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Quantum Corrections to the Newton's Law from the Galilean Limit of Quantum Gravity

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In this work the author derives the Galilean limit of the quantum gravity obtained by using the hydrodynamic approach. The result shows that the quantum interaction generates, in the limit of weak gravity, a non-zero contribution. The paper derives the small deviation from the Newtonian law due to the quantum gravity and analyzes the experimental features to validate the theoretical model. The work also shows that in the frame of the quantum gravity the equivalence principle between the inertial and gravitational mass can be violated in very extreme conditions.

Keywords: Newton law; cosmological constant; quantum gravity; Galilean gravity; dark energy; motion of galaxies.

1. INTRODUCTION

One of the main problems of the quantum gravity is to produce theoretical outputs that lead to experimental confirmation or to cosmological evidence [1-8].

The real difficulty is to find the physical ambit where the quantum mechanics and the gravitational effects are contemporaneously important. This happens because the general relativity and the quantum mechanics

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describe events of different physical scale.

The Planck scale is one of the possible contexts where they become physically coupled and it offers a possibility of investigation.

By using the quantum-gravitational equations (QGEs) obtained with the help of the hydrodynamic quantum formalism, the author showed [9] that the quantum effects play an important role in the formation of a very small black hole since the spreading of the quantum wave opposes itself to the gravitational collapse by generating a repulsive force. This fact, in the case of a very small quantity of mass (below the Planck mass) prevents the formation of the black hole [9].

Another measurable output that can come from the quantum gravity is the detailed behavior of the gravitational field of antimatter. Many and discordant are the hypotheses on the gravitational features of the antimatter [10-14] and they cannot be resolved without a defined set of quantum-gravitational equations. Actually, Cabbolet [5] claims that the CPT symmetry is incompatible with the matter-antimatter gravitational repulsion, while Villata and the author himself [15-17] show that the CPT agrees with anti-gravity.

In this paper the author derives the weak limit of the quantum gravity, compares it with the Newton law and gives physical outputs that can be experimentally verified.

The paper is organized as follows: in the section 2 the equations of the hydrodynamic QGEs are resumed; in the section 3 the Galilean limit is calculated for the gravitational potential of a particle with a pseudo-Gaussian localization; in section 4 the new features of the weak quantum gravity are discussed.

2. THE QUANTUM GRAVITATIONAL EQUATION DERIVED VIA THE HYDRODYNAMIC APPROACH

In preceding papers [9,17-18] the author has shown that the quantum-gravitational equation (QGE) in the form of the Einstein one

$$R_{\nu\mu} - \frac{1}{2} g_{\nu\mu} R_{\alpha}{}^{\alpha} = \frac{8\pi G}{c^4}\left(T_{\nu\mu} + \Lambda g_{\mu\nu}\right) \quad (1)$$

can be obtained with the help of the hydrodynamic representation of quantum mechanics [9,18], where

$$\Lambda = (\pm) - \frac{m|\psi|^2 c^2}{\gamma}, \quad (2)$$

(the minus sign refers to antimatter) and

$$T_{\mu\nu} = T_{\mu}{}^{\gamma} g_{\gamma\nu} = |\psi|^2 \mathrm{T}_{\mu}{}^{\gamma} g_{\gamma\nu} \quad , \quad (3)$$

where the quantum energy-momentum tensor (QEMT) $\mathrm{T}_{\mu}{}^{\nu}$ is defined below by (7).

The hydrodynamic representation of quantum mechanics (HQM) [9,17-19], describes the quantum evolution of a mass distribution $\rho = |\psi|^2$ by using a self-interaction (the so called quantum-potential V_{qu} given by (8) [18-20]) leading to the mass motion with a moment $p_{\mu} = (\frac{E}{c}, -p_i) = -\frac{\partial S}{\partial q^{\mu}}$, where S is the action of the mass distribution described by the wave function

$$\psi = |\psi| \exp[\frac{iS}{\hbar}] \ . \quad (4)$$

The hydrodynamic quantum equations are described in the Euclidean space by the equatin of motion [9]

$$(\pm) - \frac{mc^2}{\gamma}\sqrt{1 - \frac{V_{qu}}{mc^2}}\frac{du_{\mu}}{ds} = \frac{\partial \mathrm{T}_{\mu}{}^{\nu}}{\partial q^{\nu}} \quad (5)$$

coupled to the continuity equation of the mass

$$\frac{\partial}{\partial q_{\mu}}\left(|\psi|^2 \frac{\partial S}{\partial q^{\mu}}\right) = -\frac{\partial J_{\mu}}{\partial q_{\mu}} = 0 \quad (6)$$

where $\gamma = \left(\sqrt{1 - \frac{\dot{q}_i \dot{q}^i}{c^2}}\right)^{-1}$, where $J_{\mu} = (c\rho, -J_i) = \frac{i\hbar}{2m}(\psi^* \frac{\partial \psi}{\partial q^{\mu}} - \psi \frac{\partial \psi^*}{\partial q^{\mu}})$ is the 4-current (mJ_i is the mass current and $m\rho$ is the mass density) where the QEMT $\mathrm{T}_{\mu}^{\ \nu}$ reads [9,18-19]

$$\mathrm{T}_{\mu}^{\ \nu} = (\pm) - \frac{mc^2}{\gamma}\sqrt{1-\frac{V_{qu}}{mc^2}}\left(u_{\mu}u^{\nu} - \delta_{\mu}^{\ \nu}\right) \quad (7)$$

and where the quantum potential V_{qu} reads

$$V_{qu} = -\frac{\hbar^2}{m}\frac{\partial_{\mu}\partial^{\mu}|\psi|}{|\psi|}\,. \quad (8)$$

where, in the hydrodynamic notation, the 4-velocity reads [9,18-19]

$$u_{\mu} = \left(1-\frac{V_{qu}}{mc^2}\right)^{-1/2}\frac{p_{\mu}}{mc}\,. \quad (9)$$

It is worth noticing that in the classical limit (i.e., $\hbar \to 0$) the right hand part of (1) leads to the classical energy-momentum density tensor so that the Einstein equation is obtained.

The HQM in non-euclidean space, coupled to the Einstein equation (1), is given in appendix.

3. THE GALILEAN LIMIT OF QUANTUM GRAVITY

By using the quantum energy-momentum density tensor (QEMDT) (3) that, for a scalar, uncharged, particle reads [19]

$$T_{\mu\nu\,\pm} = (\pm) - \frac{m|\psi_{\pm}|^2 c^2}{\gamma}\sqrt{1-\frac{V_{qu}}{mc^2}}\left(u_{\mu}u_{\nu} - g_{\mu\nu}\right)\,. \quad (10)$$

where ψ_{+} and ψ_{-} are the wave functions of the matter and antimatter, respectively, the QGE for particles read [9-17]

$$R_{\nu\mu} - \frac{1}{2}g_{\nu\mu}R_{\alpha}^{\ \alpha} = \frac{8\pi G}{c^4}\left(T_{\nu\mu +} - \frac{m|\psi_{+}|^2 c^2}{\gamma}g_{\mu\nu}\right) \quad (11)$$

By introducing, the Galilean limit (i.e., low energy and low velocity limit: $u_{\mu} = (u_0, -u_i) \cong (1,(0,0,0))$ and

$$\frac{V_{qu}}{mc^2} \ll 1 \quad (12)$$

in (11), it follows that [17,23]

$$R_0^{\ 0} = \frac{4\pi Gm}{c^2}|\psi|^2\sqrt{1-\frac{V_{qu}}{mc^2}}\left[\left(2u_0u^0 - 1\right) + 1 - \frac{1}{\sqrt{1-\frac{V_{qu}}{mc^2}}}\right]$$
$$= \frac{4\pi Gm}{c^2}\sqrt{1-\frac{V_{qu}}{mc^2}}\,|\psi|^2\left[2 - \frac{1}{\sqrt{1-\frac{V_{qu}}{mc^2}}}\right] \quad (13)$$

where *G* is the gravitational constant.

Furthermore, following the procedure in ref [23], the dependence of φ from the component g_{00} of the metric tensor reads

$$g_{00} \cong 1 + \frac{2\varphi}{c^2}\,, \quad (14)$$

whose trace can be approximated as

$$g_{\alpha\alpha} \cong -2\,. \quad (15)$$

Moreover, since in the Galilean limit it holds

$$R_0^{\ 0} = R_{00} = \frac{\partial \Gamma^{\alpha}_{\ 00}}{\partial q^{\alpha}} \approx -\frac{1}{2}\frac{\partial g_{\gamma\gamma}\frac{\partial g_{00}}{\partial q^{\alpha}}}{\partial q^{\alpha}} = \frac{1}{c^2}\frac{\partial}{\partial q^{\alpha}}\frac{\partial \varphi}{\partial q^{\alpha}} \quad (16)$$

it follows that

$$\frac{\partial}{\partial q^{\alpha}}\frac{\partial \varphi}{\partial q^{\alpha}} = 4\pi Gm|\psi|^2\left[2\sqrt{1-\frac{V_{qu}}{mc^2}} - 1\right]. \quad (17)$$

The deviation from the Newton's law of (17) is due to the contribution coming from quantum potential V_{qu} that in the classical limit is null [24]. Usually the classical condition (i.e., equation (8) put equal to zero) is heuristically assumed for macroscopic large masses (compared to the value of the Plank constant). Nevertheless, since macroscopic bodies are actually granular (i.e., made of atoms of elementary particles of very small masses) a more mathematically correct condition has to be assumed.

Actually, the classical properties come from the fact that the quantum potential has a finite range of interaction [9,24-28] and it goes to zero at infinity so that, when the mean particle distance is larger than the range of quantum potential, they are quantum decoupled and lead to a classical matter phase [24-28].

More precisely, it is possible to show that for pseudo-Gaussian particles [24-28] with tails that go to zero more slowly than the Gaussian law does, in the large-scale physics the classical macroscopic behavior is warranted if the condition

$$lim_{q\to\infty} V_{qu} = -\frac{\hbar^2}{m}\frac{\partial_\mu \partial^\mu |\psi|}{|\psi|} = 0 \qquad (18)$$

is satisfied. It is noteworthy to mention that the condition (18) is not fulfilled by linear systems, given that in this case the quantum potential is quadratic and diverges at infinity [24-27].

Since the Newtonian gravity is the weak limit of the *classical* general relativity, we have to warrant the classical limit (18) in order to retrieve the Newtonian gravity in the more general quantum gravity.

To this end, we consider the case of pseudo-Gaussian particles localized in a sphere of radius ΔR located in R with spatial densities of type

$$|\psi_{(r)}|^2 = \mathrm{n}_0\, exp[-\frac{(r-R)^2}{2\Delta R^2[1+[\frac{(r-R)^2}{\Delta^2 f(r-R)}]]}] \qquad (19)$$

where $f(r-R)$ is an appropriate regular (smoothly varying) function that obeys the conditions

1) $f(0)=1$ (20)

2) $lim_{|r-R|\to\infty} f(r-R) << \frac{(r-R)^2}{\Delta^2}$. (21)

where Δ is the Gaussian range of the particle and where, for $\Delta > \Delta R$, the normalization condition leads to

$$\mathrm{n}_0 \cong \frac{1}{(2\pi)^{3/2}\Delta R^3} = \frac{\sqrt{2}}{3\pi^{1/2} V} \qquad (22)$$

where

$$V = \frac{4}{3}\pi \Delta R^3 . \qquad (23)$$

For small distances, such as $(r-R)^2 << \Delta^2$, $|\psi|^2$ is physically indistinguishable from the Gaussian behavior

$$|\psi|^2 = \mathrm{n}_0\, exp[-\frac{(r-R)^2}{2\Delta R^2}] , \qquad (24)$$

while for large distances such as $|r-R| \gg \Delta$, we obtain the behavior

$$lim_{|r-R|\to\infty} |\psi|^2 = \mathrm{n}_0\, exp[-\frac{\Delta^2 f(r-R)}{2\Delta R^2}] \qquad (25)$$

In order to give outputs connected to a real physical case, we consider, in the following, the large-distance behavior of the wave function [28,29]

$$lim_{|r'|\to\infty} |\psi| \propto \frac{a^{-1/2}}{r'^m} \qquad (26)$$

Where

$$a \sim \left(\mathrm{n}_0 \Delta R^{2m}\right)^{-1} = \frac{\pi^{3/2}}{\Delta R^{2m-3}} \qquad (27)$$

that is obtained by posing

$$f(r') = 1 + ln\left(1 + \frac{r'^{2m}}{\Delta R^{2m}}\right), \qquad (28)$$

$$\Delta = \sqrt{2}\Delta R , \qquad (29)$$

where $r' = (r-R)$, and where $m=2$ in order to have a normalizable wave function squared modulus in 3-d space.

3.1 Galilean Gravitational Potential

By using equation (17), in the Galilean limit, we can write

$$\frac{\partial}{\partial q^{\alpha}}\frac{\partial \varphi}{\partial q^{\alpha}} \cong 4\pi G m |\psi|^2 \left(1-\frac{V_{qu}}{mc^2}\right) \qquad (30)$$

that, for radially symmetric mass distribution (19) and its associated quantum potential

$$\frac{V_{qu}}{mc^2} = -\frac{\hbar^2}{m^2c^2}\frac{\partial_\mu \partial^\mu |\psi|}{|\psi|} = -\frac{\hbar^2}{m^2c^2}\frac{1}{|\psi|}\frac{1}{r'^2}\frac{\partial}{\partial r'}\left(r'^2 \frac{\partial |\psi|}{\partial r'}\right)$$
$$= -\frac{\hbar^2}{m^2c^2}\left[\left(\frac{\partial H}{\partial r'}\right)^2 + \frac{2}{r'}\frac{\partial H}{\partial r'} + \frac{\partial^2 H}{\partial r'^2}\right] \qquad (31)$$

where

$$H = -\frac{r'^2}{4\Delta R^2 \left[1+\frac{1}{\Delta^2}\frac{r'^2}{1+ln\left(1+\frac{r'^4}{\Delta R^4}\right)}\right]} = ln\frac{|\psi|}{\mathrm{n}_0} . \qquad (32)$$

(and where $\frac{\partial H}{\partial r'}$ and $\frac{\partial^2 H}{\partial r'^2}$ are given in ref. [30]) can be integrated to lead to

$$\frac{\partial \varphi}{\partial r} = \frac{Gm}{r^2}\int_0^r |\psi|^2 \left(1-\frac{V_{qu}}{mc^2}\right) dr' . \qquad (33)$$

Thence, from (30,33) it follows that

$$\frac{\partial}{\partial r'^{\alpha}}\frac{\partial \varphi}{\partial r'^{\alpha}} = \left(\mathrm{n}_0\, exp[\,2H\,]\left[1+\frac{\hbar^2}{m^2c^2}\left(\left(\frac{\partial H}{\partial r'}\right)^2 + \frac{2}{r'}\frac{\partial H}{\partial r'} + \frac{\partial^2 H}{\partial r'^2}\right)\right]\right) \qquad (34)$$

and that

$$\frac{\partial \varphi}{\partial r} = \frac{Gm}{r^2}\int_0^r \mathrm{n}_0\, exp[\,2H\,]\left(1+\frac{\hbar^2}{m^2c^2}\left(\left(\frac{\partial H}{\partial r'}\right)^2 + \frac{2}{r'}\frac{\partial H}{\partial r'} + \frac{\partial^2 H}{\partial r'^2}\right)\right) dr' . \qquad (35)$$

The expression (35) can be split in a Newtonian part and in a quantum-gravitational one such as

$$\frac{\partial \varphi}{\partial r} = \frac{\partial \varphi_N}{\partial r} + \frac{\partial \varphi_q}{\partial r} , \qquad (36)$$

Where

$$\frac{\partial \varphi_N}{\partial r} = \frac{Gm}{r^2}\int_0^r \mathrm{n}_0\, exp[\,2H\,]\, dr \qquad (37)$$

is the classical Newtonian term, derived from the mass distribution of the particle, and where

$$\frac{\partial \varphi_q}{\partial r} = \frac{Gm}{r^2}\int_0^r 2\mathrm{n}_0\, exp[\,2H\,]\left(\frac{\hbar^2}{2m^2c^2}\left[\left(\frac{\partial H}{\partial r'}\right)^2 + \frac{2}{r'}\frac{\partial H}{\partial r'} + \frac{\partial^2 H}{\partial r'^2}\right]\right) d^3 r' \qquad (38)$$

is the quantum-gravitational contribution.

3.1.1 Large-distance GQG

In order to evaluate the gravitational potential at large distance $r' \gg \Delta R$ we derive the limiting expression of the modulus of the wave function and the quantum potential that read

$$\begin{aligned} lim_{\frac{r'}{\Delta R}>>1} |\psi|^2 = lim_{\frac{r'}{\Delta R}>>1} \mathrm{n}_0 \, exp[\,2H\,] = \mathrm{n}_0 \, exp[-\frac{\Delta^2}{2\Delta R^2} ln\left(\frac{r'^4}{\Delta R^4}\right)] \\ = \mathrm{n}_0 \left(\frac{r'^4}{\Delta R^4}\right)^{-\frac{\Delta^2}{2\Delta R^2}} \cong \mathrm{n}_0 \left(\frac{r'^4}{\Delta R^4}\right)^{-1} \end{aligned} \tag{39}$$

$$lim_{\frac{r'}{\Delta R}>>1} \frac{\partial H}{\partial r'} = -\frac{\Delta^2}{\Delta R^2}\frac{1}{r'} = -\frac{2}{r'} \tag{40}$$

$$lim_{\frac{r'}{\Delta R}>>1} \frac{\partial^2 H}{\partial r'^2} = \frac{\Delta^2}{\Delta R^2}\frac{1}{r'^2} = \frac{2}{r'^2} \tag{41}$$

$$\begin{aligned} lim_{\frac{r'}{\Delta R}>>1} \frac{V_{qu}}{mc^2} = -\frac{\hbar^2}{m^2c^2} lim_{\frac{r'}{\Delta R}>>1} \left[\left(\frac{\partial H}{\partial r'}\right)^2 + \frac{2}{r'}\frac{\partial H}{\partial r'} + \frac{\partial^2 H}{\partial r'^2}\right] \\ = -\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\left[\left(\frac{\Delta^2}{\Delta R^2}\right)^2 - \frac{\Delta^2}{\Delta R^2}\right] = -2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2} \end{aligned} \tag{42}$$

leading to the Laplacian of the gravitational potential

$$\frac{\partial}{\partial r'^{\alpha}}\frac{\partial \varphi}{\partial r'^{\alpha}} = 4\pi G m |\psi|^2 \left[1 + 2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\right] \tag{43}$$

The fact that the mass distribution $|\psi|^2$ imperceptibly deviates by the Gaussian shape (24) is not significant at large distance. Thence, (as is usually done for a Gaussian mass distribution) it can be approximated by a delta-like distribution (see (67) in section 3.4).

Moreover, since almost all of the particle mass is contained in the “Gaussian core” (a sphere of radius few times ΔR) by posing $r' = \alpha\Delta R$ with $\alpha > 1$ we can write

$$lim_{\frac{\Delta R^2}{r'^2}\to 0} \frac{\partial}{\partial r'^{\alpha}}\frac{\partial \varphi}{\partial r'^{\alpha}} = 4\pi G m \begin{pmatrix} \Theta_{(\alpha\Delta R - |r'|)} |\psi|^2 \left(1 - \frac{V_{qu}}{mc^2}\right) \\ + \Theta_{(|r'| - \alpha\Delta R)} f_{(\alpha\Delta R)} \frac{\Delta R}{\pi^{3/2} r'^4}\left[1 + 2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\right] \end{pmatrix} \tag{44}$$

(where $\Theta_{(x)} = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$) that, being both the mass distribution and the quantum potential central-symmetric, leads to

$$\frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2} \left(\begin{array}{l} 4\pi \int_0^{\alpha \Delta R} |\psi|^2 \left(1 - \frac{V_{qu}}{mc^2}\right) r'^2 \, dr' \\ + \int_{\alpha \Delta R}^{r'} f_{(\alpha \Delta R)} \frac{4 \Delta R}{\pi^{1/2} r'^2} \left[1 + 2 \frac{\hbar^2}{m^2 c^2} \frac{1}{r'^2}\right] dr' \end{array} \right) . \quad (45)$$

Moreover, by using for $r' \leq \alpha \Delta R$ the approximation (24) we can write

$$\frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2} \left(\begin{array}{l} 4\pi \int_0^{\alpha \Delta R} \left[\begin{array}{l} \mathrm{n}_0 \, exp[-\frac{r'^2}{2 \Delta R^2}] \\ + \left| \left(|\psi|^2 - \mathrm{n}_0 \, exp[-\frac{(r-R)^2}{2 \Delta R^2}] \right) \right. \end{array} \right] \left(1 - \frac{V_{qu}}{mc^2}\right) r'^2 \, dr' \\ + \int_{\alpha \Delta R}^{r'} f_{(\alpha \Delta R)} \frac{4 \Delta R}{\pi^{1/2} r'^2} \left[1 + 2 \frac{\hbar^2}{m^2 c^2} \frac{1}{r'^2}\right] dr' \end{array} \right) \quad (46)$$

that, since $|\psi|^2 - \mathrm{n}_0 \, exp[-\frac{r'^2}{2 \Delta R^2}] \cong 0$ for $r' < \alpha \Delta R$, leads to

$$\frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2} \left(\begin{array}{l} 4\pi \int_0^{\alpha \Delta R} \mathrm{n}_0 \, exp[-\frac{r'^2}{2 \Delta R^2}] \left(1 - \frac{V_{qu}}{mc^2}\right) r'^2 \, dr' \\ + \int_{\alpha \Delta R}^{r'} f_{(\alpha \Delta R)} \frac{4 \Delta R}{\pi^{1/2} r'^2} \left[1 + 2 \frac{\hbar^2}{m^2 c^2} \frac{1}{r'^2}\right] dr' \end{array} \right) \quad (47)$$

and, on macroscopic distances (i.e., $\frac{\Delta R}{r'} \to 0$), to

$$lim_{\frac{\Delta R^2}{r'^2} \to 0} \frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2} \left(\begin{array}{l} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \frac{\Delta m}{m} \ \delta_{3(\mathbf{r}')} \left(1 - \frac{V_{qu}}{mc^2}\right) d^3 \mathbf{r}' \\ + \int_{\alpha \Delta R}^{r'} f_{(\alpha \Delta R)} \frac{4 \Delta R}{\pi^{1/2} r'^2} \left[1 + 2 \frac{\hbar^2}{m^2 c^2} \frac{1}{r'^2}\right] dr' \end{array} \right) \quad (48)$$

where

$$\Delta m = 4\pi \int_0^{\alpha \Delta R} |\psi|^2 \, r'^2 \, dr' \cong m \quad (49)$$

is the mass contained in the sphere of radius $r' = \alpha \Delta R$. Moreover, by writing

$$lim_{\frac{\Delta R^2}{r'^2}\to 0} \frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2}\left(\begin{array}{l} \int_0^\infty\int_0^\infty\int_0^\infty \delta_{3(\mathbf{r}')}\left(1-\frac{V_{qu}}{mc^2}\right)d^3\mathbf{r}' - \int_0^\infty\int_0^\infty\int_0^\infty (1-\frac{\Delta m}{m})\delta_{3(\mathbf{r}')}\left(1-\frac{V_{qu}}{mc^2}\right)d^3\mathbf{r}' \\ +\int_{\alpha\Delta R}^{\infty} f_{(\alpha\Delta R)}\frac{4\Delta R}{\pi^{1/2}r'^2}\left[1+2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\right]dr' \\ +\int_{\infty}^{r'} f_{(\alpha\Delta R)}\frac{4\Delta R}{\pi^{1/2}r'^2}\left[1+2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\right]dr' \end{array} \right) \quad (50)$$

and given that for $r' \to \infty$ and $\hbar = 0$ (see section 3.4) the Newton law must be obtained, it follows that

$$-\int_0^\infty\int_0^\infty\int_0^\infty (1-\frac{\Delta m}{m})\delta_{3(\mathbf{r}')}d^3\mathbf{r}' + \int_{\alpha\Delta R}^{\infty} f_{(\alpha\Delta R)}\frac{4\Delta R}{\pi^{1/2}r'^2}dr' = 0 \; . \quad (51)$$

Hence, by (12) (being $r' \gg \Delta R \gg l_c$) that

$$-\int_0^\infty\int_0^\infty\int_0^\infty (1-\frac{\Delta m}{m})\delta_{3(\mathbf{r}')}\left(1-\frac{V_{qu}}{mc^2}\right)d^3\mathbf{r}' + \int_{\alpha\Delta R}^{\infty} f_{(\alpha\Delta R)}\frac{4\Delta R}{\pi^{1/2}r'^2}\left[1+2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\right]dr' \cong 0 \, , \quad (52)$$

we finally obtain

$$lim_{\frac{\Delta R^2}{r'^2}\to 0} \frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2}\left(\begin{array}{l} \int_0^\infty\int_0^\infty\int_0^\infty \delta_{3(\mathbf{r}')}\left(1-\frac{V_{qu}}{mc^2}\right)d^3\mathbf{r}' \\ +\int_{\infty}^{r'} f_{(\alpha\Delta R)}\frac{4\Delta R}{\pi^{1/2}r'^2}\left[1+2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\right]dr' \end{array} \right) \; . \quad (53)$$

This leads to

$$\frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2}\left(\begin{array}{l} \left(1-\frac{V_{qu(0)}}{mc^2}\right) + \\ +\int_{\infty}^{r'} f_{(\alpha\Delta R)}\frac{4\Delta R}{\pi^{1/2}r'^2}\left[1+2\frac{\hbar^2}{m^2c^2}\frac{1}{r'^2}\right]dr' \end{array} \right) \quad (54)$$

where

$$\begin{aligned} f_{(\alpha\Delta R)} &= \pi^{-1/2}\alpha^4\Delta R^3 \,|\psi|^2_{(r'=\alpha\Delta R)} \\ &= \frac{\alpha^4}{2\sqrt{2}\pi^2} exp[-\frac{\alpha^2}{2[1+[\frac{\alpha^2}{2\left(1+ln\left(1+ln\,\alpha^4\right)\right)}]]}] \\ &\cong \frac{\alpha^4}{2\sqrt{2}\pi^2} exp[-\frac{\alpha^2}{2}\left(\frac{1+\frac{\alpha^2}{2}}{\left(1+ln\left(1+ln\,\alpha^4\right)\right)}\right)] \end{aligned} \quad (55)$$

warrants that $|\psi|^2$ is continuous at $r' = \alpha\Delta R$.

In order that (55) furnishes the best value of the parameter f for the approximated formulas (44,54) we must require that for $r' < \alpha\Delta R$ the mass density distribution $|\psi|^2$ is well approximated by the Gaussian behavior (24) while for $r' > \alpha\Delta R$ it is well approximated by the hyperbolic behavior (26). The search for best value of α, that can be found by a least-mean squares procedure, leads to $\alpha \sim 2$ and hence to

$$f \sim 0{,}1 \quad (56)$$

Moreover, applying the condition

$$r' \gg \Delta R \gg \frac{\hbar}{mc} \quad (57)$$

at lowest order in r^{-1}, it follows that

$$\frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2}\left(\left(1-\frac{V_{qu(0)}}{mc^2}\right)+ \int_{\infty}^{r'} f_{(\alpha\Delta R)} \frac{4\Delta R}{\pi^{1/2} r'^2} dr'\right). \quad (58)$$

and that

$$\frac{\partial \varphi}{\partial r'} \cong \frac{Gm}{r'^2}\left(\left(1-\frac{V_{qu(0)}}{mc^2}\right)-\frac{4f}{\pi^{1/2}}\frac{\Delta R}{r}\right) \qquad r > \Delta R; \quad (59)$$

that, since

$$\left(1-\frac{V_{qu(0)}}{mc^2}\right) \cong 1-\frac{3l_c^2}{2\Delta R^2}, \quad (60)$$

leads to

$$\varphi = -\frac{Gm}{r}\left(\left(1-\frac{3l_c^2}{2\Delta R^2}\right)-\varepsilon\frac{\Delta R}{r}\right), \quad (61)$$

where $\varepsilon \cong \frac{4f}{\pi^{1/2}} \cong 0{,}23$, and, finally, to

$$lim_{\frac{\Delta R}{l_c}\to\infty} \varphi = -\frac{Gm}{r}\left(1-\varepsilon\frac{\Delta R}{r}\right). \quad (62)$$

Expression (62) contains a small repulsive quantum contribution that goes like $\propto \frac{1}{r^2}$ in addition to the Newtonian potential.

From (62) we observe that, for highly localized particles for which we have for instance

$$\frac{V_{qu}}{mc^2} = \frac{3l_c^2}{2\Delta R^2} \sim 0{,}1, \quad (63)$$

the gravitational potential at large distance does not converge to the Newtonian one and reads

$$\varphi = -\frac{Gm}{r}\left(1-\frac{3l_c^2}{2\Delta R^2}\right) \quad (64)$$

where the term $m\left(1-\frac{3l_c^2}{2\Delta R^2}\right)$, basically, is the gravitational mass of the particle, showing that, in the frame of the quantum gravity, the breaking of the equivalence principle, between the inertial and gravitational mass may happen for very localized particles.

3.2 The Classical Limit

In the classical limit, that is obtained for $\frac{l_c}{r} \to 0$ and $\frac{\Delta R}{r} \to 0$, the identity (42) leads to

$$lim_{l_c\to 0} \frac{V_{qu}}{mc^2} = -2\left(\frac{l_c}{r}\right)^2 = 0 \quad (65)$$

and to

$$lim_{l_c\to 0} \frac{\partial V_{qu}}{\partial r'} = 4\left(\frac{l_c^2}{r^3}\right) = 0, \quad (66)$$

while (19) leads to the mass density distribution on the large-scale that reads

$$lim_{\Delta R^2 \to 0} |\psi|^2 = lim_{\Delta R^2 \to 0} \frac{1}{\sqrt{(2\pi)^3}\Delta R^3} exp[-\frac{r'^2}{[2\Delta R^2 + \frac{r'^2}{1+ln\left(1+\frac{r'^4}{\Delta R^4}\right)}]}] \quad (67)$$

$$= lim_{\Delta R^2 \to 0} \frac{1}{\sqrt{(2\pi)^3}\Delta R^3} exp[-\frac{r'^2}{2\Delta R^2}] \cong \delta(r')$$

that finally leads to

$$\frac{\partial}{\partial r^\alpha}\frac{\partial \varphi_q}{\partial r^\alpha} = 4\pi G m \delta(r) \quad (68)$$

and, by integrating, to

$$\varphi = -\frac{Gm}{r} \quad (69)$$

that represents the classical Newtonian law.

3.3 Weak Quantum-gravity in a Fluctuating Environment

If we consider the gravitational potential in an open environment, we must take into account for the spontaneous localization of particles due to thermal fluctuations [24-28].

The quantum coherence in a fluctuating environment is maintained up to distances of order of the De Broglie thermal length λ_c [24-28,31,32] and it is possible to show [24-28], for particles interacting through a weak potential, that the quantum localization $2\Delta R$ is of order of

$$2\Delta R \le \lambda_c = 4\pi \frac{\hbar}{\sqrt{mkT}} = 4\pi l_c \sqrt{\frac{mc^2}{kT}}, \quad (70)$$

and that the upper limit of the spontaneous enlargement of a pseudo-Gaussian wave packet when subject to external fluctuations (stationary condition at infinite time) is [24-28]

$$Max\{2\Delta R\} \approx 4\pi \frac{\hbar}{\sqrt{mkT}} = 4\pi l_c \sqrt{\frac{mc^2}{kT}} \quad (71)$$

Moreover, by the indetermination principle that requires $2\Delta R \Delta p \ge \hbar$, where for the thermal fluctuations $\Delta p = \sqrt{2mkT}$, the minimum limit for ΔR [31] is

$$\Delta R \ge \pi \frac{\hbar}{\sqrt{2mkT}}, \quad (72)$$

that, for $T \ll T_c = 4\pi^2 \frac{mc^2}{k} = m\ x\ 2{,}5x10^{45}\ {}^\circ K$, leads to

$$\Delta R \gg l_c. \quad (73)$$

Condition (73) is practically always fulfilled for any physically attainable temperature. In fact, we have that $T_c \cong 3{,}5x10^{18}\ {}^\circ K$ for a proton, $T_c \cong 2{,}2x10^{15}\ {}^\circ K$ for an electron and $T_c \cong 3x10^{11}\ {}^\circ K$ for a neutrino.

3.3.1 Weak gravity at large-distance in a fluctuating environment

In this case, from (61,71) the gravitational potential reads

$$\varphi \cong -\frac{Gm}{r}\left(\left(1-\frac{3kT}{8\pi^2 mc^2}\right) - 2\pi\varepsilon \frac{\hbar}{\sqrt{mkT}}\frac{1}{r'}\right) \quad (74)$$

that, for $kT \ll mc^2$ (i.e., $l_c \ll \Delta R$), leads to

$$\varphi \cong -\frac{Gm}{r'}\left(1 - 2\pi\varepsilon \frac{\hbar}{\sqrt{mkT}}\frac{1}{r'}\right).$$

$$r' \gg 2\pi \frac{\hbar}{\sqrt{mkT}} \quad (75)$$

Table 1. Values of $\frac{kT}{mc^2}$ and $\Delta R \cong 2\pi \frac{\hbar}{\sqrt{mkT}}$ for various particles

	m **(Kg)**	$l_c = \frac{\hbar}{mc}$ **(m)**	$\frac{kT}{mc^2}$	$\Delta R \cong 2\pi \frac{\hbar}{\sqrt{mkT}}$ **(m)**
Proton	1,6726 x 10^{-27}	2 x 10^{-18}	T x 2,7x 10^{-11}	$\frac{1}{\sqrt{T}}$ x 4,4x 10^{-9}
Neutron	1,6749 x 10^{-27}	2 x 10^{-18}	T x 2,7x 10^{-11}	$\frac{1}{\sqrt{T}}$ x 4,4x 10^{-9}
Electron	9,1 x 10^{-31}	3,5 x 10^{-15}	T x 4,7 x 10^{-8}	$\frac{1}{\sqrt{T}}$ x 1,8x 10^{-7}
Neutrino	1,2 x 10^{-35}	2,7 x 10^{-10}	T x 3,6 x 10^{-3}	$\frac{1}{\sqrt{T}}$ x 5 x 10^{-5}

In Table 1, for instance, the values of $\frac{kT}{mc^2}$ and $\Delta R \cong 2\pi \frac{\hbar}{\sqrt{mkT}}$ for various particles are reported.

4. CHARACTERISTICS OF THE GALILEAN QUANTUM-GRAVITY AND ITS EXPERIMENTAL VALIDATION

In order to build up experimental tests that can validate the theory, it is helpful to discuss some features of the Galilean quantum gravitational potential (GQGP) that are quite different from those of the classical limit.

The most important one is that, actually, the GQGP is not a real gravitational potential as we mean in the classical theory.

In fact, the GQGP has been derived (see section 3) just for an isolated particle and it is not independent of the test particle that has to be introduced in order to experience it.

This fact is a consequence of the presence of the quantum potential in the Ricci tensor component $R_0^{\ 0}$ (13) that changes with the evolution of the quantum wave function.

In the presence of another particle (e.g., the test one or a colliding one) due to their mutual interaction, the wave function undergoes the quantum coupled evolution so that the quantum potential, as well as the gravitational potential, are not assigned functions of the coordinates but change according to the quantum interaction of the particles (e.g., the radial symmetry of the quantum potential of a single particle (used to derive (31-35)) is not applicable).

On the other hand, if the distance of the particles r fulfils the condition $r \gg \Delta R$, for a pseudo-Gaussian distribution (19), the quantum potential reads

$$\lim_{\frac{r}{\Delta R}\to\infty, \frac{\Delta R}{l_c}\to\infty} \frac{V_{qu}}{mc^2} = -\frac{l_c^2}{\Delta R^2}\frac{2\Delta R^2}{r^2} = 0 \qquad (76)$$

and can be neglected under the spontaneous mass localization $\Delta R \cong 2\pi \frac{\hbar}{\sqrt{mkT}} \gg l_c$ due to the environmental fluctuations. In this case, it follows that the particles acquire the classical behavior and the gravitational potential becomes classic.

Generally speaking, we observe that the GQGP contains additional non-classic parameters, such as the localization ΔR and Compton's length of the particle, with respect to the classical Newtonian gravity.

In the case of a pseudo-Gaussian particle, the first order correction to the Newtonian potential is given in Table 2.

Among the cases in Table 2, we can retrieve the Newtonian law by imposing the classical conditions (i.e., $\hbar \to 0$ (and, hence, both $l_c \to 0$ and $\Delta R \to 0$ with $\frac{\Delta R}{l_c} = 2\pi\sqrt{\frac{mc^2}{kT}} \to \infty$) and $r \gg \Delta R$).

Table 2. Newton law, with first order quantum correction, for free particles (i.e., $lim \frac{\Delta R}{l_c} \to \infty$) and for highly localized particles ($\Delta R \sim l_c$)

	$\Delta R \sim l_c$	$lim \frac{\Delta R}{l_c} \to \infty$
$r \gg \Delta R$	$\varphi = -\frac{Gm}{r}\left(\left(1-\frac{3l_c^2}{2\Delta R^2}\right)-\varepsilon\frac{\Delta R}{r}\right)$	$\varphi = -\frac{Gm}{r}\left(1-\varepsilon\frac{\Delta R}{r}\right)$

Moreover, the localization of the particle is determined by the environmental fluctuations (i.e., $\Delta R \cong 2\pi\frac{\hbar}{\sqrt{mkT}}$), hence at first order it follows that

$$\varphi = -\frac{Gm}{r}\left(1-2\pi\varepsilon\frac{\hbar}{\sqrt{mkT}}\frac{1}{r}\right) \qquad (77)$$

with

$$r \gg 2\pi\frac{\hbar}{\sqrt{mkT}} \qquad (78)$$

Expression (77) has to be considered the generalization of the Newton law (at the first order in $\frac{\Delta R}{r}$) for particles with free quantum localization.

4.1 The Breaking of Equivalence Principle in Quantum-Gravity

From Table 2 we can see that for particles whose spatial localization is a few times its Compton's length (e.g., $\Delta R \sim 0{,}1\ l_c$) the Galilean limit of quantum gravity (64) breaks the classical principle of equivalence between the inertial and gravitational mass.

Even if it is very interesting to experimentally verify this theoretical output, such a goal faces the quite difficult problem of fixing the particle localization.

From a general point of view, the spatial localization can be achieved by the use of a physical potential (e.g., a square well potential of infinite height) or by modulating the amplitude of the stochastic noise [24,31].

If the first method can be used for particles that carry a (repulsive) force in addition to the gravitational one (e.g., the electromagnetic one) the weakness of the gravitational constant poses some difficulties in measuring the gravitational interaction.

On the other hand, for a particle sensible just to the gravitational force, its localization can only be achieved by modulating the thermal fluctuations of the vacuum [24,31].

Even if theoretically possible, the physical conditions are practically unrealizable even on cosmic scale. In fact, for instance, for the neutron a temperature of 10^{17} °K is required for the breaking of the equivalence principle (see table 1), while for the neutrino, the lowest critical temperature, is of order 10^{10} °K.

5. DISCUSSION AND CONCLUSION

The present work shows that the quantum gravitational effects, stemming from (2) that is able to justify the astronomical observations on the velocity of galaxies rotation [18], lead to corrections of the classical Newtonian gravity that in principle can be measured in a particle-particle interaction experiment on earth laboratories.

At very short distance the gravity force and the quantum mechanics are coupled to each other and the definition of a classic-type of potential is not possible.

At large distances, and in a fluctuating environment, the first order correction to the

Newtonian law is proportional to the ratio between the quantum localization of the particle and the distance from its position.

In freely localized particles, a slight temperature dependence of the gravitational potential is introduced by the first-order correction.

When the particle localization is very high, the quantum contribution becomes physically important: In this case, the theory output shows that the equivalence principle between the gravitational and inertial mass can be violated. The paper shows that this occurrence can happen when the particle localization is a small fraction of Compton's length.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

The quantum hydrodynamic description in non-euclidean space

Since equation (1) (containing the QEMDT (3)) determines the metric of the space, equations (5-8) or (10 -13) have to be generally written in the non-euclidean co-ordinates and they read [9]

$$\frac{du_\mu}{ds} - \frac{1}{2}\frac{\partial g_{\lambda\kappa}}{\partial q^\mu} u^\lambda u^\kappa = -u_\mu \frac{d}{ds}\left(ln\sqrt{1-\frac{V_{qu}}{mc^2}} \right) + \frac{\partial}{\partial q^\mu}\left(ln\sqrt{1-\frac{V_{qu}}{mc^2}} \right) \tag{A.1}$$

with the conservation equation that reads

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial q^\mu}\sqrt{-g}\left(g^{\mu\nu} \,|\psi|\, \frac{\partial S}{\partial q^\nu} \right) = 0 \tag{A.2}$$

where

$$V_{qu} = -\frac{\hbar^2}{m}\frac{1}{|\psi|\sqrt{-g}}\partial^\mu \sqrt{-g}\left(g^{\mu\nu}\partial_\nu |\psi| \right), \tag{A.3}$$

where $g_{\nu\mu}$ is the metric tensor and where $g = |g_{\nu\mu}|^{-1} = -J_{ac}{}^{-2}$, where J_{ac} is the Jacobian of the transformation of the Galilean co-ordinates to non-euclidean ones [23].

Connection with the standard quantum mechanics

It is noteworthy to observe that, due to the biunique relation between the quantum hydrodynamic approach and the standard quantum mechanics [20-22,24-26], the solutions of the coupled equations (1-3,5-8) are equivalent to the QGE (1) coupled (through (13)) to the Klein-Gordon equation that in non-euclidean space reads

$$\partial^\mu \psi_{;\mu} = \frac{1}{\sqrt{-g}}\partial^\mu \sqrt{-g}\left(g^{\mu\nu}\partial_\nu \psi \right) = -\frac{m^2c^2}{\hbar^2}\psi \tag{A.4}$$

Moreover, the fully independence of the quantum gravity equation by the hydrodynamic approach is achieved by deriving the quantum energy impulse tensor density (QEMDT) $T_{\mu\nu}$ (3) as a function of the wave function (as given in reference [19]) that reads

$$T_\mu{}^\nu = m|\psi|^2 c^2 \left(\frac{\frac{\hbar}{2im^2c^2}\partial\, ln[\frac{\psi}{\psi^*}]}{\partial t} \right)^{-1} \left(\left(\frac{\hbar}{2mc}\right)^2 \frac{\partial\, ln[\frac{\psi}{\psi^*}]}{\partial q^\mu}\frac{\partial\, ln[\frac{\psi}{\psi^*}]}{\partial q_\nu} + \left(1-\frac{V_{qu}}{mc^2}\right)\delta_\mu{}^\nu \right) \tag{A.5}$$

However, the quantum field approach does not contradict the Schrödinger one. Both define a field that becomes quantized by external imposed conditions [33] (i.e., quantization). In the quantum field theory, the quantization condition is introduced in a more clear and standardized way.

The same happens for the quantum hydrodynamic approach. The equations define a field (that contains also non-quantum solutions). The restriction to the quantum ones is obtained by the quantization conditions [26].

The analysis how the quantization is introduced in the hydrodynamic approach to quantum mechanics (and how it modifies the minimum action principle) is matter of underway work whose objective is to derive the quantum gravity equation by a minimum action principle [34]. When we do that, a cosmological term as given by (2), that is able to solve the conundrum of the huge order of maglitude of the quantum vacuum energy, appears.

Once this aspect is analitically and quantitatively made clear, the comparison with the outputs in modified quantum field approach can be fruitfully carried out.

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