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# A New Decision Making Method on Interval Valued Fuzzy Soft Matrix (IVFSM)

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Author MZ designed the study, wrote the first draft of the manuscript and managed literature searches. Author MS managed the analysis of the study. Both authors read and approved the final manuscript.*

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## Abstract

Interval valued fuzzy soft set and IVFSM are those mathematical tools which deal with problems involving uncertainties and imprecise or incomplete data. IVFSM may be useful for functions whose membership values vary. In this paper, we study basic definitions of IVFSM with some properties and prove commutative laws, associative laws and De-Morgan laws by using And-Operation and Or-Operation on IVFSM. We propose a new decision making method on IVFSM named as "interval valued fuzzy soft max-min decision making method" (IVFSMmDM) with the help of interval valued fuzzy soft max-min decision making function. Finally, we apply IVFSMmDM method for decision making to solve those problems which involving uncertainties by using data from [19].

*Keywords: Interval valued fuzzy set (IVFS); interval valued soft set (IVSS); interval valued fuzzy soft set (IVFSS); IVFSM; IVFSMmDM.*

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# 1 Introduction

In recent time, classical methods are used to solve different problems which are faced in medical sciences, engineering, social sciences etc. Many theories like fuzzy set theory, probability theory, rough set theory etc. are used to solve these uncertainties, but these theories have their own problems which were attensioned by Zadeh. First of all (Zadeh) in [1] initiated the concept of fuzzy sets by the extension of classical notion on sets. Nowadays fuzzy set theory is rapidly progressing, but in particular cases we face some limitations such as how to adjust the membership functions in this theory. A new concept of SS introduced for the solution of these limitations (Molodtsov) [2].

In these days, mathematician plays a vital role in the soft set (SS) fuzzification. After fuzzification of SS a new theory introduced which is known as fuzzy soft set (FSS) with different types and properties (Maji et al.) [3]. The SS theory was reviewed [4] and used this theory for decision making, and also introduced different types of SS with examples and defined some operations such as And-operation, Or-operation, union, intersection, complement etc. on SS [5]. After some time the work on SS was extended by using the definition of parameterization reduction [6] and used it in decision making problem. Some new notions on SS [7] (Ali et al.) are introduced, such as restricted union, restricted intersection, restricted difference and extended intersection with examples and properties.

Generalized FSS [8] was proposed with some properties and used (Majumdar et al.) generalized FSS for decision making problem and for the diagnosis of Pneumonia. (Neog and Sut) Some propositions were proposed on fuzzy soft union and intersection with proof and examples on FSS and verify the De Morgan Laws for a family of fuzzy soft sets with examples [9].

Matrix representation play a vital role in engineering and science, but sometimes classical matrices fails to solve the problems. (Yang and Chenli ji) Fuzzy soft matrix (FSM) [10] was introduced for the solution of those problems which can not be handled by classical matrices by the product of FSM and used it for decision making. The product of soft matrices and fuzzy soft matrices are introduced with operations and some properties [11,12] (Cagman, N. and Enginoglu, S) and also proposed new decision making methods which are known as soft max–min decision making method and fuzzy soft max–min decision making method. (Bohra et al.) Further advanced the FSM theory and its applicability in different fields of life in 2012 and also used it for decision making by product of fuzzy soft matrices [13].

The idea of IVFSS is introduced [14] by combining the IVFS and SS, (Yang et al.) IVFSS is used for decision making and define And-operation; Or-operation also proved De Morgan laws, associative laws and distributive laws on IVFSS. (Chetia et al.) IVFSS is used for medical diagnosis in 2010 [15]. (Rajarajeshwari and Dhanalakshmi) A new concept is proposed by the combination of IVFSS and soft matrices with examples and different properties which are called IVFSM [16], they also proposed new definitions on IVFSM with examples. They introduced some new operations on IVFSM such as arithmetic mean, weighted arithmetic mean, geometric mean, weighted geometric mean, harmonic mean and weighted harmonic mean with some properties of IVFS-matrices in decision making.

By the extension of IVFSM (Sarala and Prabhavathi) [17] initiated the Sanchez’s approach for diagnosis of Dengue and Chikangunya also introduced union and intersection of IVFSM. IVFSM is used for checking the performance of different cities of a country (Zulqarnain and Saeed) [18]. They also used FSM and IVFSM for decision making and redefined the product of IVFSM [19] and compare the result of FSM and IVFSM for decision making and observed that the FSM is more appropriate for decision making.

In this paper, we proved commutative laws, associative laws and De-Morgan laws by using And-Operation and Or-Operation of IVFSM also defined IVFSMmDM method with the help of interval valued fuzzy soft max-min decision function. We construct an algorithm for IVFSMmDM in this paper and use this method for decision making.

## 2 Preliminaries

In this section, we review some concepts and definitions of IVFSM with examples, which will be needed in the sequel.

M represents the universal set and E represents the set of parameters in this whole paper.

**Definition 2.1 [2]:** A pair (F, A) is called a SS over M if A is any subset of E, and there exist a mapping from A to P (M) is F, P (M) is the parameterized family of subsets of the M but not a set.

**Definition 2.2 [1]:** A fuzzy set A in M is characterized by a membership function $f_A(y_i)$ which associates with each object of M in the interval [0, 1], with the value of $f_A(y_i)$ where $y_i$ representing the grade of membership of y in A.

**Definition 2.3 [3]:** A pair (F, A) is called FSS over M, and there exist a mapping from A to P (M) is F, P (M) is the collection of fuzzy subsets of M.

**Definition 2.4 [10]:** A pair ($f_A$, E) is called a SS over M if A is any subset of E. Then a subset $R_A$ of M×E is defined as $R_A = \{(m, y): y \in A, m \in f_A(y)\}$, is the relation form of $(f_A, E)$ the characteristic function of $R_A$ is written by

$$\chi_{RA} : U \times E \rightarrow \{0, 1\}, \quad \chi_{RA}(m, y) = \begin{cases} 1, & (m, y) \in R_A \\ 0, & (m, y) \notin R_A \end{cases} \tag{1}$$

It can be written in matrix form such as.

$$[a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

This is called an SM of the SS ($f_A$, E) over M of order m×n.

**Definition 2.5 [10]:** A pair (F, A) is called FSS in the fuzzy soft class (M, E). Then (F, A) is represented in a matrix form such as

$$A_{mxn} = [a_{ij}]_{mxn} \text{ or } A = [a_{ij}] \quad (i = 1 \rightarrow m), (j = 1 \rightarrow n)$$

Where

$$a_{ij} = \begin{cases} \mu_j(b_j) & if\ y_j \in A \\ 0 & \text{if } y_j \notin A \end{cases} \tag{2}$$

**Example 2.1:** Let M = {$Z_1$, $Z_2$, $Z_3$, $Z_4$} be a universal set and E = {$y_1$, $y_2$, $y_3$, $y_4$} be the set of parameters, A= {$y_3$, $y_4$} ⊆ E then FSS can be written as

(F, A) = {F ($y_3$) = {($Z_1$, 0.51), ($Z_2$, 0.26), ($Z_3$, 0.56), ($Z_4$, 0.28)},

F ($y_4$) = {($Z_1$, 0.25), ($Z_2$, 0.51), ($Z_3$, 0.49), ($Z_4$, 0.23)}}

This FSS can be represented in FSM by using equation 2 such as

$$(F, A) = \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{matrix} \begin{bmatrix} 0.0 & 0.0 & 0.51 & 0.25 \\ 0.0 & 0.0 & 0.26 & 0.51 \\ 0.0 & 0.0 & 0.56 & 0.49 \\ 0.0 & 0.0 & 0.28 & 0.23 \end{bmatrix}$$

**Definition 2.6 [14]:** A pair (F, A) is called IVFSS over M where F is a mapping such that

$$F: A \to I^M$$

Where $I^M$ represent the all interval valued fuzzy subsets (IVFSbS) of M.

**Definition 2.7 [16]:** A pair (F, A) is called IVFSS over M, where F is a mapping such that

$$F: A \to I^M$$

Where $I^M$ represent all IVFSbS of M. Then the IVFSS can be expressed in matrix form as

$$A_{mxn} = [a_{ij}]_{mxn}$$

Or

$$A = [a_{ij}] \qquad (i = 1 \to m), (j = 1 \to n)$$

Where

$$a_{ij} = \begin{cases} [\mu_{jL}(b_i), \mu_{jU}(b_i)] & \text{if } y_j \in A \\ [0, 0] & \text{if } y_j \notin A \end{cases} \tag{3}$$

Where $[\mu_{jL}(b_i), \mu_{jU}(b_i)]$ represent the membership of $b_i$ in the IVFS $F(y_j)$.

**Example 2.2:** Let $M = \{V_1, V_2, V_3, V_4\}$ be a universal set and $E = \{y_1, y_2, y_3, y_4\}$ be the set of parameters. Consider

$$A = \{y_3, y_4\} \subseteq E \qquad F: A \to P(M)$$

Then IVFSS (F, A) is

$$(F, A) = \{F(y_3) = \{(V_1, [0.5, 0.6]), (V_2, [0.2, 0.3]), (V_3, [0.5, 0.6]), (V_4, [0.2, 0.3])\},$$
$$F(y_4) = \{(V_1, [0.2, 0.3]), (V_2, [0.5, 0.6]), (V_3, [0.4, 0.5]), (V_4, [0.2, 0.3])\}\}$$

This IVFSS can be represented in IVFSM by using equation 3.

$$(F, A) = \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} \begin{bmatrix} [0.0, 0.0] & [0.0, 0.0] & [0.5, 0.6] & [0.2, 0.3] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.5, 0.6] \\ [0.0, 0.0] & [0.0, 0.0] & [0.5, 0.6] & [0.4, 0.5] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

**Definition 2.8 [16]:** A and B are two IVFS-matrices then A is said to be IVFS sub matrix of B if $\mu_{AL} \leq \mu_{BL}$ and $\mu_{AU} \leq \mu_{BU}$ for all i and j, it is denoted by $A \subseteq B$

**Example 2.3:** Let A, B ⊆ E such that A ⊆ B
A= $\{y_3, y_4\}$ and B = $\{y_2, y_3, y_4\}$

Then

(F, A) = {F ($y_3$) = {($V_1$, [0.3, 0.4]), ($V_2$, [0.2, 0.3]), ($V_3$, [0.5, 0.6]), ($V_4$, [0.2, 0.3])},
F ($y_4$) = {($V_1$, [0.2, 0.3]), ($V_2$, [0.3, 0.4]), ($V_3$, [0.4, 0.5]), ($V_4$, [0.2, 0.3])}}

(F, B) = {F ($y_2$) = {($V_1$, [0.4, 0.5]), ($V_2$, [0.6, 0.7]), ($V_3$, [0.8, 0.9]), ($V_4$, [0.3, 0.4])},
F ($y_3$) = {($V_1$, [0.3, 0.4]), ($V_2$, [0.2, 0.3]), ($V_3$, [0.5, 0.6]), ($V_4$, [0.2, 0.3])},
F ($y_4$) = {($V_1$, [0.2, 0.3]), ($V_2$, [0.3, 0.4]), ($V_3$, [0.4, 0.5]), ($V_4$, [0.2, 0.3])}}

These IVFSS can be represented in IVFSM such as by using equation 3.

$$(F, A) = \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} \begin{bmatrix} [0.0, 0.0] & [0.0, 0.0] & [0.3, 0.4] & [0.2, 0.3] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.3, 0.4] \\ [0.0, 0.0] & [0.0, 0.0] & [0.5, 0.6] & [0.4, 0.5] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

and

$$(F, B) = \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} \begin{bmatrix} [0.0, 0.0] & [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] \\ [0.0, 0.0] & [0.6, 0.7] & [0.2, 0.3] & [0.3, 0.4] \\ [0.0, 0.0] & [0.8, 0.9] & [0.5, 0.6] & [0.4, 0.5] \\ [0.0, 0.0] & [0.3, 0.4] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

Where (F, A) ⊆ (F, B).

**Definition 2.9 [16]:** A = $[a_{ij}]$ be an IVFSM of order mxn, then transpose of IVFSM can be defined as $A^T$ = $[a_{ji}]$ of order n×m,

Where

$[a_{ij}] = [\mu_{jL}(b_i), \mu_{jU}(b_i)]$, ($i = 1 \rightarrow m$) and ($j = 1 \rightarrow n$).

**Example 2.4:** Consider A be IVFSM of order 2×2

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ then}$$

$$A^T = \begin{bmatrix} [0.2, 0.5] & [0.2, 0.4] \\ [0.4, 0.6] & [0.3, 0.8] \end{bmatrix}$$

**Definition 2.10 [16]:** The addition of two IVFS-matrices A and B is conformable if their order is same, it is defined as

$A+B = [\max(\mu_{AL}, \mu_{BL}), \max(\mu_{AU}, \mu_{BU})]$ for all i and j.

**Example 2.5:** A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A+B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

**Definition 2.11 [17]:** The subtraction of two IVFS-matrices A and B is conformable if their order is same, it is defined as

$$A-B = [\min(\mu_{AL}, \mu_{BL}), \min(\mu_{AU}, \mu_{BU})] \text{ for all i and j.}$$

**Example 2.6:** A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A-B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix}$$

**Definition 2.12 [19]:** Let A = $[a_{ij}]$ and B = $[b_{ij}]$ are two IVFS-matrices of order m×n and n×p respectively then their product defined as

$$A*B = [c_{ik}]_{m\times p} = [\max(\mu_{ALj} * \mu_{BLj}), \max(\mu_{AUj} * \mu_{BUj})] \text{ for all i, j, k.}$$

Or

$$A*B = c_{ij} = \sum_{k=1}^{n} (a_{ik} * b_{ik}),\ i = 1, 2\dots m \text{ and } j = 1, 2\dots p$$

$$A*B = [c_{ik}]_{m\times p} = [\sum_{k=1}^{n} (a_{ikL} * b_{ikL}), \sum_{k=1}^{n} (a_{ikU} * b_{ikU})],\ i = 1, 2\dots m \text{ and } k = 1, 2\dots p \text{ for all i, j, k.}$$

**Example 2.7:** Consider A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A*B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} * \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A*B = \begin{bmatrix} [0.08, 0.35] + [0.12, 0.36] & [0.14, 0.40] + [0.28, 0.54] \\ [0.08, 0.28] + [0.09, 0.48] & [0.14, 0.32] + [0.21, 0.72] \end{bmatrix}$$

$$A*B = \begin{bmatrix} [0.12, 0.36] & [0.28, 0.54] \\ [0.09, 0.48] & [0.21, 0.72] \end{bmatrix}$$

**Definition 2.13 [17]:** A= $[a_{ij}]$ be an IVFSM of order m×n, where $a_{ij} = [\mu_{jL}(b_i), \mu_{jU}(b_i)]$

Then its complement is defined as $A^C = [b_{ij}]$ where $b_{ij} = [1- \mu_{jU}(b_i), 1-\mu_{jL}(b_i)]$ for all i, j.

**Example 2.8:** Let A be an IVFSM.

$$A = \begin{bmatrix} [0.4, 0.6] & [0.3, 0.7] \\ [0.3, 0.5] & [0.5, 0.7] \end{bmatrix}$$

Then its complement is defined as

$$A^C = \begin{bmatrix} [0.4, 0.6] & [0.3, 0.7] \\ [0.5, 0.7] & [0.3, 0.5] \end{bmatrix}$$

**Proposition: 2.1:** $(A^C)^C = A$ for all A where A is any IVFSM.

**Proof (1)**

Let $A = [\mu_{AL}, \mu_{AU}] \in$ IVFSM, then

**L.H.S =** $(A^C)^C$
$= ([\mu_{AL}, \mu_{AU}]^C)^C$
$= ([1 - \mu_{AU}, 1 - \mu_{AL}])^C$
$= [1 - (1 - \mu_{AL}), 1 - (1 - \mu_{AU})]$
$= [1 - 1 + \mu_{AL}, 1 - 1 + \mu_{AU}]$
$= [\mu_{AL}, \mu_{AU}]$
= **R.H.S.**

**Definition 2.14 [16]:** An IVFSM of any order is called null-IVFSM in which $[a_{ij}] = [0, 0]$ for all i, j. it can be represented as

$$O = \begin{bmatrix} [0, 0] & [0, 0] & [0, 0] & [0, 0] \\ [0, 0] & [0, 0] & [0, 0] & [0, 0] \\ [0, 0] & [0, 0] & [0, 0] & [0, 0] \end{bmatrix}$$

**Definition 2.15 [16]:** An IVFSM of any order is called universal-IVFSM in which $[a_{ij}] = [1, 1]$ for all i, j. it can be represented as

$$U = \begin{bmatrix} [1, 1] & [1, 1] & [1, 1] & [1, 1] \\ [1, 1] & [1, 1] & [1, 1] & [1, 1] \\ [1, 1] & [1, 1] & [1, 1] & [1, 1] \end{bmatrix}$$

**Definition 2.16 [19]:** $A = [a_{ij}]$ and $B = [b_{ik}]$ are two IVFSM of same order m×n then And-product is defined as

$\wedge: A\times B \rightarrow C_{m\times n^2}$, $[a_{ij}]_{m\times n} \wedge [b_{ik}]_{m\times n} = [c_{ip}]_{m\times n^2}$ where

$c_{ip} = [\min(\mu_{ALj}, \mu_{BLj}), \min(\mu_{AUj}, \mu_{BUj})]$ for all i, j, k. such that $p = n(j - 1) + k$.

**Example 2.9:** Consider A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix}_{2\times 2} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}_{2\times 2}$$

Then

$$A \wedge B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \wedge \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A \wedge B = [c_{ip}]_{2\times 4} = \begin{bmatrix} [0.2, 0.5] & [0.2, 0.5] & [0.4, 0.6] & [0.4, 0.6] \\ [0.2, 0.4] & [0.2, 0.4] & [0.3, 0.6] & [0.3, 0.8] \end{bmatrix}$$

**Definition 2.17 [19]:** $A = [a_{ij}]$ and $B = [b_{ik}]$ are two IVFS-matrices of same order m×n then Or-product is defined as

$\vee$: $A\times B \rightarrow C_{m\times n^2}$, $[a_{ij}]_{m\times n} \vee [b_{ik}]_{m\times n} = [c_{ip}]_{m\times n^2}$ where

$c_{ip} = [\max(\mu_{ALj}, \mu_{BLj}), \max(\mu_{AUj}, \mu_{BUj})]$ for all i, j, k. such that $p = n(j-1) + k$.

**Example 2.10:** Consider A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A \vee B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \vee \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A \vee B = [c_{ip}]_{2\times 4} = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] & [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] & [0.3, 0.8] & [0.7, 0.9] \end{bmatrix}$$

De Morgan laws, commutative laws, domination laws and involution laws are hold in And-product and Or-product of IVFSM, the proof of these laws can be seen in [19].

**Definition 2.18 [17]:** Let $A = [a_{ij}]$ and $B = [b_{ij}]$ are two IVFS-matrices of same order m×n then their union defined as

$A_{m\times n} \tilde{\cup} B_{m\times n} = C_{m\times n} = [c_{ij}]_{m\times n}$, where

$c_{ij} = [a_{ij}] \tilde{\cup} [b_{ij}] = [\mu_{AL}, \mu_{BL}] \tilde{\cup} [\mu_{AU}, \mu_{BU}] = [\mu_{AL} + \mu_{BL} - \mu_{AL}*\mu_{BL}, \mu_{AU} + \mu_{BU} - \mu_{AU}*\mu_{BU}]$ for all i, j.

**Example 2.11:** Consider A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A \tilde{\cup} B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \tilde{\cup} \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A\ \tilde{U}\ B = \begin{bmatrix} [0.2+0.4-0.2*0.4, 0.5+0.7-0.5*0.7] & [0.4+0.7-0.4*0.7, 0.6+0.8-0.6*0.8] \\ [0.2+0.3-0.2*0.3, 0.4+0.6-0.4*0.6] & [0.3+0.7-0.3*0.7, 0.8+0.9-0.8*0.9] \end{bmatrix}$$

$$A\ \tilde{U}\ B = \begin{bmatrix} [0.6-0.08, 1.2-0.35] & [1.1-0.28, 1.4-0.48] \\ [0.5-0.06, 1.0-0.24] & [1.0-0.21, 1.7-0.72] \end{bmatrix}$$

$$A\ \tilde{U}\ B = \begin{bmatrix} [0.52, 0.85] & [0.82, 0.92] \\ [0.44, 0.76] & [0.79, 0.98] \end{bmatrix}$$

**Definition 2.19 [17]:** Let A= $[a_{ij}]$ and B= $[b_{ij}]$ are two IVFS-matrices of same order m×n then their intersection defined as

$A_{m\times n} \cap B_{m\times n} = C_{m\times n} = [c_{ij}]_{m\times n}$, where

$c_{ij} = [a_{ij}] \cap [b_{ij}] = [\mu_{AL}, \mu_{BL}] \cap [\mu_{AU}, \mu_{BU}] = [\mu_{AL}*\mu_{BL}, \mu_{AU}*\mu_{BU}]$ for all i, j.

**Example 2.12:** Consider A and B are two IVFS-matrices.

$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix}$ and

$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$

Then

$$A \cap B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \cap \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A \cap B = \begin{bmatrix} [0.2*0.4, 0.5*0.7] & [0.4*0.7, 0.6*0.8] \\ [0.2*0.3, 0.4*0.6] & [0.3*0.7, 0.8*0.9] \end{bmatrix}$$

$$A \cap B = \begin{bmatrix} [0.08, 0.35] & [0.28, 0.48] \\ [0.06, 0.24] & [0.21, 0.72] \end{bmatrix}$$

**Proposition: 2.2**

**(1)** A U [I] = [I]
**(2)** A ∩ [O] = [O]

**Proof (1)**

Let $A = [\mu_{AL}, \mu_{AU}] \in$ IVFSM, and $I = [\mu_{IL}, \mu_{IU}] = [1, 1]$ be an IVFSM, then

**L.H.S = A** U I

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ be an IVFSM, and $I = [\mu_{IL}, \mu_{IU}] = [1, 1]$ be an Universal IVFSM, then
$A \cup I = [\mu_{AL}, \mu_{AU}] \cup [\mu_{IL}, \mu_{IU}]$
$= [\mu_{AL} + \mu_{IL} - \mu_{AL} * \mu_{IL}, \mu_{AU} + \mu_{IU} - \mu_{AU} * \mu_{IU}]$
$= [\mu_{AL} + \mu_{IL} - \mu_{AL}, \mu_{AU} + \mu_{IU} - \mu_{AU}]$
$= [\mu_{IL}, \mu_{IU}]$
= **R.H.S.**

**Proof (2)**

Let $A = [\mu_{AL}, \mu_{AU}] \in$ IVFSM, and $O = [\mu_{OL}, \mu_{OU}] = [0, 0]$ be an Null IVFSM, then

**L.H.S = A** ∩ O

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ be an IVFSM, and $O = [\mu_{OL}, \mu_{OU}] = [0, 0]$ be an Null IVFSM, then

$A \cap O = [\mu_{AL}, \mu_{AU}] \cap [\mu_{OL}, \mu_{OU}]$
$= [\mu_{AL} * \mu_{OL}, \mu_{AU} * \mu_{OU}]$
$= [\mu_{OL}, \mu_{OU}]$
= **R.H.S.**

**Proposition: 2.3**

**(1)** A U [O] = A
**(2)** A ∩ [I] = A

**Proof (1)**

Let $A = [\mu_{AL}, \mu_{AU}] \in$ IVFSM, and $O = [\mu_{OL}, \mu_{OU}] = [0, 0]$ be an Null IVFSM, then

**L.H.S = A** U O
$= [\mu_{AL}, \mu_{AU}] \text{ U } [\mu_{OL}, \mu_{OU}]$
$= [\mu_{AL} + \mu_{OL} - \mu_{AL} * \mu_{OL}, \mu_{AU} + \mu_{OU} - \mu_{AU} * \mu_{OU}]$
$= [\mu_{AL} - \mu_{OL}, \mu_{AU} - \mu_{OU}]$
$= [\mu_{AL}, \mu_{AU}]$
= A
= **R.H.S.**

**Proof (2)**

Let $A = [\mu_{AL}, \mu_{AU}] \in$ IVFSM, and $I = [\mu_{IL}, \mu_{IU}] = [1, 1]$ be an IVFSM, then

**L.H.S = A** ∩ I

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ be an IVFSM, and $I = [\mu_{IL}, \mu_{IU}] = [1, 1]$ be an Universal IVFSM, then

$A \cap I = [\mu_{AL}, \mu_{AU}] \cap [\mu_{IL}, \mu_{IU}]$
$= [\mu_{AL} * \mu_{IL}, \mu_{AU} * \mu_{IU}]$
$= [\mu_{AL}, \mu_{AU}]$
= A
= **R.H.S.**

**Definition 2.20 [16]:** Let $A = [a_{ij}]_{m\times n}$ be an IVFSM and K be any scalar, where $a_{ij} = [\mu_{jL}(b_i), \mu_{jU}(b_i)]$. Than scalar multiplication of IVFSM is defined as

$KA = K * [\mu_{jL}(b_i), \mu_{jU}(b_i)]$

$= [K*\mu_{jL}(b_i), K*\mu_{jU}(b_i)]$, where K varies from 0 to 1.

**Example 2.13:** Consider A be IVFSM of order 2×2

$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix}$ and K = 0.3 be any scalar.

Than

$$KA = 0.3*\begin{bmatrix}[0.2, 0.5] & [0.4, 0.6]\\ [0.2, 0.4] & [0.3, 0.8]\end{bmatrix}$$

$$=\begin{bmatrix}[0.06, 0.15] & [0,12, 0.18]\\ [0.06, 0.12] & [0.09, 0.24]\end{bmatrix}.$$

**Definition 2.21 [12]:** $C_{ip}$ be an FSM of order $m\times n^2$, where $I_k = \{p: \exists\ i, c_{ip} \neq 0, (k-1)\, n < p \leq kn\}$ for all $k \in I = \{1, 2,..., n\}$. Then fs-max-min decision function, denoted Mm, is defined as follows

$$Mm: FSM_{m\times n^2} \rightarrow FSM_{m\times 1},$$

$$Mm\ [c_{ip}] = [d_{i1}] = [\max \{t_{ik}\}]$$

Where

$$t_{ik} = \begin{cases} \min_{p\in I_k}\{c_{ip}\} & \text{if } I_k \neq 0 \\ 0.0 & \text{if } I_k = 0 \end{cases} \tag{4}$$

The one column fs-matrix Mm $[c_{ip}]$ is called max-min decision fs-matrix.

**Definition 2.22 [12]:** M is a universe and $[d_{i1}]$ is a max-min decision fs-matrix. Then optimum fuzzy set on M is defined as

$$Opt\ [d_{i1}]\ (M) = \{d_{i1}/ m_i: m_i \in M,\ d_{i1} \neq 0\}$$

Now, using above definitions we can construct an IVFSMmDM method by the following.

# 3 Operations on IVFSM

In this section, we prove the De-Morgan laws, Associative laws and commutative laws by using the definitions of And-Operation and Or-Operation of IVFSM.

**Definition 3.1 [20]:** Let $A= [a_{ij}]_{m\times n}$ and $B= [b_{ij}]_{m\times n}$ are two IVFS-matrices then their OR-operation is defined as

$$A \vee B = [c_{ij}]_{m\times n}, \text{ where } c_{ij} = [\max(\mu_{AL}, \mu_{BL}), \max(\mu_{AU}, \mu_{BU})] \text{ for all } i, j.$$

**Proposition: 3.1 (Commutative law)**

$$A \vee B = B \vee A$$

**Proof:**

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ and $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ are two IVFS-matrices of order m×n, than

**L.H.S =** $A \vee B = [\mu_{AL}, \mu_{AU}] \vee [\mu_{BL}, \mu_{BU}]$
$= [\max (\mu_{AL}, \mu_{BL}), \max (\mu_{AU}, \mu_{BU})]$
$= [\max (\mu_{BL}, \mu_{AL}), \max (\mu_{BU}, \mu_{AU})]$
$= B \vee A$
= **R.H.S**

**Proposition: 3.2 (Associative law)**

$(A \vee B) \vee C = A \vee (B \vee C)$

**Proof:**

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$, $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ and $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ are IVFS-matrices of order m×n, than

**L.H.S =** $(A \vee B) \vee C = ([\mu_{AL}, \mu_{AU}] \vee [\mu_{BL}, \mu_{BU}]) \vee [\mu_{CL}, \mu_{CU}]$
$= ([\max(\mu_{AL}, \mu_{BL}), \max(\mu_{AU}, \mu_{BU})]) \vee [\mu_{CL}, \mu_{CU}]$
$= [\max(\mu_{AL}, \mu_{BL}, \mu_{CL}), \max(\mu_{AU}, \mu_{BU}, \mu_{CU})]$
$= [\mu_{AL}, \mu_{AU}] \vee ([\max(\mu_{BL}, \mu_{CL}), \max(\mu_{BU}, \mu_{CU})]$
$= A \vee (B \vee C)$
= **R.H.S**

**Definition 3.2 [20]:** Let $A= [a_{ij}]_{m\times n}$ and $B= [b_{ij}]_{m\times n}$ are two IVFS-matrices then AND-operation is defined as

$A \wedge B = [c_{ij}]_{m\times n}$, where $c_{ij} = [\min(\mu_{AL}, \mu_{BL}), \min(\mu_{AU}, \mu_{BU})]$ for all i, j.

**Proposition: 3.3 (Commutative law)**

$A \wedge B = B \wedge A$

**Proof:**

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ and $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ are two IVFS-matrices of order m×n, than

**L.H.S =** $A \wedge B = [\mu_{AL}, \mu_{AU}] \wedge [\mu_{BL}, \mu_{BU}]$
$= [\min(\mu_{AL}, \mu_{BL}), \min(\mu_{AU}, \mu_{BU})]$
$= [\min(\mu_{BL}, \mu_{AL}), \min(\mu_{BU}, \mu_{AU})]$
$= B \wedge A$
= **R.H.S**

**Proposition: 3.4 (Associative law)**

$(A \wedge B) \wedge C = A \wedge (B \wedge C)$

**Proof:**

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$, $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ and $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ are IVFS-matrices of order m×n, than

**L.H.S =** $(A \wedge B) \wedge C = ([\mu_{AL}, \mu_{AU}] \wedge [\mu_{BL}, \mu_{BU}]) \wedge [\mu_{CL}, \mu_{CU}]$
$= ([\min(\mu_{AL}, \mu_{BL}), \min(\mu_{AU}, \mu_{BU})]) \wedge [\mu_{CL}, \mu_{CU}]$
$= [\min(\mu_{AL}, \mu_{BL}, \mu_{CL}), \min(\mu_{AU}, \mu_{BU}, \mu_{CU})]$
$= [\mu_{AL}, \mu_{AU}] \wedge ([\min(\mu_{BL}, \mu_{CL}), \min(\mu_{BU}, \mu_{CU})]$
$= A \wedge (B \wedge C)$
= **R.H.S**

**Theorem: 3.1 (De Morgan laws)**

Let $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ and $D = [d_{ij}] = [\mu_{DL}, \mu_{DU}]$ are two IVFS-matrices of order m×n, than

**(1)** $(C \vee D)^C = C^C \wedge D^C$
**(2)** $(C \wedge D)^C = C^C \vee D^C$

**Proof: 1**

**L.H.S** $= (C \vee D)^C$
$= ([\mu_{CL}, \mu_{CU}] \vee [\mu_{DL}, \mu_{DU}])^C$
$= [\max(\mu_{CL}, \mu_{DL}), \max(\mu_{CU}, \mu_{DU})]^C$
$= [1- \max(\mu_{CU}, \mu_{DU}), 1- \max(\mu_{CL}, \mu_{DL})]$
$= [\min(1- \mu_{CU}, 1- \mu_{DU}), \min(1- \mu_{CL}, 1- \mu_{DU})]$
$= C^C \wedge D^C$
= **R.H.S**

**Proof: 2**

**L.H.S** $= (C \wedge D)^C$
$= ([\mu_{CL}, \mu_{CU}] \wedge [\mu_{DL}, \mu_{DU}])^C$
$= [\min(\mu_{CL}, \mu_{DL}), \min(\mu_{CU}, \mu_{DU})]^C$
$= [1- \min(\mu_{CU}, \mu_{DU}), 1- \min(\mu_{CL}, \mu_{DL})]$
$= [\max(1- \mu_{CU}, 1- \mu_{DU}), \max(1- \mu_{CL}, 1- \mu_{DU})]$
$= C^C \vee D^C$
= **R.H.S**

Similarly we can prove other related laws by using the above definitions.

# 4 IVFS-Max-Min Decision Making

In this section, we construct an IVFS-max-min decision making (IVFSMmDM) method by using IVFS-max-min decision function which is defined here. Before this method FSMmDM method was introduced by (Cagman and Enginoglu) in [12]. By the help of FSMmDM method we introduced a new decision making method which is known by IVFSMmDM method. The method selects optimum alternatives from the set of the alternatives.

**Definition 4.1:** Let $[c_{ip}] \in IVFSM_{m\times n^2}$, $I_k = \{P: \exists\ i, ci_P \neq 0, (k-1)\ n < P \leq kn\}$ for all $k\in I = \{1, 2, 3, \ldots, n\}$. Then IVFS-max-min decision function denoted by Mm and defined as

$$Mm: IVFSM_{m\times n^2} \rightarrow IVFSM_{m\times 1}$$

$$Mm\,[c_{ip}] = [d_{i1}] = [\max\{t_{ik}\}]$$

Where

$$t_{ik} = \begin{cases} \min_{p\in I_k}\{c_{ip}\} & \text{if } I_k \neq 0 \\ [0.0, 0.0] & \text{if } I_k = 0 \end{cases} \quad (5)$$

Where $c_{iP} = [\min(\mu_{ALj}, \mu_{BLj}), \min(\mu_{AUj}, \mu_{BUj})]$ for all i, j, k. such that $p = n\,(j - 1) + k$

Which is one column IVFSM Mm $[c_{ip}]$ is called max-min decision IVFSM.

**Definition 4.2:** $M = \{m_1, m_2, m_3, \ldots, m_n\}$ be a universal set and max-min $[c_{ip}] = [d_{i1}]$. Then optimum fuzzy set on U is defined as

$$Opt\,[d_{i1}]\,(M) = \{d_{i1}/m_i: m_i\in M, d_{i1}\neq 0\}$$ which is called optimum IVFS on M.

## 4.1 Generalized IVFSMs in decision making problem

Two friends A and B want to take a decision that which city is more appropriate for living from any country of the world according to these parameters such as education, health, local bodies and devolvement programs i.e., parameters (E). Both friends have freedom to take decision and evaluation of parameters related to the chosen object or may option the same set of parameters. Here it is assumed that the evaluation of parameters by the decision makers must be generalized fuzzy and may be presented in linguistic form or generalized FSS format, alternatively, in the form of generalized fuzzy soft matrices.

Now, by using the definition of IVFSMmDM we construct an algorithm for decision making.

**Algorithm:**

1. Select the appropriate subsets from the set of parameters.
2. Construct the IVFSM by selected set of parameters.
3. Take the product of the IVFS-matrices by And product.
4. Find max-min decision IVFSM.
5. Finally we find optimum IVFSM on M.

**Application:**

Let M = {Q, P, K, S} be a universal set which consists on four cities of any country the world.

Let E = {$y_1$, $y_2$, $y_3$, $y_4$} be the set of parameter where each $y_i$ stand for Education, health, local bodies and development program respectively.

**Example 4.1:** Two friends A and B are wants to live in any city of M, if both friends have their own set of parameters. Then we select a city on the basis of the sets of friend's parameters by using the IVFSMmDM as follows.

Suppose M = {Q, P, K, S} be a universal set and E = {$y_1$, $y_2$, $y_3$, $y_4$} be a set of parameters.

**Step 1**

First of all A and B both friends chose the set of parameters,

A = {$y_1$, $y_2$, $y_4$} and B = {$y_1$, $y_2$, $y_3$}.

**Step 2**

Now, we construct the IVFS-matrices by using equation 3 according to selected parameters of both friends.

$$A = [a_{ij}] = \begin{bmatrix} [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.6] & [0.0, 0.0] & [0.5, 0.6] \\ [0.8, 0.8] & [0.7, 0.8] & [0.0, 0.0] & [0.4, 0.5] \\ [0.3, 0.4] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \end{bmatrix}$$

$$B = [b_{ik}] = \begin{bmatrix} [0.6, 0.7] & [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] \\ [0.7, 0.8] & [0.6, 0.7] & [0.2, 0.3] & [0.0, 0.0] \\ [0.8, 0.9] & [0.8, 0.9] & [0.5, 0.6] & [0.0, 0.0] \\ [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] & [0.0, 0.0] \end{bmatrix}$$

**Step 3**

Now, we find product of IVFS-matrices by using And-product

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.6] & [0.0, 0.0] & [0.5, 0.6] \\ [0.8, 0.8] & [0.7, 0.8] & [0.0, 0.0] & [0.4, 0.5] \\ [0.3, 0.4] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \end{bmatrix} \wedge \begin{bmatrix} [0.6, 0.7] & [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] \\ [0.7, 0.8] & [0.6, 0.7] & [0.2, 0.3] & [0.0, 0.0] \\ [0.8, 0.9] & [0.8, 0.9] & [0.5, 0.6] & [0.0, 0.0] \\ [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] & [0.0, 0.0] \end{bmatrix}$$

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} [.4,.5] & [.4,.5] & [.3,.4] & [.0,.0] & [.3,.4] & [.3,.4] & [.3,.4] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.2,.3] & [.2,.3] & [.2,.3] & [.0,.0] \\ [.7,.8] & [.6,.7] & [.2,.3] & [.0,.0] & [.6,.6] & [.6,.6] & [.2,.3] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.5,.6] & [.5,.6] & [.2,.3] & [.0,.0] \\ [.8,.8] & [.8,.8] & [.5,.6] & [.0,.0] & [.7,.8] & [.7,.8] & [.5,.6] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.4,.5] & [.4,.5] & [.4,.5] & [.0,.0] \\ [.3,.4] & [.3,.4] & [.2,.3] & [.0,.0] & [.3,.4] & [.3,.4] & [.2,.3] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.0,.0] & [.2,.3] & [.2,.3] & [.2,.3] & [.0,.0] \end{bmatrix}$$

**Step 4**

To calculate Mm $([a_{ij}] \wedge [b_{ik}]) = [d_{i1}]$, in this step we find $d_{i1}$ for all i=1, 2, 3, 4.
First of all we find $d_{11}$.
$d_{11} = \max \{t_{1k}\} = \max \{t_{11}, t_{12}, t_{13}, t_{14}\}$ where $k \in \{1, 2, 3, 4\}$
For $d_{11}$ we also find $t_{1k}$ for all $k \in \{1, 2, 3, 4\}$.
When k = 1 and n = 4 then $t_{11}$ is $I_1 = \{P: c_{iP} \neq 0, 0<P\leq 4\}$ and
When we find $t_{12}$ then k = 2, n = 4 and $I_2 = \{P: c_{iP} \neq 0, 4 <P \leq 8\}$.
Similarly for k = 3, $I_3 = \{P: c_{Ip} \neq 0, 8 <P \leq 12\}$ and for k = 4, $I_4 = \{P: c_{iP} \neq 0, 12 < P \leq 16\}$.
$t_{11} = \min \{c_{11}, c_{12}, c_{13}\}$, where $c_{ip} = [\min \{\mu_{ALJ}, \mu_{BLJ}\}, \min \{\mu_{AUJ}, \mu_{BUJ}\}]$
$t_{11} = \min \{[0.4, 0.5], [0.4, 0.5], [0.3, 0.4]\}$
$t_{11} = [0.3, 0.4]$
Similarly we obtain other values
$t_{12} = \min \{c_{15}, c_{16}, c_{17}\} = \min \{[0.3, 0.4], [0.3, 0.4], [0.3, 0.4]\}$
$t_{12} = [0.3, 0.4]$
$t_{13} = [0.0, 0.0]$
$t_{14} = \min \{c_{213}, c_{214}, c_{215}\} = \min \{[0.2, 0.3], [0.2, 0.3], [0.2, 0.3]\}$
$t_{14} = [0.2, 0.3]$
So
$d_{11} = \max \{t_{11}, t_{12}, t_{13}, t_{14}\} = \max \{[0.3, 0.4], [0.3, 0.4], [0.0, 0.0], [0.2, 0.3]\}$
$\mathbf{d_{11} = [0.3, 0.4]}$
Similarly we find $d_{21}$, $d_{31}$, $d_{41}$.
$d_{21} = \max \{t_{2k}\} = \max \{t_{21}, t_{22}, t_{23}, t_{24}\}$ where $k \in \{1, 2, 3, 4\}$
$t_{21} = \min \{c_{21}, c_{22}, c_{23}\} = \min \{[0.7, 0.8], [0.6, 0.7], [0.2, 0.3]\}$
$t_{21} = [0.2, 0.3]$
$t_{22} = \min \{c_{25}, c_{26}, c_{27}\} = \min \{[0.6, 0.6], [0.6, 0.6], [0.2, 0.3]\}$
$t_{22} = [0.2, 0.3]$
$t_{23} = [0.0, 0.0]$
$t_{24} = \min \{c_{213}, c_{214}, c_{215}\} = \min \{[0.5, 0.6], [0.5, 0.6], [0.2, 0.3]\}$
$t_{24} = [0.2, 0.3]$
So
$d_{21} = \max \{t_{21}, t_{22}, t_{23}, t_{24}\} = \max \{[0.2, 0.3], [0.2, 0.3], [0.0, 0.0], [0.2, 0.3]\}$
$\mathbf{d_{21} = [0.2, 0.3]}$
$d_{31} = \max \{t_{3k}\} = \max \{t_{31}, t_{32}, t_{33}, t_{34}\}$ where $k \in \{1, 2, 3, 4\}$
$t_{31} = \min \{c_{31}, c_{32}, c_{33}\} = \min \{[0.8, 0.8], [0.8, 0.8], [0.5, 0.6]\}$
$t_{31} = [0.5, 0.6]$
$t_{32} = \min \{c_{35}, c_{36}, c_{37}\} = \min \{[0.7, 0.8], [0.7, 0.8], [0.5, 0.6]\}$
$t_{32} = [0.5, 0.6]$
$t_{33} = [0.0, 0.0]$
$t_{34} = \min \{c_{313}, c_{314}, c_{315}\} = \min \{[0.4, 0.5], [0.4, 0.5], [0.4, 0.5]\}$
$t_{34} = [0.4, 0.5]$
So
$d_{31} = \max \{t_{31}, t_{32}, t_{33}, t_{34}\} = \max \{[0.5, 0.6], [0.5, 0.6], [0.0, 0.0], [0.4, 0.5]\}$

**$d_{31}$ = [0.5, 0.6]**
Finally we find $d_{31}$
$d_{41}$= max {$t_{4k}$} = max {$t_{41}$, $t_{42}$, $t_{43}$, $t_{44}$} where k ∈ {1, 2, 3, 4}
$t_{41}$ = min {$c_{41}$, $c_{42}$, $c_{43}$} = min {[0.3, 0.4], [0.3, 0.4], [0.2, 0.3]}
$t_{41}$ = [0.2, 0.3]
$t_{42}$ = min {$c_{45}$, $c_{46}$, $c_{47}$} = min {[0.3, 0.4], [0.3, 0.4], [0.2, 0.3]}
$t_{42}$ = [0.2, 0.3]
$t_{43}$ = [0.0, 0.0]
$t_{44}$ = min {$c_{413}$, $c_{414}$, $c4_{15}$} = min {[0.2, 0.3], [0.2, 0.3], [0.2, 0.3]}
$t_{44}$ = [0.2, 0.3]
So
$d_{41}$ = max {$t_{41}$, $t_{42}$, $t_{43}$, $t_{44}$} = max {[0.2, 0.3], [0.2, 0.3], [0.0, 0.0], [0.2, 0.3]}
**$d_{41}$ = [0.2, 0.3]**
Finally we obtain the IVFSM by IVFSMmDM.

$$\text{Mm}\ ([a_{ij}] \wedge [b_{ik}]) = [d_{i1}] = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \end{bmatrix}$$

$$[d_{i1}] = \begin{bmatrix} [0.3, 0.4] \\ [0.2, 0.3] \\ [0.5, 0.6] \\ [0.2, 0.3] \end{bmatrix} \tag{6}$$

Finally, we obtain an optimum IVFSM on M according to above matrix.

$$\text{Opt Mm}\ ([a_{ij}] \wedge [b_{ik}])\ (M) = \{[0.3, 0.4]/Q, [0.2, 0.3]/P, [0.5, 0.6]/K, [0.2, 0.3]/S\}$$

Where K is optimum city of any country of the world for living of A and B.

Similarly we can apply Or-product for other problems.

## 5 Conclusions

In this paper, we study basic definitions of IVFSM with some properties and prove commutative laws, associative laws and De-Morgan laws by using And-Operation and Or-Operation on IVFSM. In [12] (Cagman and Enginoglu) introduced FSMmDM method by using fuzzy soft max-min decision making function on FSM. We have extended the approach of Cagman and Enginoglu and proposed a new decision making method on IVFSM named as “interval valued fuzzy soft max-min decision making method” (IVFSMmDM) with the help of interval valued fuzzy soft max-min decision making function. Finally, we applied IVFSMmDM method for decision making by using data from [19].

## Competing Interests

Authors have declared that no competing interests exist.

## References

[1] Zadeh LA. Fuzzy sets. Journal of Information and Control. 1995;8(3):338-353.

[2] Molodtsov DA. Soft set theory-first result. Computers and Mathematics with Applications. 1999;37:19-31.

[3] Maji PK, Roy AR, Biswas R. Fuzzy soft sets. Journal of Fuzzy Mathematics. 2001;9(3):589-602.

[4] Maji PK, Roy AR, Biswas R. An application of soft sets in a decision making problem. Computers and Mathematics with Applications. 2002;44(8-9):1077–1083.

[5] Maji PK, Roy AR. Soft set theory. Computers and Mathematics with Applications. 2003; 45:555–562

[6] Chen D, Tsang ECC, Yeung DS, Wang X. The parameterization reduction of soft sets and its applications. Computers & Mathematics with Applications. 2005;49:5-6.

[7] Ali MI, Feng F, Liu XY, Min WK, Shabir M. On some new operations in soft set theory. Computers and Mathematics with Applications. 2009;57:1547–1553.

[8] Majumdar P, Samanta SK. Generalized fuzzy soft sets. Computers and Mathematics with Applications. 2010;59(4):1425-1432.

[9] Neog TJ, Sut DK. On union and intersection of fuzzy soft sets. Int. J. Comp. Tech. Appl. 2011; 2(5):1160-1176.

[10] Yang Y, Chenli Ji. Fuzzy soft matrices and their applications. Lecture Notes in Computer Science. 2011;7002:618–627.

[11] Cagman N, Enginoglu S. Fuzzy soft set theory and its applications. Iranian Journal of Fuzzy Systems. 2011;8(3):137-147.

[12] Cagman N, Enginoglu S. Fuzzy soft matrix theory and its application in decision making. Iranian Journal of Fuzzy Systems. 2012;9(1):109-119.

[13] Borah MJ, Neog TJ, Sut DK. Fuzzy soft matrix theory and its decision making. International Journal of Modern Engineering. 2012;2(2):121-127.

[14] Yang X, Lin TY, Yang J, Li Y, Dongiun Y. Combination of interval-valued fuzzy set and soft set. Computers and Mathematics with Applications. 2009;58(3):521-527.

[15] Chetia B. An application of interval valued fuzzy soft sets in medical diagnosis. Int. J. Contemp. Math. Sciences. 2010;5(38):1887-1894.

[16] Rajarajeswari P, Dhanalakshmi P. Interval valued fuzzy soft matrix theory. Annals of Pure and Applied Mathematics. 2014;7(2):61-72.

[17] Sarala N, Prabharathi M. An application of interval valued fuzzy soft matrix in medical diagnosis. IOSR Journal of Mathematics. 2015;11(1):01-06.

[18] Zulqarnain M, Saeed M. An application of interval valued fuzzy soft matrix in decision making. Science International (Lahore). 2016;28(3):2261-2264.

[19] Zulqarnain M, Saeed M, Tabasum F. Comparison between fuzzy soft matrix (FSM) and interval valued fuzzy soft matrix (IVFSM) in decision making. Science International (Lahore). 2016;28(5): 4277-4283.

[20] Sarala N, Prabharathi M. An application of interval valued fuzzy soft matrix in decision making problem. International Journal of Mathematics Trends and Technology. 2015;21(1).




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