, the extended tanh-method, we study a (2+1)-dimensional generalized Kadomtsov-Petviashvili-Benjamin-Bona-Mahony equation. It is shown that this class gives compactons solutions, solitary patterns solutions and periodic solutions. The change of the physical structure of the solutions is caused by variation of the exponents and the coefficients of the derivatives.

Keywords: Solitary patterns solution, compactons, periodic solution, the (2+1)-dimensional GKP-BBM equation.

1 Introduction

Recently, Tang et al. [1] considered the following (2+1)-dimensional generalized Kadomtsov-Petviashvili-Benjamin-Bona-Mahony (GKP-BBM) equation:

$$[(u^n)_t + (u^n)_x - a(u^{m+1})_x - b(u^n)_{xxt}]_x + k(u^n)_{yy} = 0,\ \mathrm{n} > \mathrm{m} \geq 1, \tag{1.1}$$

where a, b, k are constants, n, m are positive integers. Specially, when n = 2,m = 1,system (1.1) becomes the standard KP-BBM equation [2]. With the development of soliton theory, there exist many different approaches to search for exact solutions of nonlinear partial differential equations, such as mapping method [3], the trigonometric function series method [4], (G'/G)-expansion method [5], improved Fan subequation method [6], the bifurcation method and qualitative theory of dynamical systems method[7,8] and so on. Practically, there is no unified technique that can be employed to handle all types of nonlinear differential equations.

The sine-cosine and the extended tanh algorithms, that provides a systematic framework for many nonlinear dispersive equations, which will be employed to back up our analysis to determine solitary traveling wave solution, compactons , solitary pattern traveling wave solution and

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periodic traveling wave solution.
Let $u(x, y, t) = \phi(x + y - ct) = \phi(\xi)$, where c is the wave speed. Then (1.1) becomes to

$$[(1-c)(\phi^n)' - a(\phi^{m+1})' + bc(\phi^n)^m]' + k(\phi^n)'' = 0, \tag{1.2}$$

Where "'" is the derivative with respect to ξ .

Integrating twice and using the constants of integration to be zero we find

$$(k+1-c)\phi^2 - a\phi^{m-n+3} + bcn[(n-1)\phi'^2 + \phi\phi''] = 0. \tag{1.3}$$

The paper is organized as follows: In section 2, the sine-cosine method and the tanh method are briefly discussed. In Section 3, represents exact analytical solutions of (1.1) by using the tanh method and the extended tanh method. In Section 4, represents exact analytical solutions of (1.1) by using the sine-cosine method. In the last section, we conclude the paper and give some discussions.

2 Analysis of the Two Methods

The sine-cosine method, the tanh method and the extended tanh method have been applied for a wide variety of nonlinear problems. The main features of the two methods will be reviewed briefly.

For both methods, we first use the wave variable $\xi = x + y - ct$ to carry a PDE in two independent variables

$$P(u, u_t, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, ...) = 0, \tag{2.1}$$

into an ODE

$$Q(u, u', u'', u''', ...) = 0, \tag{2.2}$$

Eq. (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

2.1 The Sine-Cosine Method Admits the Use of the Solution in the Form or in the Form

$$u(x, y, t) = \begin{cases} \lambda \cos^{\beta}(\mu\xi), & |\mu\xi| < \dfrac{\pi}{2} \\ 0, & \text{otherwise}, \end{cases}, \tag{2.3}$$

or into the form

$$u(x,y,t)=\begin{cases}\lambda\sin^{\beta}(\mu\xi), & |\mu\xi|<\pi\\ 0, & \text{otherwise,}\end{cases}, \tag{2.4}$$

where λ,μ and β are parameters that will be determined.

We substitute (2.3) or (2.4) into the reduced ordinary differential equation obtained above in (2.2), balancing the terms of the cosin functions when (2.3) is used, or balancing the terms of the sine functions when (2.4) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations to obtain all possible values of the parameters λ,μ and β.

2.2 The Tanh Method and the Extended Tanh Method

The standard tanh method introduced in [9,10] where the tanh is used as a new variable, since all derivatives of a tanh are represented by a tanh itself. We use a new independent variable

$$Y=\tanh(\mu\xi), \tag{2.5}$$

that leads to the change of derivatives:

$$\frac{d}{d\xi}=\mu(1-Y^2)\frac{d}{dY},$$

$$\frac{d^2}{d\xi^2}=\mu^2(1-Y^2)[-2Y\frac{d}{dY}+(1-Y^2)\frac{d^2}{dY^2}]. \tag{2.6}$$

We then apply the following finite expansion:

$$u(\mu\xi)=S(Y)=\sum_{k=0}^{M}a_kY, \tag{2.7}$$

and

$$u(\mu\xi)=S(Y)=\sum_{k=0}^{M}a_kY^k+\sum_{k=1}^{M}b_kY^{-k}, \tag{2.8}$$

where M is a positive integer that will be determined to derive a closed form analytic solution. However, if M is not an integer, a transformation formula is usually used. Substituting (2.5) and (2.6) into the simplified ODE (2.2) results in an equation in powers of Y. To determine the parameter M, we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. With M determined, we collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters a_k, b_k, μ *and* c .Having determined these parameters, knowing that M is a positive integer in most cases, and using (2.7) or (2.8) we obtain an analytic solution $u(x,t)$ in a closed form.

3 Using the Sine-Cosine Method

Substituting (2.3) into (1.3) yields

$$-a\lambda^{m+1}\cos^{(m+1)\beta}(\mu\xi)+(k+1-c-bcn^2\mu^2\beta^2)\lambda^n\cos^{n\beta}(\mu\xi)+bcn\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi)=0. \quad (3.1)$$

Eq. (3.1) is satisfied only if the following system of algebraic equations holds:

$$n\beta \neq 1, n\beta-2=(m+1)\beta, k+1-c=bcn^2\mu^2\beta^2, a\lambda^{m+1}=bcn\mu^2\lambda^n\beta(n\beta-1). \quad (3.2)$$

Solving the system (3.2) gives

$$\beta=\frac{-2}{m-n+1}, \mu^2=\frac{(k+1-c)(m-n+1)^2}{4bcn^2}, \lambda^{m-n-1}=\frac{2an}{(k+1-c)(m+n+1)}. \quad (3.3)$$

The results (3.3) can be easily obtained if we also use the sine method (2.4). Combining (3.3) with (2.3) and (2.4), the following compacton solutions [11,12]:

$$u_1=\begin{cases} \pm\left[\frac{2an}{(k+1-c)(m+n+1)}\right]\cos^2\frac{m-n+1}{2n}\sqrt{\frac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}} \\ |x+y-ct|<\frac{n\pi}{m-n+1}\sqrt{\frac{bc}{k+1-c}}, n-m-1=2l, l\in Z^+, abc>0, \\ 0, \qquad \text{otherwise} \end{cases} \quad (3.4)$$

$$u_2=\begin{cases} \left[\frac{2an}{(k+1-c)(m+n+1)}\right]\cos^2\frac{m-n+1}{2n}\sqrt{\frac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}} \\ |x+y-ct|<\frac{n\pi}{m-n+1}\sqrt{\frac{bc}{k+1-c}}, n-m-1=2l+1, l\in Z^+, ab(k+1-c)>0, \\ 0, \qquad \text{otherwise} \end{cases} \quad (3.5)$$

and

$$u_3=\begin{cases}\pm[\dfrac{2an}{(k+1-c)(m+n+1)}]\sin^2\dfrac{m-n+1}{2n}\sqrt{\dfrac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}} \\ |x+y-ct|<\dfrac{n\pi}{m-n+1}\sqrt{\dfrac{bc}{k+1-c}},n-m-1=2l,l\in Z^+,abc>0, \\ 0, \qquad \text{otherwise} \end{cases} \qquad (3.6)$$

$$u_4=\begin{cases}[\dfrac{2an}{(k+1-c)(m+n+1)}]\sin^2\dfrac{m-n+1}{2n}\sqrt{\dfrac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}} \\ |x+y-ct|<\dfrac{n\pi}{m-n+1}\sqrt{\dfrac{bc}{k+1-c}},n-m-1=2l+1,l\in Z^+,ab(k+1-c)>0, \\ 0, \qquad \text{otherwise} \end{cases} \qquad (3.7)$$

However, for $bc(k+1-c)<0$, we obtain the following solitary patterns solutions:

$$u_5=\pm[\frac{2an}{(k+1-c)(m+n+1)}]\cosh^2\frac{m-n+1}{2n}\sqrt{\frac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}}$$
$$n-m-1=2l,l\in Z^+,a(k+1-c)>0, \qquad (3.8)$$

$$u_6=[\frac{2an}{(k+1-c)(m+n+1)}]\cosh^2\frac{m-n+1}{2n}\sqrt{\frac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}}$$
$$n-m-1=2l+1,l\in Z^+, \qquad (3.9)$$

and

$$u_7=\pm[\frac{-2an}{(k+1-c)(m+n+1)}]\sinh^2\frac{m-n+1}{2n}\sqrt{\frac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}}$$
$$n-m-1=2l,l\in Z^+,a(k+1-c)>0, \qquad (3.10)$$

$$u_8=[\frac{-2an}{(k+1-c)(m+n+1)}]\sinh^2\frac{m-n+1}{2n}\sqrt{\frac{k+1-c}{bc}}(x+y-ct)]^{\frac{1}{n-m-1}}$$
$$n-m-1=2l+1,l\in Z^+, \qquad (3.11)$$

4 Using the Extended Tanh Method

Second In Eq.(1.3), balancing $\phi^{m-n+3} with\ \phi\phi''$ we find

$$M(m-n+3) = M + M + 2, \tag{4.1}$$

so that

$$M = \frac{2}{m-n+1}, m-n+1 \neq 0. \tag{4.2}$$

To get a closed form analytic solution, the parameter M should be an integer. A transformation formula

$$\phi = \psi^{\frac{1}{m-n+1}}, \tag{4.3}$$

should be used to achieve our goal. This in turn transforms Eq.(1.3) to

$$(k+1-c)\psi^2 - a\psi^3 + \frac{bcn(2n-m-1)}{(m-n+1)^2}\psi'^2 + \frac{bcn}{m-n+1}\psi\psi'' = 0 \, . \tag{4.4}$$

Balancing $\phi^3$and $\psi\psi'' gives$ $\mathrm{M} = 2.$ The tanh method allows us to use the substitution

$$\phi(\xi) = a_0 + a_1 Y + a_2 Y^2 + b_1 Y^{-1} + b_2 Y^{-2}. \tag{4.5}$$

Substituting (4.5) into (4.4), collecting the coecients of each power of Y, and solve the resulting system of algebraic equations to find the following three sets of solutions:

$$A_1 = B_1 = B_2 = 0,\ A_0 = -A_2 = \frac{(k+1-c)(m+n+1)}{2an},$$

$$c^2 = c^2, \mu^2 = -\frac{(k+1-c)(m-n+1)^2}{4bcn^2}, \tag{4.6}$$

$$A_1 = B_1 = A_2 = 0,\ A_0 = -B_2 = \frac{(k+1-c)(m+n+1)}{2an},$$

$$c^2 = c^2, \mu^2 = -\frac{(k+1-c)(m-n+1)^2}{4bcn^2}, \tag{4.7}$$

$$A_1 = B_1 = 0, A_2 = B_2 = -\frac{1}{2}A_0 = -\frac{(k+1-c)(m+n+1)}{8an},$$

$$c^2 = c^2, \mu^2 = -\frac{(k+1-c)(m-n+1)^2}{16bcn^2}, \tag{4.8}$$

Noting that $\phi = \psi^{\frac{1}{m-n+1}}$, for $bc(k+1-c) < 0$, we obtain the following solitary patterns solutions

$$u_9 = \{A*[1-\tanh^2(\Omega_1(x+y-ct))]\}^{\frac{1}{m-n+1}}, \tag{4.9}$$

$$u_{10} = \{A*[1-\coth^2(\Omega_1(x+y-ct))]\}^{\frac{1}{m-n+1}}, \tag{4.10}$$

and

$$u_{11} = \{\frac{A*}{2}[1-\frac{1}{2}(\tanh^2\Omega_1(x+y-ct)+\coth^2\Omega_1(x+y-ct))]\}^{\frac{1}{m-n+1}}, \tag{4.11}$$

where $A* = \frac{(k+1-c)(m+n+1)}{2an}, \Omega_1 = \frac{m-n+1}{2n}\sqrt{\frac{-(k+1-c)}{bc}}$.

However, for $bc(k+1-c) > 0$, we obtain the following periodic solutions

$$u_{12} = \{A*[1-\tan^2(\Omega_2(x+y-ct))]\}^{\frac{1}{m-n+1}}, \tag{4.12}$$

$$u_{13} = \{A*[1-\cot^2(\Omega_2(x+y-ct))]\}^{\frac{1}{m-n+1}}, \tag{4.13}$$

And

$$u_{14} = \{\frac{A*}{2}[1+\frac{1}{2}(\tan^2\Omega_1(x+y-ct)+\cot^2\Omega_1(x+y-ct))]\}^{\frac{1}{m-n+1}}, \tag{4.14}$$

where $\Omega_2 = \frac{m-n+1}{4n}\sqrt{\frac{k+1-c}{bc}}$.

5 Conclusions

The sine-cosine method and the extended tanh method were used to investigate the (2+1)-dimensional GKP-BBM equation. The study revealed compactons, solitary patterns solutions and periodic traveling wave solutions for some examined variants. The study emphasized the fact that the two methods are reliable in handling nonlinear problems. The obtained results clearly demonstrate the efficiency of the two methods used in this work. Moreover, the methods are capable of greatly minimizing the size of computational work compared to other existing techniques. This emphasizes the fact that the two methods are applicable to a wide variety of nonlinear problems.

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Competing Interests

Authors have declared that no competing interests exist.

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