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# Jeans Instability of Self-gravitating Porous Medium under the Effect of Electron Plasma Frequency and Coriolis Force

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***Authors' contributions***

*This work was carried out in collaboration between all authors. Author AK designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author DLS discussed and analyzed the result obtained in the manuscript. The all work is done under the guidance and supervision of author RKP. All authors read and approved the final manuscript.*

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## ABSTRACT

This paper deals with the theoretical investigation of the combined effect of electron plasma frequency and Coriolis force on the hydromagnetic waves through a self gravitating porous medium in the presence of fine dust particles subjected to a transverse uniform magnetic field. A general dispersion relation is obtained using the normal mode analysis with the help of relevant linearized perturbation equation of motion. This dispersion relation is reduced for longitudinal and transverse modes of propagation. Dispersion relations for two modes are further reduced for the axis of rotation parallel and perpendicular to the direction of the magnetic field. We find that Jean's

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criterion of instability remains valid but the expression of the critical Jean's wave number is modified. The numerical analysis is performed to show the effect of thermal conductivity, relaxation time, rotation, Stokes drag and porosity of the medium on the growth rate of instability.

*Keywords: Electron plasma frequency; suspended particles; magnetic field; thermal conductivity; porosity; permeability.*

## 1. INTRODUCTION

Investigation of the self-gravitational instability in recent times have acquired considerable momentum. Major reason for the spurt in the interest is due to it's an important role in star formation in magnetic dusty clouds by gravitation collapsing process. The gravitational instability of an infinite homogeneous, self-gravitating gaseous medium was first discovered by Jean's [1]. It was shown that the disturbances would grow if their wavelength exceeded a certain minimum wavelength $\lambda_j$ , given by $\lambda_j = \left(\frac{\pi c^2}{G\rho}\right)^{\frac{1}{2}}$ where $\rho$ and $c$ denote the density and sound velocity respectively and G is the gravitational constant. Since several authors have investigated this problem under varying assumption of hydrodynamics and hydromagnetics and a comprehensive account of all these investigations has been given by Chandrasekhar [2] in his monograph on the problem of hydrodynamics stability. He has shown that Jean's criterion remains unaffected by the separate or simultaneous presence of a uniform rotation and uniform magnetic field. Several authors Alfven [3], Lehnert [4-6], Cadez [7], Ali and Bhatia [8], Bhatia and Hazarika [9], Lima et al. [10], Sheikh et al. [11], Oberoi [12], Cohen [13] have investigated the problem of the gravitational instability of infinite homogeneous gaseous plasma with different physical parameter such as viscosity, magnetic field, rotation, electrical conductivity, thermal conductivity, resistivity and Hall current.

In addition to this, it has been established as fact that the Coriolis force plays an important role in astrophysics. The effect of Coriolis force is very important to understand the relation between angular momentum and rotational kinetic energy. The magnetic force may be viewed as a kind of Coriolis force due to Thomas rotation, induced by successions of noncollinear Lorentz boots. In this direction many investigators, Bhatia [14], Herrengger [15], Lehnert [16], Mattei [17] have discussed the effect of Coriolis force in some astrophysical problems.

On other hands, it is well known that in astrophysical situations, the fluid is often not pure but contains suspended particles. Scanlon and Segel [18] have considered the effect of suspended particles on the onset of Bénard convention. Sharma and Sharma [19] have studied the suspended particles and the gravitational instability of a rotating plasma. Recently Kumar [20] discussed Hall current effect on the thermal instability of porous compressible viscoelastic dusty fluid and pointed out that Hall current and suspended particles have destabilizing effects whereas compressibility and the magnetic field has stabilizing effects on the system. The electron inertia parameter is important in the problem of magnetic reconnection processes, and it gives fundamental information about the wave propagation in the system with a finite plasma frequency. Uberoi [12] have pointed out that the finite electron inertia effects modify the Alfven compressional wave and it gives new mode of wave known as inertial compressional Alfven wave (ICW). This inertial compressional Alfven wave when interacts with acoustic and gravitational modes, interesting characteristic changes in Jean's gravitational instability are observed for non-rotating and rotating systems. Along with this, the flow through porous media is of considerable interest in geophysical fluid dynamics. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law. In this connection, many researchers Kumar and Singh [21], Lapwood [22], Kumar and Singh [23] and Kumar et al. [24] have discussed Jean's instability of porous medium with different parameters.

Pensia et al. [25] have discussed, the role of Coriolis force and suspended particles in the fragmentation of matter in the central region of the galaxy.

In view of the above investigations, we attempt here to discuss the influence of electron plasma frequency and Coriolis force on the propagation of hydrodynamics waves in the self gravitating porous medium. Further, we address the problem in the thermally conducting medium in the presence of transverse magnetic field to determine a magnetohydrodynamical model suitable for a geophysical system. From the point of view of astrophysical problems, the present study can serve a theoretical support for experimental investigations. This problem, to the best of our knowledge, has not been investigated yet.

## 2. LINEARIZED PERTURBATION EQUATIONS

We consider an infinite homogeneous, viscous, self-gravitating, rotating, ionized plasma composed of gas and the fine dust particles incorporating thermal conductivity and finite electron inertia, flowing through a porous medium.

$$\frac{\partial\vec{v}}{\partial t} = -\frac{\vec{\nabla}\delta P}{\rho} + \vec{\nabla}\delta\phi + \frac{k_s N(\vec{u}-\vec{v})}{\rho} + \frac{1}{4\pi\rho}\left(\vec{\nabla}\times\vec{b}\right)\times\vec{B} + \vartheta\nabla^2\vec{v} - \frac{\vartheta}{k_1}\vec{v} + 2\left(\vec{v}\times\vec{\Omega}\right) \quad (1)$$

$$\varepsilon\frac{\partial\delta\rho}{\partial t} + \frac{\rho\vec{\nabla}.\vec{v}}{\rho} = 0 \quad (2)$$

$$\delta P = C^2\delta P \quad (3)$$

$$\nabla^2\delta\phi + 4\pi G\delta\rho = 0 \quad (4)$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{u} = \vec{v} \quad (5)$$

$$\lambda\nabla^2\delta T = \rho C_p\frac{\partial\delta t}{\partial t} - \frac{\partial\delta P}{\partial t} \quad (6)$$

$$\frac{\delta P}{P} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho} \quad (7)$$

$$\frac{\partial\vec{b}}{\partial t} = \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) + \frac{C^2}{\omega_{pe}^2}\frac{\partial}{\partial t}\nabla^2\vec{b} \quad (8)$$

Where,
$\vec{v}(v_x, v_y, v_z), \vec{u}(u_x, u_y, u_z)$,
$N, \rho, P, \phi, \vec{B}(0,0,B), \vec{\Omega}(\Omega_x, 0, \Omega_z)T, G, \vartheta, C_p, \lambda, R, k_1, m, \rho_s, \omega_{pe}, k_s(6\pi\eta r) and\ \vec{b}(b_x, b_y, b_z)$, denote respectively, the gas velocity, the particle velocity, the number density of the particle, density of the gas, pressure of the gas, gravitational potential, magnetic field, rotation, temperature, Gravitational constant, kinematic viscosity, specific heat at constant pressure, thermal conductivity, gas constant, permeability, particles density, plasma frequency of electron, the constant in the Stokes drag formula and perturbation in magnetic field.

## 3. DISPERSION RELATION

We analyze these perturbations with normal oscillation technique. In a uniform system, we can find a plane wave solution, for all variables varying as,

$$exp\{i(k_x x + k_z z + \omega t)\} \quad (9)$$

Where $k_x\ and\ k_z$ are the components of wave numbers of perturbation along the x and z-axis respectively, so that $k_x^2 + k_z^2 = k^2$ and $\omega$ the frequency of harmonic disturbances, Using equations (1) to (9), we obtain the following algebraic equations for the components.

$$D_1 v_x - 2\Omega_z v_y + \frac{ik_x}{k^2}\Omega_T^2 s = 0 \quad (10)$$

$$2\Omega_z v_x + D_2 v_y - 2\Omega_x v_z = 0 \quad (11)$$

$$2\Omega_x v_y + \xi_1 v_z + \frac{ik_z}{k^2}\Omega_T^2 s = 0 \quad (12)$$

The divergence of (1) with the aid of (2)-(9) gives

$$\frac{ik_x k^2 V^2}{a_1} v_x + 2i(k_x\Omega_z - k_z\Omega_x)v_y - D_3 s = 0 \quad (13)$$

Where, $s = \frac{\delta\rho}{\rho}$ is the condensation of the medium, $\gamma = \frac{C_p}{C_v} = \frac{C^2}{C'^2}$ a ratio of the specific heat, $V = \frac{B}{\sqrt{4\pi\rho}}$ is the Alfven velocity, $a = \frac{k_s N}{\rho}$ has the dimension of frequency, $\tau = \frac{m}{k_s}$ is the relaxation time and $m$ is the mass of the particle.

$\tau a = \frac{\rho_s}{\rho}$ is the mass concentration, $\sigma = i\omega$ is the growth rate of perturbation, $\theta = \frac{\lambda}{\rho C_p}$ is the thermometric conductivity, $\Omega_\vartheta = \vartheta\left(k^2 - \frac{1}{k_1}\right)$, $a_1 = \sigma f$, $f = \left(1 + \frac{C^2 k}{\omega_{pe}^2}\right)$, C and C' are the adiabatic and isothermal velocities of sound.

$$\xi_1 = \left(\sigma + \Omega_\vartheta + \frac{a\sigma\tau}{\sigma\tau+1}\right), \Omega_{j'}^2 = \left(C'^2k^2 - 4\pi G\rho\right), \Omega_j^2 = \left(C^2k^2 - 4\pi G\rho\right), \quad \Omega_T^2 = \left(\frac{\sigma\Omega_j^2 + \gamma_k\Omega_{j'}^2}{\sigma+\gamma_k}\right),$$

$$D_1 = \left(\xi_1 + \frac{V^2k_x^2}{a_1}\right), D_2 = \left(\xi_1 + \frac{V^2k_z^2}{a_1}\right), D_3 = \left(\varepsilon\sigma\xi_1 + \Omega_T^2\right), \ \gamma_k = \gamma\theta k^2, \quad V^2 = \frac{B^2}{4\pi\rho}, \ k^2 = k_x^2 + k_z^2$$

For the nontrivial solution of equations (10)-(13), the determinant of coefficients of $v_x, v_y,\ v_z\ and\ s$ should vanish, leading to the following dispersion relation.

$$\xi_1\left[\sigma\xi_1\left(D_1D_2+4\Omega^2\right)+\frac{4\Omega_x^2k^2V^2\sigma}{a_1}+\frac{\Omega_T^2}{\varepsilon}\left(D_2^2+4\Omega^2-\frac{4(k_x\Omega_z-k_z\Omega_x)^2}{k^2}\right)\right]=0 \tag{14}$$

Equation (14) gives the general dispersion relation and it represent the combined influence of finite electron plasma frequency, Coriolis force, presence of fine dust particles, porosity of the medium, viscosity, permeability of the medium and thermal conductivity on the propagation of the hydromagnetic waves when the system is subjected to the transverse magnetic field. We find that in this dispersion relation the term due to finite plasma frequency has entered through the factor f. This dispersion relation will be able to predict all information about the wave and instability of the hydromagnetic fluid plasma considered. The above dispersion relation is very lengthy and investigates the effect of each parameter we now reduce the dispersion relation (14) for two modes of propagation.

## 4. ANALYSIS OF THE DISPERSION RELATION

Now we shall discuss the dispersion relation given by equation (14) for the following modes. Longitudinal propagation i.e. - $k_x=0\,,\ k_z=k$**,** Transverse propagation i.e.-$k_x=k,\ \ k_z=0$.

### 4.1 Longitudinal Mode of Propagation $(\boldsymbol{K}\parallel\boldsymbol{B})$

For this case, we assume that all the perturbations are longitudinal to the direction of the magnetic field $(i.e.k_x=0,k_z=k)$. Thus the dispersion relation (14) reduces to the simple form to give,

$$\sigma\xi_1\left\{\left(\xi_1+\frac{k^2V^2}{a_1}\right)^2+4\Omega^2\right\}+\frac{4\sigma k^2V^2}{a_1}\Omega_x^2+\frac{\Omega_T^2}{\varepsilon}\left\{\left(\xi_1+\frac{k^2V^2}{a_1}\right)^2+4\Omega_z^2\right\}=0 \tag{15}$$

We find that for the longitudinal mode of propagation the dispersion is modified due to the presence of fine dust particles, finite plasma frequency, viscosity, porosity and permeability of the medium, magnetic field, thermal conductivity and Coriolis force. This dispersion relation is further reduced for rotational axes parallel and perpendicular to the direction of the magnetic field for simplicity.

### 4.2 Axis of Rotation Parallels to the Magnetic Field $(\boldsymbol{\Omega}\parallel\boldsymbol{B})$

When the axis of rotation is along the magnetic field, i.e. $\Omega_x=0\ and\ \Omega_z=\Omega$. Then (15) reduces to as,

$$\left(\sigma\xi_1+\frac{\Omega_T^2}{\varepsilon}\right)\left[\left(\sigma+\Omega_\vartheta+\frac{\beta\sigma}{\sigma\tau+1}+\frac{k^2V^2}{a_1}\right)^2+4\Omega^2\right]=0 \tag{16}$$

This dispersion relation shows the simultaneous effects of, finite electron plasma frequency, Coriolis force, the presence of fine dust particles, magnetic field, thermal conductivity, viscosity, porosity and permeability of the medium on the propagation of hydromagnetic waves through the fluid. The dispersion relation (16) is the product of two independent factors, each representing a different mode of propagation. The first factor of equation (16) on substituting the values of$\xi_1$ and $\Omega_T^2$, gives the following 4-degree polynomial equation.

$$\sigma^4\varepsilon\tau+\sigma^3\varepsilon(1+\beta+\Omega_\vartheta\tau+)+\sigma^2\tau\gamma_k\left[\varepsilon\{\gamma_k(1+\beta+\Omega_\vartheta\tau)+\Omega_\vartheta\}+\tau\Omega_{j'}^2\right]+\sigma\left(\varepsilon\Omega_\vartheta\gamma_k+\Omega_j^2+\gamma_k\tau\Omega_{j'}^2\right)+\gamma_k\Omega_{j'}^2=0 \tag{17}$$

The dispersion relation (17) represents a stable damped mode modified by the viscosity, porosity, thermal conductivity and fine dust particles. The second factor of (16) gives, on substituting the value of $a_1$ the following six-degree polynomial equation.

$$\begin{aligned}\tau^2f^2\sigma^6&+2\tau\sigma^5f^2[1+\tau(\beta+\Omega_\vartheta)]+\sigma^4\left[f^2\{1+\tau(\beta+\Omega_\vartheta)\}^2+2\tau(\Omega_\vartheta f+\tau k^2V^2)+4\tau^2f^2\Omega^2\right]\\&+\sigma^3\left[2\tau fk^2V^2+8\tau f^2\Omega^2+2f(f\Omega_\vartheta+\tau k^2V^2)\{1+\tau(\beta+\Omega_\vartheta)\}\right]\\&+\sigma^2\left[(f\Omega_\vartheta+\tau k^2V^2)^2+2fk^2V^2\{1+\tau(\beta+\Omega_\vartheta)\}+4f^2\Omega^2\right]\\&+\sigma[2k^2V^2(f\Omega_\vartheta+\tau k^2V^2)]+k^4V^4=0\end{aligned} \tag{18}$$

The dispersion relation (18) represents a non-gravitating Alfven mode modified by the presence of Coriolis force, finite electron plasma frequency, fine dust particles, viscosity and permeability of the medium.

### 4.3 Axis of Rotation Perpendicular to the Magnetic Field $(\boldsymbol{\Omega} \perp \boldsymbol{B})$

In the case of a rotation axis perpendicular to the magnetic field, we put $\Omega_x = \Omega \, and \Omega_z = 0$ in the dispersion relation (15) and this gives,

$$\left(\xi_1 a_1 + k^2V^2\right)\left[\left(\xi_1 a_1 + k^2V^2\right)\left(\sigma\xi_1 + \frac{\Omega_T^2}{\varepsilon}\right) + 4\sigma a_1 \Omega^2\right] = 0 \tag{19}$$

This dispersion relation is the product of two independent factors. These factors show the mode of propagations incorporating different parameters as discussed below. The first factor of equation (19) also represents a stable non-gravitating Alfven mode modified by fine dust particles, finite electron inertia, and viscosity but this mode is not affected by Coriolis force. The second factor of equation (19) gives on substituting the values of $\xi_1, a_1$and$\Omega_T^2$, the following seven-degree polynomial equation.

$$\begin{aligned}
\tau^2\sigma^7 f + \sigma^6[\tau f\{2 + \tau(2\Omega_\vartheta + 2\beta + \gamma_k)\}] \\
&+ \sigma^5\Bigg[f\{1 + 2\tau(2\Omega_\vartheta + \beta + \gamma_k)\}\\
&+ \tau^2\left\{f\frac{\Omega_j^2}{\varepsilon} + k^2V^2 + 4f\Omega^2 + f(\beta + \Omega_\vartheta)(\Omega_\vartheta + \beta + 2\gamma_k)\right\}\Bigg]\\
&+ \sigma^4\Bigg[f(2\Omega_\vartheta + \gamma_k) + 2\tau\left\{f\frac{\Omega_j^2}{\varepsilon} + k^2V^2 + 4f\Omega^2 + f(\beta + \Omega_\vartheta)(\Omega_\vartheta + \gamma_k) + f\Omega_\vartheta\gamma_k\right\}\\
&+ \tau^2\left\{\gamma_k\left(f\frac{\Omega_{j'}^2}{\varepsilon} + k^2V^2 + 4f\Omega^2\right) + (\beta + \Omega_\vartheta)\left(f\frac{\Omega_j^2}{\varepsilon} + k^2V^2 + f\beta\gamma_k + f\Omega_\vartheta\gamma_k\right)\right\}\Bigg]\\
&+ \sigma^3\Bigg[\left(f\frac{\Omega_j^2}{\varepsilon} + k^2V^2 + 4f\Omega^2 + f\Omega_\vartheta^2 + 2f\Omega_\vartheta\gamma_k\right)\\
&+ \tau\Bigg\{(\beta + \Omega_\vartheta)\left(f\frac{\Omega_j^2}{\varepsilon} + k^2V^2 + 2f\Omega_\vartheta\gamma_k\right) + \Omega_\vartheta\left(f\frac{\Omega_j^2}{\varepsilon} + k^2V^2\right)\\
&+ 2\gamma_k\left(f\frac{\Omega_{j'}^2}{\varepsilon} + k^2V^2 + 4f\Omega^2\right)\Bigg\} + \tau^2\left\{k^2V^2\frac{\Omega_j^2}{\varepsilon} + \gamma_k(\beta + \Omega_\vartheta)\left(f\frac{\Omega_{j'}^2}{\varepsilon} + k^2V^2\right)\right\}\Bigg]\\
&+ \sigma^2\Bigg[\Omega_\vartheta\left(f\frac{\Omega_j^2}{\varepsilon} + k^2V^2 + f\Omega_\vartheta\gamma_k\right) + \gamma_k\left(f\frac{\Omega_{j'}^2}{\varepsilon} + k^2V^2 + 4f\Omega^2\right)\\
&+ \tau\left\{2k^2V^2\frac{\Omega_j^2}{\varepsilon} + 2\Omega_\vartheta\gamma_k\left(f\frac{\Omega_{j'}^2}{\varepsilon} + k^2V^2\right)\right\} + \tau^2k^2V^2\gamma_k\frac{\Omega_{j'}^2}{\varepsilon} + \tau\beta\gamma_k\left(f\frac{\Omega_{j'}^2}{\varepsilon} + k^2V^2\right)\Bigg]\\
&+ \sigma\left[k^2V^2\frac{\Omega_j^2}{\varepsilon} + \Omega_\vartheta\gamma_k\left(f\frac{\Omega_{j'}^2}{\varepsilon} + k^2V^2\right) + 2\tau k^2V^2\gamma_k\frac{\Omega_{j'}^2}{\varepsilon}\right] + k^2V^2\gamma_k\frac{\Omega_{j'}^2}{\varepsilon} = 0
\end{aligned} \tag{20}$$

The dispersion relation (20) represents the effect of the simultaneous inclusion of the finite electron plasma frequency, fine dust particles, Coriolis force, viscosity, thermal conductivity, magnetic field, permeability and porosity of the medium on the propagation of hydromagnetic waves propagating through the medium. The condition of instability and the expression of the critical Jean's wave number are obtained from the constant term of equation (20). In the astrophysical situation, we study the effects of thermal conductivity, the presence of fine dust particles, Coriolis force and finite electron inertia on the growth rate of an unstable mode by choosing the arbitrary values of these parameters in the present problem. We write the dispersion relation (20) in nondimensional form as.

$$
\begin{aligned}
&\sigma^{*7}\tau^{*2}f^* + \sigma^{*6}\left[2\tau^* f^* + \tau^{*2} f^*\left(2k_s^* + 2\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)+\lambda^*\right)\right] \\
&\quad + \sigma^{*5}\Bigg[\!\!\Bigg[ f^* + 2\tau^* f^*\left\{k_s^* + 2\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right) + \lambda^*\right\} + \tau^{*2}\Bigg[ f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + 4f^*\Omega^{*2} \\
&\quad + f^*\left\{k_s^* + \vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right\}\left\{k_s^* + \vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right) + 2\lambda^*\right\}\Bigg]\Bigg]\!\!\Bigg] \\
&\quad + \sigma^{*4}\Bigg[ f^*\left(2\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)+\lambda^*\right) \\
&\quad + 2\tau^*\Bigg\{\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + 4f^*\Omega^{*2}\right) \\
&\quad + f^*\left(k_s^* + \vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)\left(\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)+\lambda^*\right) + f^*\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\Bigg\} \\
&\quad + \tau^{*2}\Bigg\{\lambda^*\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + 4f^*\Omega^{*2}\right) \\
&\quad + \left(k_s^* + \vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + k_s^* f^*\lambda^* + f^*\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\right)\Bigg\}\Bigg] \\
&\quad + \sigma^{*3}\Bigg[ f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + 4f^*\Omega^{*2} + f^*\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right) + 2f^*\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^* \\
&\quad + \tau^*\Bigg\{\left(k_s^* + 2\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + 2f^*\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\right) \\
&\quad + \vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2}\right)\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2}\right)\Bigg\}\Bigg] \\
&\quad + \sigma^{*2}\Bigg[\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + f^*\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\right) \\
&\quad + \lambda^*\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2} + 4f^*\Omega^{*2}\right) \\
&\quad + \tau^*\left\{2\frac{(k^{*2}-1)}{\varepsilon}k^{*2}V^{*2} + 2\vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2}\right)\right\} \\
&\quad + \tau^{*2}k^{*2}V^{*2}\lambda^*\frac{(k^{*2}-1)}{\varepsilon} + \tau^* k_s^*\lambda^*\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2}\right)\Bigg]\sigma^*\Bigg[\frac{(k^{*2}-1)}{\varepsilon}k^{*2}V^{*2} \\
&\quad + \vartheta^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\left( f^*\frac{(k^{*2}-1)}{\varepsilon} + k^{*2}V^{*2}\right) + 2\tau^*\lambda^*\frac{(k^{*2}-1)}{\varepsilon}k^{*2}V^{*2}\Bigg] \\
&\quad + \lambda^*\frac{(k^{*2}-1)}{\varepsilon}k^{*2}V^{*2} = 0 \qquad (21)
\end{aligned}
$$

Where, the various nondimensional parameters are defined as,

$$
\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho}}, k_s^* = \frac{k_s N}{\rho\sqrt{4\pi G\rho}}, \quad \lambda^* = \frac{\lambda}{\rho C_p\sqrt{4\pi G\rho}}, \quad k^* = \frac{kC}{\sqrt{4\pi G\rho}}, \quad \vartheta^* = \frac{\vartheta\sqrt{4\pi G\rho}}{C^2},
$$

$$
k_1^* = \frac{k_1\sqrt{4\pi G\rho}}{C^2}, \quad \tau^* = \tau\sqrt{4\pi G\rho}, \quad V^* = \frac{V\sqrt{4\pi G\rho}}{C}, \quad \Omega^* = \frac{\Omega}{\sqrt{4\pi G\rho}} \qquad (22)
$$

In Figs. 1-3 we have depicted the nondimensional growth rate versus nondimensional wave number for various arbitrary values of the thermal conductivity ($\lambda^*$), relaxation time ($\tau^*$), and rotation ($\Omega^*$).

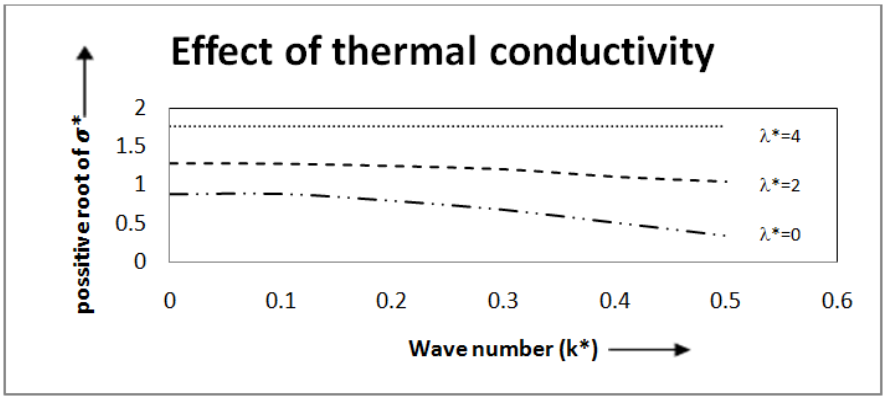


**Fig. 1. The growth rate of instability is plotted against the dimensionless wave number $k^*$ with variation in the thermal conductivity $\lambda^*$= 0, 2, 4, with taking the values of $k_s^*$, $\vartheta^*$, $V^*$, $k_1^*$, $\Omega^*$ and $\tau^*$ as unity**

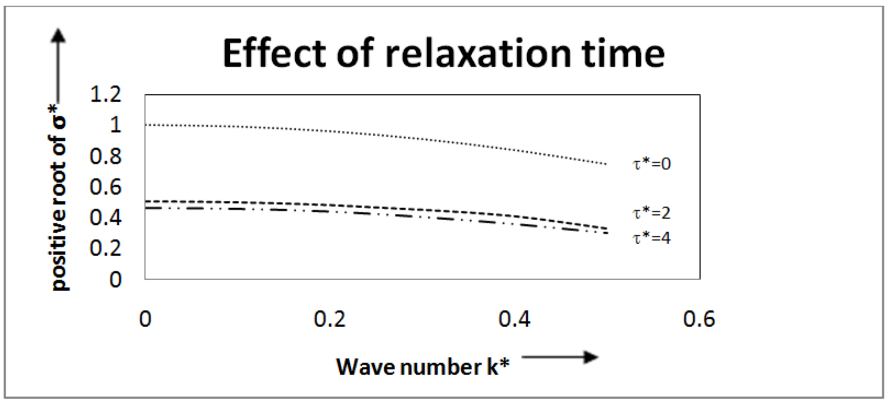


**Fig. 2. The growth rate of instability is plotted against the dimensionless wave number $k^*$ with variation in the relaxation time $\tau^*$= 0, 2, 4, with taking the values of $k_s^*$, $V^*$, $\vartheta^*$, $k_1^*$, $\Omega^*$ and $\lambda^*$ as unity**

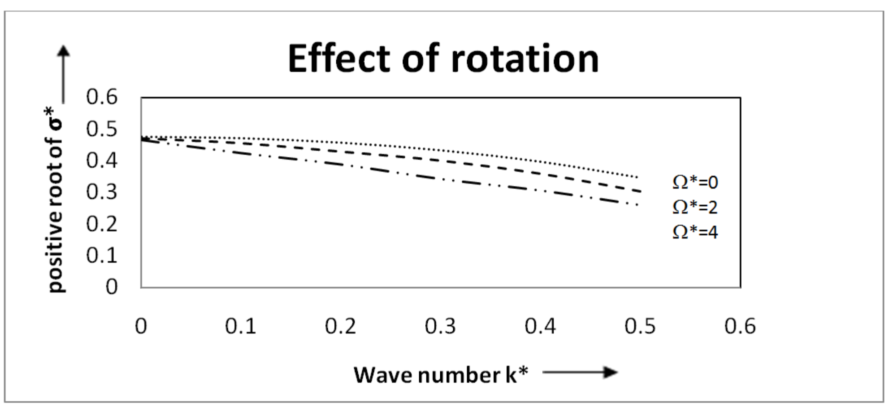


**Fig. 3. The growth rate of instability is plotted against the dimensionless wave number $k^*$ with variation in the Rotation $\Omega^* = 0, 2, 4$, with taking the values of $\lambda^*$, $V^*$, $k_s^*$, $\vartheta^*$, $k_1^*$ and $\tau^*$ as unity**

Fig. 1 the growth rate of an unstable mode (the positive real root of $\sigma^*$) is plotted against the wave number ( $k^*$) for different values of thermal conductivity parameter ($\lambda^*$). From the curves, we find that the growth rate of instability increases with increase in thermal conductivity. The peak value of the growth rate increases by increasing $\lambda^*$. The peak value of the growth rate is minimum for a nonthermal conducting medium ( $\lambda^* = 0$ ). Thus we conclude that thermal conductivity destabilizes the system in the longitudinal mode when the axis of rotation is perpendicular to the magnetic field.

Fig. 2 we have depicted the growth rate $(\sigma^*)$ of instability against wavenumber $(k^*)$ for the different value of relaxation time $(\tau^*)$ parameter. It is observed that the relaxation time parameter has a reverse effect on the growth rate, as compared to that of the thermal conductivity parameters. In other words, due to an increase in the relaxation time parameter, the growth rate of the instability decreases. Thus the relaxation time parameter has a stabilize the system. Also, the peak value of the growth rate decreases by increasing $(\tau^*)$.

Fig. 3 the growth rate of instability $(\sigma^*)$ is plotted against wavenumber $(k^*)$ for different values of the rotation parameter $(\Omega^*)$. From the curves, we find that the rotation has a reverse effect on the growth rate compared to that of thermal conductivity parameter. The growth rate of instability decreases with increase in rotation parameter. Thus, the rotation parameter has a damping effect on the growth rate of the system. The peak value of the growth rate is unaffected by the presence of the rotation parameter and it is the same for all the values of $\Omega^*$. The growth rate is maximum for the case of a non- rotating medium (i.e. for $\Omega^* = 0$).Thus we conclude that rotation stabilizes the system in a longitudinal mode of propagation when the axis of rotation is perpendicular to the magnetic field. Hence the rotation has a stabilizing effect on the growth rate of instability.

## 5. TRANSVERSE MODE OF PROPAGATION $(\boldsymbol{K \perp B})$

For this case, we assume all the perturbations transverse to the direction of the magnetic field $(i.e. k_x = k, \quad k_z = 0)$, the dispersion relation (14) gives,

$$\sigma\left[\xi_1\{D_1\xi_1 + 4\Omega^2\} + \frac{4\Omega_x^2 V^2 k^2}{a_1}\right] + \frac{\Omega_T^2}{\varepsilon}\left(\xi_1^2 + 4\Omega_x^2\right) = 0 \quad (23)$$

The dispersion relation (23) represents the effect of simultaneous inclusion of the finite electron plasma frequency, the presence of finite dust particles, Coriolis force, viscosity, magnetic field, thermal conductivity, permeability and porosity of the medium on the propagation of the hydromagnetic wave through the system for the transverse mode of propagation. Now we discuss this dispersion relation (23) in the case of rotation axes parallel and perpendicular to the magnetic field.

### 5.1 Axis of Rotation Parallels to the Magnetic Field $(\boldsymbol{\Omega \parallel B})$

When the axis of rotation is along the magnetic field, we put $\Omega_x = 0 \; and \; \Omega_z = \Omega$ and then the dispersion relation (23) becomes,

$$\xi_1\left[\xi_1\left\{\sigma D_1 + \frac{\Omega_T^2}{\varepsilon}\right\} + 4\sigma\Omega^2\right] = 0 \quad (24)$$

The dispersion relation (24) has two independent factors, each representing a different mode of propagation. The first factor of this dispersion relation is stable mode as discussed in the previous case and the second factor of the dispersion relations (24) after simplification can be written as,

$$\begin{aligned}
&\tau^2\sigma^6 f + \sigma^5\tau f[2\{1+\tau(\beta+\Omega_\vartheta)\} + \tau\gamma_k] \\
&+ \sigma^4\left[\tau^2\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + 4\Omega^2 f\right) + 2\tau\Omega_\vartheta f + \{1+\tau(\beta+\Omega_\vartheta)\}\{1+\tau(\beta+\Omega_\vartheta+2\gamma_k)\}f\right] \\
&+ \sigma^3\left[\tau^2\gamma_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2 + 4\Omega^2 f\right) + \tau\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + 8\Omega^2 f + 2\Omega_\vartheta\gamma_k f\right)\right. \\
&\left. + \{1+\tau(\beta+\Omega_\vartheta)\}\left\{(2\Omega_\vartheta+\gamma_k)f + \tau\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + \gamma_k\Omega_\vartheta f + \beta\gamma_k f\right)\right\}\right] \\
&+ \sigma^2\left[f\left(\Omega_\vartheta^2 + 4\Omega^2\right) + \{1+\tau(\beta+\Omega_\vartheta)\}\left\{\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + 2\Omega_\vartheta\gamma_k f + \tau\gamma_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right)\right\}\right. \\
&\left. + \tau\left\{\Omega_\vartheta\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2\right) + \gamma_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2 + 8\Omega^2 f\right)\right\}\right] \\
&+ \sigma\left[\Omega_\vartheta\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + \Omega_\vartheta\gamma_k f\right) + \tau\gamma_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right) + \gamma_k\{1+\tau(\beta+\Omega_\vartheta)\}\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right)\right] \\
&+ \Omega_\vartheta\gamma_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right) = 0 \qquad (25)
\end{aligned}$$

The dispersion relation (25) is the six degree polynomial equation and represents the effect of simultaneous inclusion of finite electron plasma frequency, presence of fine dust particles, thermal conductivity, viscosity, magnetic field, permeability and porosity of the medium on the propagation of hydromagnetic wave propagating through the medium for transverse propagation with the axis of rotation parallel to the direction of magnetic field. The condition of instability and the expression for the critical Jean wave number are obtained from the constant term of equation (25) and are written as,

$$(C^2k^2 - 4\pi G\rho) + \frac{\varepsilon}{f}\, k^2V^2 < 0 \tag{26}$$

and

$$k_j = \sqrt{\frac{4\pi G\rho}{C^2 + \frac{\varepsilon V^2}{f}}} \tag{27}$$

From above condition, we find that Jean's wave number is modified by a magnetic field, finite electron plasma frequency and porosity of the medium for the transverse mode of propagation with the axis of rotation parallel to the direction of magnetic field.

## 5.2 Axis of Rotation Perpendicular to the Magnetic Field$(\boldsymbol{\Omega} \perp \boldsymbol{B})$

In the case of a rotation axis perpendicular to the magnetic field i.e., $\Omega_x = \Omega \, and \, \Omega_z = 0$, the dispersion relation (23) reduces to

$$\left(\xi_1^2 + 4\Omega^2\right)\left\{\sigma D_1 + \frac{\Omega_T^2}{\varepsilon}\right\} = 0 \tag{28}$$

The dispersion relation (28) has two independent factors, each representing a different mode of propagation. The first factor of this dispersion relation (28) gives,

$$\sigma^4\tau^2 + \sigma^3 2\tau\{1+\tau(\beta+\Omega_\vartheta)\} + \sigma^2\left[\{1+\tau(\beta+\Omega_\vartheta)\}^2 + 2\tau\Omega_\vartheta + \tau^2 4\Omega^2\right] + 2\sigma\left[\Omega_\vartheta\{1+\tau(\beta+\Omega_\vartheta)\} + \tau 4\Omega^2\right] + \Omega_\vartheta^2 + 4\Omega^2 = 0 \tag{29}$$

This represents a rotating mode of propagation which is independent of magnetic field, self-gravitation, thermal conductivity, the porosity of the medium and finite electron plasma frequency. This is also a stable mode which represents a damping effect due to viscosity, permeability, presence of fine dust particles and rotation. In order to see the effects of various parameters on the growth rate of instability, we write the dispersion relation (29) in nondimensional form in terms of self-gravitation as,

$$\begin{aligned}\sigma^{*4}\tau^* + \sigma^{*3}&\left[2\tau^* + 2\tau^{*2}\left\{k_s^* + \vartheta^*\left(k^{*2} - \frac{1}{k_1^*}\right)\right\}\right] \\ &+ \sigma^{*2}\left[\left\{1 + \tau^*\left(k_s^* + \vartheta^*\left(k^{*2} - \frac{1}{k_1^*}\right)\right)\right\}^2 + 2\tau^*\vartheta^*\left(k^{*2} - \frac{1}{k_1^*}\right) + 4\tau^{*2}\Omega^{*2}\right] \\ &+ 2\sigma^*\left[\vartheta^*\left(k^{*2} - \frac{1}{k_1^*}\right)\left\{1 + \tau^*\left(k_s^* + \vartheta^*\left(k^{*2} - \frac{1}{k_1^*}\right)\right)\right\} + 4\tau^{*2}\Omega^{*2}\right] + \left(\vartheta^*\left(k^{*2} - \frac{1}{k_1^*}\right)\right)^2 \\ &+ 4\Omega^{*2} = 0\end{aligned} \tag{30}$$

Where the various nondimensional parameters are defined as,

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho}},\; k_s^* = \frac{k_s N}{\rho\sqrt{4\pi G\rho}}, \quad k^* = \frac{kC}{\sqrt{4\pi G\rho}}, \quad \vartheta^* = \frac{\vartheta\sqrt{4\pi G\rho}}{C^2}, \quad k_1^* = \frac{k_1\sqrt{4\pi G\rho}}{C^2}, \quad \tau^* = \tau\sqrt{4\pi G\rho}, \quad \Omega^* = \frac{\Omega}{\sqrt{4\pi G\rho}} \tag{31}$$

In order to see the effects of the particular parameter on the growth rate of instability, numerical calculations of equation (30) have been performed by varying one parameter and keeping all other parameters fixed. From the above-normalized dispersion relation (30), we take the real positive root among all other roots of $\sigma^*$. The positive roots of $\sigma^*$are plotted against wavenumber $k^*$to observe the behavior of instability for several values of the different parameters involved.

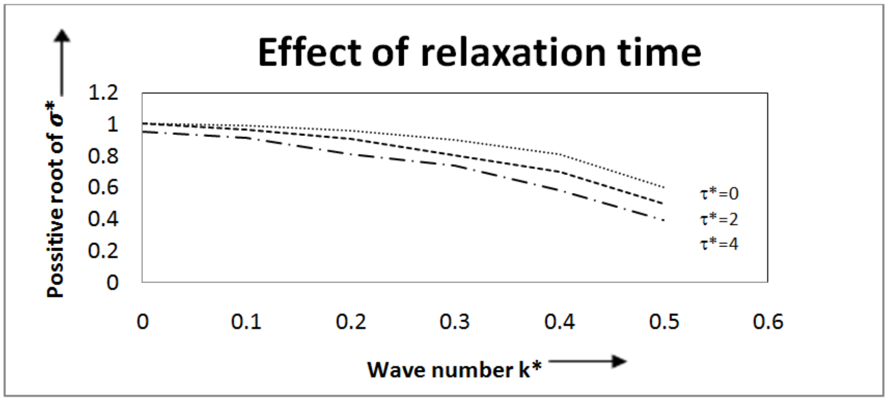


**Fig. 4. The growth rate of instability is plotted against the dimensionless wave number $k^*$ with variation in the relaxation time $\tau^*$= 0, 2, 4, with taking the values of $k_s^*, k_1^*, \Omega^*$, and $\vartheta^*$as unity**

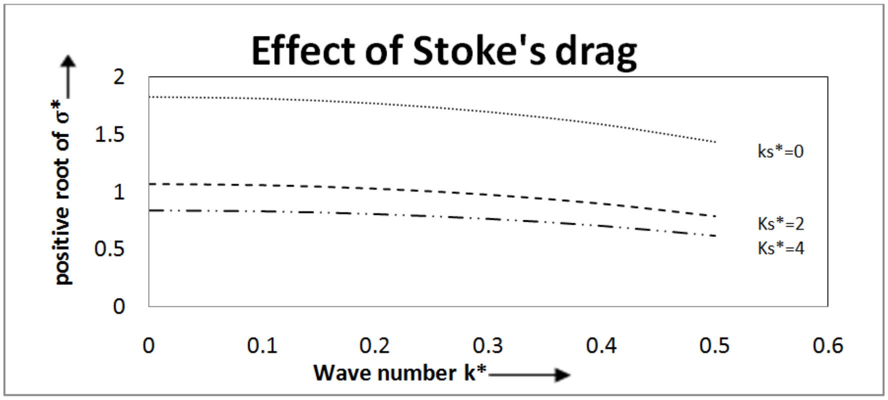


**Fig. 5. The growth rate of instability is plotted against the dimensionless wave number $k^*$ with variation in the Stokes' drag constant $k_s^*$ = 0, 2, 4, with taking the values of $k_1^*, \Omega^*, \vartheta^*$ and $\tau^*$ as unity**

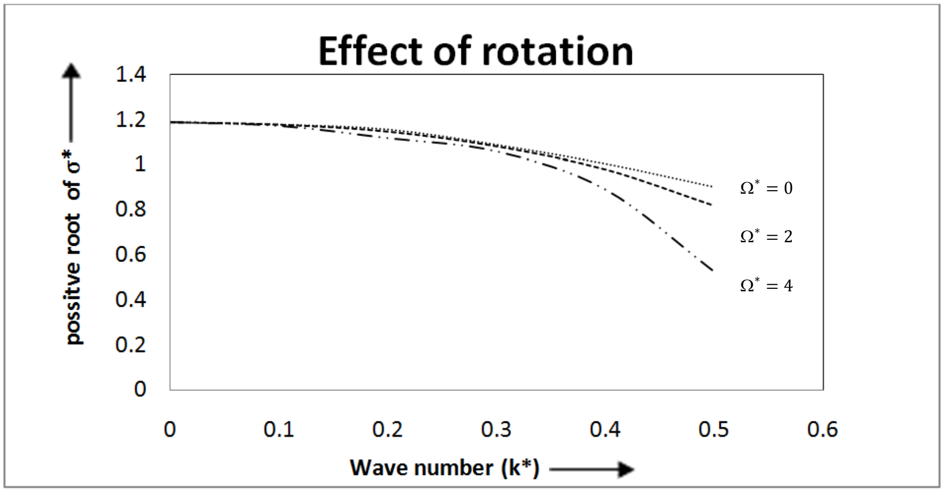


**Fig. 6. The growth rate of instability is plotted against the dimensionless wave number $k^*$ with variation in the rotation $\Omega^*$= 0, 2, 4, with taking the values of $k_1^*$, $k_s^*$, $\vartheta^*$ and $\tau^*$ as unity**

Fig. 4 it is clear that the effect of the relaxation ($\tau^*$) parameter on the growth rate of instability is same as shown in Fig. 2 in the case of longitudinal propagation with an axis of rotation perpendicular to the magnetic field. Thus the effect of the relaxation time ($\tau^*$) parameter is found to stabilize the system in both the longitudinal and transverse modes of propagation.

Fig. 5 the effect of the Stoke drag parameter on the growth rate is shown by the depicting the curves between ($\sigma^*$) and ($k^*$) for the various values of $k_s^*$. From the curves, we see that Stokes drag parameter shows the similar effect as shown by relaxation time parameter ($\tau^*$). Thus Stokes drag force has a stable influence on the self-gravitational instability of the system.

Fig. 6 shows the variation of the growth rate ($\sigma^*$) of instability against wavenumber ($k^*$) for different value of rotation $\Omega^*$ parameter. It is obvious that with an increase the value of rotation than the decrease in the growth rate of the system. The peak value of the growth rate gets increased due to the nonrotating system ($\Omega^* = 0$). The value of rotation increasing then the growth rate of instability is decreasing. Thus the rotation has also a stabilizing influence in both the transverse and longitudinal mode.

$$\sigma^4\tau f + \sigma^3 f\{1 + \tau(\beta + \Omega_\vartheta + \gamma_k)\} + \sigma^2\left[(\Omega_\vartheta + \gamma_k)f + \tau\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + \beta\gamma_k f + \gamma_k\Omega_\vartheta f\right)\right] + \sigma\left[\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + \Omega_\vartheta\gamma_k f\right) + \tau\gamma_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right)\right] + \gamma_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right) = 0 \quad (32)$$

This is a four-degree polynomial equation and shows the combined influence of various parameters, fine dust particles, viscosity, porosity, magnetic field, thermal conductivity and electron plasma frequency in the transverse mode of propagation when the axis of rotation perpendicular to the magnetic field.

## 6. CONCLUSIONS

To summarize, we have dealt with the hydromagnetic waves in the self-gravitating porous medium under the effect of electron plasma frequency and Coriolis force. A general dispersion relation is obtained which is modified due to the presence of these parameters. It is found that the viscosity, permeability, and suspended particles have a dissipative effect but do not affect Jean's expression. The dispersion relation is reduced for longitudinal and transverse modes of propagation, which are further reduced for axes of rotation parallel and perpendicular to the direction of the magnetic field. The dispersion relation is converted into the non-dimensional form where the physical parameters are put in the dimensionless form. The growth rate of instability is obtained analytically as a function of the physical parameters of the system considered. For some special cases, numerical solution is obtained to explain the roles that the variables of the problem play. Some curves are plotted and discussed. From the curves, it is found that thermal conductivity has a destabilizing influence, while the relaxation time and Coriolis force have a stabilizing role on the growth rate of the system. It is also found that Jean's criterion of instability is modified due to the presence of magnetic field and electron plasma frequency in the transverse mode of propagation. The Alfven mode is converted into inertial compression Alfven mode, which shows the remarkable change in Jean's criteria of instability. The interstellar medium is approximated to behave like an MHD (Magneto-hydrodynamics) fluid in the central region of our galaxy. Thus as a whole, the application of present problem of hydromagnetic waves in the central region of our galaxy will help to study the outflow of matter from this region.

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## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

1. Jeans JH. The stability of spherical nebula. Phil. Trans. Roy. Soc. London 1902; A199:1.
2. Chandrasekhar S. Hydrodynamic and hydromagnetic stability. Oxford Clarendon Press; 1961.

3. Alfven H. On the origin of the solar system. Oxford Clarendon Press. 1954;B.
4. Lehnert B. Magnetohydrodynamic waves under the action of the coriolis force. Astrophys. J. 1954;119:647.
5. Lehnert B. Magnetohydrodynamic waves under the action of the coriolis force II. Astrophys. J. 1955;121:481.
6. Lehnert B. On the equilibrium and stability of rotating high-beta plasmas cosmic electrodynamics. 1970;397:1.
7. Cadez VM. Gravitational instability of a homogeneous gas cloud with radiation. Astron. Astrophys. 1990;235:1.
8. Ali A, Bhatia PK. Instability of a gravitating partially ionized plasma. Astrophys. Space. Sci. 1992;195:38.
9. Bhatia PK, Hazarika ABR. Gravitational instability of partially ionized plasma in an oblique magnetic field. Physica Scripta. 1995;51:775–779.
10. Lima JAS, Silva R, Santos J. Jeans' gravitational instability and nonextensive kinetic theory. Astron. Astrophys. 2002; 396:309.
11. Sheikh S, Khan A, Bhatia PK. Thermally conducting partially ionized plasma in a variable magnetic field. Plasma Physics. 2007;47:147.
12. Oberoi C. Electron-inertia effects on the transverse gravitational instability. J. Plasma Fusion Res. 2009;8.
13. Cohen EGD. Statistical mechanics and its applications statistics and dynamics. Physica A. 2002;305:19.
14. Bhatia PK. Gravitational instability of a rotating anisotropic plasma. Physics. of Fluids.1967;10:1652-1653.
15. Herrengger FJ. Effects of collisions and gyroviscosity on gravitational instability in a two-component plasma. J. Plasma Physics.1972;8:393-400.
16. Lehnert B. Magnetohydrodynamic waves under the action of the coriolis force. Astrophys. J. 1954;119:647-654.
17. Mattei G. Wave propagation and instabilities in a rotating anisotropic plasma. Meccanica. 1968;3:214–230.
18. Scanlon JW, Segel LA. Some effects of suspended particles on the onset of Bénard convection. Phys. Fluids. 1973;16: 1573.
19. Sharma RC, Sharma KC. Suspended particles and the gravitational instability of a rotating plasma. Astrophys. Space Sci. 1979;71:325.
20. Kumar P. Hall current effect on thermal instability of compressible viscoelastic dusty fluid in porous medium. Heat Transfer Research. 2012;43:167-185.
21. Kumar P, Singh M. Instability of two rotating viscoelastic (walters B') superposed fluids with suspended particles in porous medium. J. Thermal Sci. 2007; 11:93-102.
22. Lapwood ER. Convection of a fluid in a porous medium. Proc. Camb. Phil. Soc. 1948;44:508-521.
23. Kumar P, Singh M. Hydrodynamic and hydromagnetic stability of viscous-viscoelastic superposed fluids in presence of suspended particles. Phys. Sep. Sci. Eng. 2007;10:1155.
24. Kumar P, Lal R, Singh GJ. Thermal instability of walters B' viscoelastic fluid permeated with suspended particles in hydromagnetics in porous medium. J. Ther. Sci. Yugoslavia. 2004;8:51-61.
25. Pensia RK. Patidar AK. Shrivastava V. Kumar V, Prajapat V. The role of coriolis force and suspended particles in the fragmentation of matter in the central region of galaxy. Journal of Bangladesh Academy of Sciences. 2012;36:1-12.



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