



# Structure Equations, Permitted Movement of Relativistic Continuum and Sagnac's, Erenfest's and Bell's Paradoxes

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## Authors' contributions

*This work was carried out in collaboration between all authors. Author SAP designed the study, created the structure equations, solved the Bell's and Sagnac's paradoxes and wrote the first draft of the manuscript. Author JF designed the study and proposed the solution of the Erenfest's paradox. Author ERM managed the literature searches and physical interpretation of the Erenfest's paradox. All authors read and approved the final manuscript.*

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## ABSTRACT

From obtained structure equations, restrictions on a space-time geometry for possible solutions of relativistic continua are studied. The Minkowski space proved to be "cramped" to describe the continuum if except for the medium motion equations one imposes rigidity and rotation conditions. The continuum is a basis of noninertial reference frames (NRF) where one studies different physical processes. For example, bases of simplest NRF are constructed: 1. Relativistic globally uniformly accelerated Born rigid NRF. 2. Relativistic Born rigid uniformly rotating reference frame (RF) without a horizon. 3. Rigid irrotational spherically symmetrical quasi-Einstein's NRF. One can't describe bases of these systems in the Minkowski space, the Riemannian space-time is needed. The space-time of these RF is not directly connected with the general relativity theory (GRT), though it imposes conditions on some solutions of the Einstein equations. A solution of the Sagnac's, Erenfest's and Bell's paradoxes is proposed.

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## 1. INTRODUCTION

From the physical encyclopaedia [1], "reference frames (RF) are the collections of the coordinate and clock system connected with the body relatively to which the motion (or the equilibrium) of any other mass points or bodies is studied"... . So to investigate the motion (equilibrium) of other bodies an analytical specification of the body properties (the basis of RF itself) is needed. We select a continuum as a basis body.

4-acceleration, a strain velocity tensor and a rotational velocity tensor are characteristics of a continuum at 4-space-time. The law of motion includes 4-acceleration and at specified flat metric 4-velocity field and the main continuous medium tensors are determined by the integration of the motion equation. The continuum at a force field specifies some reference frame (RF). For RF with specified properties besides the motion equations one needs to know additional conditions assigned to the main continuous medium tensors depending on 4-velocities and 4-accelerations. For example, the demand concerning the rotation and the rigidity. The number of equations to obtain 4-velocity becomes overdetermined and the integrability conditions should be fulfilled. The latter are fulfilled if both 4-velocities of the continuous medium and the metric coefficients will be sought for.

When describing properties of arbitrary deformable reference frames in the form of the continuum either field of 4-velocities (the Euler viewpoint) or the law of continuum motion determining the connection between Euler and Lagrange variables are specified. Space-time is considered either flat in the case of special relativity theory (SRT), or the Riemannian one in the case of the general relativity theory (GRT).

If one neglects by the gravitational particle interaction and an external force influencing on a body is not a gravitational one then to describe the medium motion the relativistic SRT mechanics is applied.

In SRT fields do not curve the space-time. The space-time geometry remains flat. Only "spatial sections" are curved. Such viewpoint is the routine in the relativity theory (RT). We want to prove the fallibility of such approach connected

with the existing transition from the inertial reference frame (IRF) to the noninertial reference frame (NRF).

Even for the transition to the simplest reference frames (RF) there is no a complete evidence in RT. One does not know what reference frames in SRT are the relativistic uniformly accelerated ones. On the one hand, the Möller-Rindler systems are related to such ones [2], on the other hand the Logunov's system is [3].

The Logunov's system is a motion of a charged dust in a homogeneous constant electric field with zero initial velocities. However, neither the Möller-Rindler system [2] nor the Logunov's system [3] are both Born rigid and relativistic uniformly accelerated ones! The Möller-Rindler system is a relativistic rigid one, but it is not a global uniformly accelerated system. The Logunov's system is a global uniformly accelerated one, but it does not move in a Born rigid way!

*We divide all NRF into 2 classes:*

1. *NRF with the specified law of motion.*
2. *NRF with the specified structure.*

The routine method of transition from IRF to NRF [4,5] is connected with the transformation of coordinates containing a non-linear time (that is with the law of continuum motion in the Lagrangian coordinates, which is obtained, for example, by the integration of the motion equations in the Euler variables).

It's obvious that if the motion equations are specified in the Minkowski space then no transformation of coordinates permit go beyond the scope of the flat space-time, as one can't obtain the Riemannian - Christoffel tensor differed from zero if this one is absent in IRF. *We determine a such NRF as the 1-st class NRF.*

*In the 2-nd class NRF both the knowledge of the law of continuum motion and the predetermined RF properties specified by the strain velocity tensors and the rotational velocity tensors are needed.*

The Minkowski space, for example, is "tight" to satisfy simultaneously even the elemental requirements: The Born's rigidity and uniformly acceleration. In this article the 2-nd class NRF will be considered.

## 2. STRUCTURE EQUATIONS OF RELATIVISTIC CONTINUUM

Our approach is following. Consider the flat Minkowski space-time with the signature (+---) and the continuum at rest. At some moment  $t=t_0$  any force field is switched on (except the gravitational one) and the continuum begins to move. What space-time properties are induced by the force field? According to the orthodox interpretation the space-time properties will be unchanged [4]. Our answer to this question will not be so dogmatic. We do not exclude the possibility that inclusion of the force field can change the space-time properties transforming it in the world tube limits into a curved one. We determine the structure of this space-time in accordance with the specified force field structure and with the continuum characteristics such as  $\Sigma_{\mu\nu}$  the strain velocity tensor,  $\Omega_{\mu\nu}$  the rotational velocity tensor, and the first curvature vector of world lines of medium particles  $A_\mu$

(motion equations).

For a moving continuum in four-dimensional space-time with the signature (+---) the decomposition of the covariant derivative, of the 4-velocity field into expansion, rotation and acceleration is

$$\nabla_\mu V_\nu = \Sigma_{\mu\nu} + \Omega_{\mu\nu} + V_\mu A_\nu, \quad (2.1)$$

where  $V_\mu$  is the field of 4 – velocity, satisfying the normalization condition

$$g_{\mu\nu} V^\mu V^\nu = 1, \quad (2.2)$$

$g_{\mu\nu}$  is the metric tensor at the Euler reference frame. The connection between covariant components 4-velocity  $V_\nu$  and contravariant ones  $V^\mu$  is determined by means of the metric tensor

$$V_\nu = g_{\nu\mu} V^\mu.$$

Space 4-velocity  $V^k$  corresponds to the direction of  $v^k$  three-dimensional velocity. The relativistic strain velocity tensor  $\Sigma_{\mu\nu}$  is determined by

$$\Sigma_{\mu\nu} = \nabla_{(\mu} V_{\nu)} - V_{(\mu} A_{\nu)}, \quad (2.3)$$

and the relativistic tensor of rotational velocity has the form

$$\Omega_{\mu\nu} = \nabla_{[\mu} V_{\nu]} - V_{[\mu} A_{\nu]}. \quad (2.4)$$

Covariant components of 4-acceleration  $A_\nu$  are connected with the 4-acceleration  $A^\mu$  by the ratio

$$\begin{aligned} A_\nu &= g_{\nu\mu} A^\mu \\ A_\mu &= V^\nu \nabla_\nu V_\mu. \end{aligned} \quad (2.5)$$

The Greek indices go from zero to three, the Latin ones from one to three.

One can interpret the expansion (2.1) from two viewpoints:

1. Consider that the field of 4 – velocity  $V_\mu$  is known, for example, as a result of the integration of the relativistic Euler or Navier - Stokes equation at the specified flat metric. In this case  $\Sigma_{\mu\nu}$ ,  $\Omega_{\mu\nu}$ ,  $A_\mu$  continuum characteristics can be obtained in accordance with formulae (2.3) – (2.5), and expansion (2.1) acts as a mathematical identity.
2. Consider that  $\Sigma_{\mu\nu}$ ,  $\Omega_{\mu\nu}$ ,  $A_\mu$  functions are specified. In this case expansion (2.1) converts into the differential equation system relatively to  $V_\nu$  and  $g_{\mu\nu}$ . As the number of system equations (2.1) and (2.2) ranks over the number of unknown functions some integrability conditions must be satisfied. The relation

$$\frac{\partial^2 V_\nu}{\partial x^\varepsilon \partial x^\sigma} = \frac{\partial^2 V_\nu}{\partial x^\sigma \partial x^\varepsilon}, \quad (2.6)$$

will be the integrability condition for 4-velocity components.

To obtain the connection between geometrical and kinematic continuum characteristics we will calculate the expression in explicit form

$$2\nabla_{[\epsilon}\nabla_{\sigma]}V_\nu = 2\partial_{[\epsilon}\partial_{\sigma]}V_\nu + \left(\frac{\partial\Gamma_{\epsilon\nu}^\mu}{\partial x^\sigma} - \frac{\partial\Gamma_{\sigma\nu}^\mu}{\partial x^\epsilon} + \Gamma_{\sigma\rho}^\mu\Gamma_{\epsilon\nu}^\rho - \Gamma_{\epsilon\rho}^\mu\Gamma_{\sigma\nu}^\rho\right)V_\mu.$$

From this it follows by taking into account (2.1) – (2.6), that

$$R_{\epsilon\sigma,\nu}^\mu V_\mu = 2\nabla_{[\epsilon}\Sigma_{\sigma]\nu} + 2\nabla_{[\epsilon}\Omega_{\sigma]\nu} + 2\nabla_{[\epsilon}(V_{\sigma]}A_\nu). \quad (2.7)$$

The integration of the system (2.1), (2.7), where  $R_{\epsilon\sigma,\nu}^\mu$  is the curvature tensor expressed in terms of the metric tensor in an ordinary way, permits the solution of the space-time geometry problem in which NRF with specified structure is realized. We name equations (2.7) as NRF structure equations [6].

### 3. COMPARISON OF CLASSICAL AND RELATIVISTIC UNIFORMLY ACCELERATED RIGID CONTINUUM

Let us consider the medium motion at the level of classical Newtonian mechanics. One can determine the velocity field  $V_a$  of such a system from the equations in the Cartesian coordinates

$$\left. \begin{aligned} \frac{\partial v_a}{\partial x_b} + \frac{\partial v_b}{\partial x_a} &= 0 \\ \frac{\partial v_a}{\partial x_b} - \frac{\partial v_b}{\partial x_a} &= 0 \end{aligned} \right\} \Rightarrow \frac{\partial v_a}{\partial x_b} = \frac{\partial v_b}{\partial x_a} = 0$$

$$\frac{dv_a}{dt} = \frac{\partial v_a}{\partial t} + v_b \frac{\partial v_a}{\partial x_b} = \frac{\partial v_a}{\partial t} = a_a = \text{const} \quad (3.1)$$

The first equation (3.1) means zero strain velocity tensor, i. e. it corresponds to the rigid motion. The second one reflects the absence of rotation, and the third one reflects that the 4-acceleration is constant. The solution of (3.1) has the form

$$v_a = a_a t + v_{0a}, \quad (3.2)$$

where  $v_{0a}$  is the initial velocity. If the acceleration has a constant direction and its value depends on time, the solution of (3.1) is obtained in the form

$$v_a = \int_0^t a_a(\tau) d\tau + v_{0a}. \quad (3.3)$$

Classical mechanics permits a solid-state translation with arbitrary acceleration depending on time. Non-commuted identical dust particles located at such field at equal initial velocities move as a solid.

In order to generalize the classical conception of the rigid motion Born introduced the definition consistent with SRT and GRT. According to this definition the continuum motion is called rigid (in the Born's sense) if for any pair of neighboring body particles the orthogonal interval between corresponding pairs of world lines of medium particles remains constant during the motion. The orthogonal interval is the distance between two world particle lines, measured along the hypersurface orthogonal to both world particle lines. The difference between the classical and relativistic rigidity conditions is in the selection of spatial hypersurfaces along which distances between world lines of body particles are measured.

In classical consideration the hyperplanes of simultaneous events are the hypersurfaces. The hyperplanes of simultaneity in one IRF are not a hyperplanes of simultaneity in the other one. While the Born's rigidity condition has no that lack. Obviously when rigid moving hypersurfaces orthogonal to world lines in one IRF are hyperplanes orthogonal to world lines in all other IRF that makes the Born-rigid NRF the Lorentz-covariant one as opposed to the classical rigid NRF.

The Born rigidity condition is equivalent to zero  $\Sigma_{\mu\nu}$  relativistic strain velocity tensor. Therefore one can expect in relativistic consideration  $\Sigma_{\mu\nu} = \Omega_{\mu\nu} = 0$ , and in accordance with [4] one determines "the relativistic uniformly accelerated motion as a rectilinear one at which the acceleration value  $A$  in its own (in each time instant) reference frame remains constant". Then as a result, we will obtain the field of 4-velocity  $V_\mu$  of relativistic rigid NRF at SRT.

Such a program is realized in [7]. As proper reference frame the Fermi- Walker's tetrad system [8] was used, in the basis of which the constant acceleration is specified. Such motion can't be realized at the Minkowski space as the obtained system of equations is then inconsistent.

If at the right hand side of equation (2.7)  $\Sigma_{\mu\nu} = \Omega_{\mu\nu} = 0$ , and  $g_{\mu\nu}A^\mu A^\nu = \text{const}$ , then the left side of an equation does not vanish. Consequently in the Minkowski space where the curvature tensor is identically zero, the rigid globally uniformly accelerated NRF does not exist.

The question arises. What is the motion of the assembly of identical particles if they are located in a constant uniform force field when the initial velocities of all particles are equal to zero?

Let us consider the motion of the charged dust particles in a constant uniform electric field.

To obtain the Logunov's or the Möller's metric one considers the pseudo – Euclidean interval specified in the Euler variables in the form

$$dS^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2, \quad (3.4)$$

where  $x^0 = ct, x^1, x^2, x^3$  are the Cartesian coordinates, and the law of continuum motion for the Logunov's metric has the form

$$\begin{aligned} x^1(y^1, t) &= y^1 + \left(\frac{c^2}{a_0}\right) \left[ \sqrt{1 + \frac{a_0^2 t^2}{c^2}} - 1 \right], \\ x^2 &= y^2, \quad x^3 = y^3, \quad x^0 = y^0 \end{aligned} \quad (3.5)$$

or

$$\begin{aligned} x^1(y^1, t) &= y^1 + \frac{c^2}{a_0} \left[ \cosh\left(\frac{a_0 t}{c}\right) - 1 \right], \\ x^2 &= y^2, \quad x^3 = y^3, \\ t &= \left(\frac{c}{a_0}\right) \sinh\left(\frac{a_0 t}{c}\right), \end{aligned} \quad (3.6)$$

where the time in IRF is used as a time parameter in (3.5) and in (3.6)  $\mathcal{T}$  is the proper time. A substitution of (3.5) and (3.6) into (3.4) gives [3]

$$\begin{aligned} dS^2 &= \frac{c^2 dt^2}{1 + \frac{a_0^2 t^2}{c^2}} - 2 \frac{a_0 t dt dy^1}{\left(1 + \frac{a_0^2 t^2}{c^2}\right)^{\frac{1}{2}}} \\ &\quad - (dy^1)^2 - (dy^2)^2 - (dy^3)^2, \end{aligned} \quad (3.7)$$

$$\begin{aligned} dS^2 &= c^2 (d\tau)^2 - 2 \sinh\left(\frac{a_0 \tau}{c}\right) c d\tau dy^1 \\ &\quad - (dy^1)^2 - (dy^2)^2 - (dy^3)^2 \end{aligned} \quad (3.8)$$

If from metrics (3.7), (3.8) according to [4] one constructs three-dimensional metric tensor

$$\gamma_{kl} = -g_{kl} + g_{0k} g_{0l} / g_{00},$$

then for the square of “physical space distance” element we obtain

$$dl^2 = (1 + a_0^2 t^2 / c^2) (dy^1)^2 + (dy^2)^2 + (dy^3)^2, \quad (3.9)$$

$$dl^2 = \cosh^2(a_0 \tau / c) (dy^1)^2 + (dy^2)^2 + (dy^3)^2. \quad (3.10)$$

It follows from the latter formulae that the Logunov metric is not a rigid one.

For the Möller transform the law of motion has the form

$$\begin{aligned} x^1(y^1, T) &= y^1 \cosh\left(\frac{a_0 T}{c}\right) + \frac{c^2}{a_0} \left[ \cosh\left(\frac{a_0 T}{c}\right) - 1 \right], \\ x^2 &= y^2, \quad x^3 = y^3, \\ t &= \frac{c}{a_0} \left( 1 + \frac{a_0 y^1}{c^2} \right) \sinh\left(\frac{a_0 T}{c}\right), \quad y^0 = cT, \end{aligned} \quad (3.11)$$

and the Möller metric is expressed by the interval element

$$dS^2 = \left( 1 + \frac{a_0 y^1}{c^2} \right)^2 c^2 (dT)^2 - (dy^1)^2 - (dy^2)^2 - (dy^3)^2. \quad (3.12)$$

Analysis of the Möller transform showed that in the Fermi-Walker basis (to which the accelerometer readings are related [6]) the accelerations of different particles are not identical and these ones are calculated according to the formula

$$a(y) = \frac{a_0}{\left( 1 + \frac{a_0 y}{c^2} \right)}, \quad (3.12a)$$

where  $a_0$  is the acceleration of the particle along  $y$  axis located at the origin of the Lagrangian co-moving coordinate system,  $c$  is the velocity of

light in free space. Thus, the Möller transform does not describe the transition to the global linearly accelerated NRF. Each Lagrangian particle moves with constant acceleration but these accelerations are not equal [2].

Expressing in laws of motion (3.5), (3.6), (3.11) the Lagrangian coordinates by the Euler ones, we pass from the Logunov's and Möller's metrics to the pseudo-Euclidean interval (3.4).

It is difficult to understand an origin of deformations depending on a time for co-moving observers in moving in a uniform field. Identical physical conditions for any basis particles resulted in a particle motion relatively each other.

When constructing of the relativistic rigid uniformly accelerated NRF our approach is based on the demand of the deformation absence at the medium in its translational motion without initial velocity in the homogeneous field. The approach integrates properties of the Logunov system (uniformly acceleration) and the Möller system (rigidity), but inside the world tube the space-time is not a flat one. Mathematically the problem reduces to the solution of system (2.1) provided that

$$\begin{aligned} \Sigma_{\mu\nu} = \Omega_{\mu\nu} = 0, \quad g_{\mu\nu} V^\mu V^\nu = 1, \\ g_{\mu\nu} A^\mu A^\nu = -a_0^2 / c^4 \end{aligned} \quad (3.13)$$

and system (2.7) taking into account (2.3) and (2.4) provided that the acceleration  $a_0$  and the  $c$  light velocity are constant. System (2.1) at the Euler variables reduces to the form

$$\nabla_\mu V_\nu = V_\mu A_\nu \quad (3.14)$$

Its solution is easier searched at the Lagrange accompanying reference frame where

$$V_k = V^k = 0, \quad V^0 = g_{00}^{-1/2}, \quad V_0 = g_{00}^{1/2}. \quad (3.15)$$

Let the medium moves along the Euler  $x^1$  coordinate. Then we will find the NRF metric in the Lagrange coordinates in the form

$$dS^2 = D(X^1)(dX^0)^2 - A(X^1)(dX^1)^2 - (dX^2)^2 - (dX^3)^2. \quad (3.16)$$

Independence of  $A(X^1)$  metric coefficient on the time coordinate is equivalent to zero strain

velocity tensor, and the absence of the metric  $g_{0k}$  components is equivalent to the rotation absence. The solution of system (3.14) taking into account (3.13) and (3.15) and using the Dingle formula [9] results in the relation

$$A(X^1) = \frac{c^4}{4a_0^2 D^2} \left( \frac{dD}{dX^1} \right)^2. \quad (3.17)$$

Substitution of (3.17) into the structure equations (2.7) gives an identity. If one transforms the Lagrange coordinates  $X^i$  into other Lagrange coordinates  $y^i$  in accordance with formulae

$$dy^1 = A^{1/2} dX^1, \quad X^0 = y^0, \quad X^2 = y^2, \quad X^3 = y^3,$$

one finds the expression for the metric of uniformly accelerated NRF

$$dS^2 = \exp\left(\frac{2a_0 y^1}{c^2}\right) (dy^0)^2 - (dy^1)^2 - (dy^2)^2 - (dy^3)^2, \quad (3.18)$$

where acceleration  $a_0$  is directed along the  $y^1$  axis. One can directly be convinced of the uniformly accelerated NRF (3.18)

$$\begin{aligned} A^1 = \frac{DV^1}{dS} = \frac{dV^1}{dS} + \Gamma_{00}^1 (V^0)^2 = \frac{1}{g_{00}} \Gamma_{00}^1 \\ = -\frac{g^{11}}{2g_{00}} \frac{\partial g_{00}}{\partial y^1} = \frac{a_0}{c^2}. \end{aligned} \quad (3.19)$$

The rest of the components of the 4-acceleration are equal to zero. Let us find the NRF space-time geometry using the known formula for the curvature tensor [4]

$$\begin{aligned} R_{\alpha\beta,\gamma\delta} = \frac{1}{2} \left( \frac{\partial^2 g_{\alpha\delta}}{\partial y^\beta \partial y^\gamma} + \frac{\partial^2 g_{\beta\gamma}}{\partial y^\alpha \partial y^\delta} - \frac{\partial^2 g_{\alpha\gamma}}{\partial y^\beta \partial y^\delta} - \frac{\partial^2 g_{\beta\delta}}{\partial y^\alpha \partial y^\gamma} \right) \\ + g_{\mu\nu} \left( \Gamma_{\beta\gamma}^\mu \Gamma_{\alpha\delta}^\nu - \Gamma_{\beta\delta}^\mu \Gamma_{\alpha\gamma}^\nu \right) = g_{\alpha\sigma} R_{\beta,\gamma\delta}^\sigma \end{aligned} \quad (3.20)$$

where the Christoffel symbols are calculated in accordance with the formulae

$$\Gamma_{\mu,\alpha\beta}^\mu = \frac{1}{2} \left( \frac{\partial g_{\mu\alpha}}{\partial y^\beta} + \frac{\partial g_{\mu\beta}}{\partial y^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial y^\mu} \right), \quad (3.21)$$

$$\Gamma_{\alpha\beta}^\mu = \frac{1}{2} g^{\mu\gamma} \left( \frac{\partial g_{\gamma\alpha}}{\partial y^\beta} + \frac{\partial g_{\gamma\beta}}{\partial y^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial y^\gamma} \right). \quad (3.22)$$

One independent curvature tensor component calculated in accordance with the metric (3.18) has the form

$$R_{10,10} = -\frac{1}{2} \left[ \frac{\partial^2 g_{00}}{\partial y^{12}} - \frac{1}{2g_{00}} \left( \frac{\partial g_{00}}{\partial y^1} \right)^2 \right] = -\frac{a_0^2}{c^4} \exp\left(\frac{2a_0 y^1}{c^2}\right). \quad (3.23)$$

The components of the Ricci tensor  $R_{\beta\gamma} = g^{\alpha\gamma} R_{\alpha\beta,\gamma\delta}$  can be written as

$$R_{00} = -R_{10,10}, \quad R_{11} = -\frac{a_0^2}{c^4}, \quad R_{10} = 0. \quad (3.24)$$

and the scalar curvature is

$$R = 2 \frac{a_0^2}{c^4}. \quad (3.25)$$

Thus, one can realize the relativistic rigid uniformly accelerated NRF in the space with constant curvature.

If instead of the metric (3.18) one substitutes  $g_{00} = (1 + a_0 y^1 / c^2)^2$ , corresponding to the Möller metric [2], into the right side of equation (3.23), then  $R_{10,10} = 0$ , as the Möller metric was obtained by means of transformation of coordinates from the Minkowski space. In our case the joint demand of rigidity and uniformly acceleration does not make the right side of structure equations (2.7) vanish. Hence, the Riemannian - Christoffel tensor is differed from zero. The formulas (3.18), (3.23) are obtained in [6] and repeated in [10,11].

Let us prove the lemma.

**Lemma 1**

In the Minkowski space the Born-rigid and relativistic uniformly accelerated translational continuum motion is absent.

**Proof**

According to (2.7)  $\Omega_{\mu\nu} = 0$  for the translational motion,  $\Sigma_{\mu\nu} = 0$  for the rigid motion in the Born sense. For the relativistic uniformly accelerated motion the equality is valid

$$g_{\mu\nu} A^\mu A^\nu = const. \quad (3.26)$$

Then the structural equation has the form

$$R_{\varepsilon\sigma,\nu}^\mu V_\mu = \nabla_\varepsilon (V_\sigma A_\nu) - \nabla_\sigma (V_\varepsilon A_\nu). \quad (3.27)$$

and equations (2.1-2.5) yield the relation

$$\nabla_\varepsilon V_\sigma = V_\varepsilon A_\sigma. \quad (3.28)$$

Let us examine the convolution

$$R_{\varepsilon\sigma,\nu\mu} V^\mu V^\sigma = (\delta_\varepsilon^\sigma - V_\varepsilon V^\sigma) \nabla_\sigma A_\nu - A_\varepsilon A_\nu. \quad (3.29)$$

Convolving with 4-accelerations, we have

$$R_{\varepsilon\sigma,\nu\mu} V^\mu V^\sigma A^\varepsilon A^\nu = A^\sigma A^\nu \nabla_\sigma A_\nu - (A^\nu A_\nu)^2 \\ = A^\sigma \nabla_\sigma (A^\nu A_\nu) - A^\sigma A_\nu \nabla_\sigma A^\nu - (A^\varepsilon A_\varepsilon)^2. \quad (3.30)$$

In accordance with (3.26)

$$R_{\varepsilon\sigma,\nu\mu} V^\mu V^\sigma A^\varepsilon A^\nu = -(A^\varepsilon A_\varepsilon)^2 = const \neq 0. \quad (3.31)$$

The Riemann-Christoffel tensor convolution is differed from zero. So, the Riemann-Christoffel tensor is nonzero. The lemma 1 is proved.

In structure equations (2.7) for vortex-free rigid motions, if one contracts over the first and third indices (the Ricci and curvature tensors are selected), applying the identity  $\nabla_\sigma (V_\varepsilon A^\varepsilon) \equiv 0$  and equation (3.26), we have

$$R_{\sigma\mu} V^\mu = V_\sigma \nabla_\varepsilon A^\varepsilon, \quad (3.32)$$

that is equivalent to

$$(R_{\sigma\mu} - A g_{\sigma\mu}) V^\mu = 0, \quad A \equiv \nabla_\varepsilon A^\varepsilon. \quad (3.33)$$

One can obtain metric (3.18) directly from the relations

$$R_{\sigma\mu} V^\mu V^\sigma = R_0^0 = \nabla_\varepsilon A^\varepsilon. \quad (3.34)$$

#### 4. RELATIVISTIC RIGID UNIFORMLY REVOLVING DISK

Generally when examining the revolving disk one chooses the rest-frame where the  $r_0, \varphi_0, z_0, t_0$  cylindrical coordinates are introduced and passes to the revolving reference frame  $r, \varphi, z, t$  according to the formulas:

$$\begin{aligned} r_0 &= r, & \varphi_0 &= \varphi + \Omega t, \\ z_0 &= z, & t_0 &= t, \end{aligned} \quad (4.0)$$

where the  $\Omega$  speed of rotation relatively  $z$  axis is constant. The line element is

$$dS^2 = \left(1 - \frac{\Omega^2 r^2}{c^2}\right) c^2 dt^2 - 2\Omega r^2 d\varphi dt - dz^2 - r^2 d\varphi^2 - dr^2. \quad (4.1)$$

This formula is valid if  $r\Omega/c < 1$ . Other velocity distributions restricting the disk linear velocity at  $r \rightarrow \infty$  to values less than the light velocity  $c$  and at  $\frac{\Omega r}{c} \ll 1$  with  $v = \Omega r$  are presented in [12,13,14]. But only the usual distribution law  $v = \Omega r$ ,  $\Omega = const$  satisfies the stiffness criterion, both the classic and the relativistic one (in Born's sense).

Let us find the metric of the rigid relativistic uniformly revolving NRF supposing that the strain velocity tensor is  $\Sigma_{\mu\nu} = 0$ . We demand the invariant constancy characterizing the relativistic generalization of the square of the disk rotational velocity  $\omega$ .

$$\Omega_{\mu\nu}\Omega^{\mu\nu} = \frac{2\omega^2}{c^2} = const. \quad (4.2)$$

In the Lagrangian co-moving RF connected with the revolving disk we have

$$dS^2 = D(r)c^2dt^2 - 2P(r)c dt d\varphi - dz^2 - r^2d\varphi^2 - dr^2, \quad (4.3)$$

$$A^1 = \frac{1}{2D} \frac{dD}{dr}, \quad A^2 = A^3 = A^0 = 0. \quad (4.4)$$

After the calculations [6,15,16,17,18] we have two independent equations

$$\frac{P}{D} \frac{dD}{dr} - \frac{dP}{dr} = -2 \frac{\omega}{c} (Dr^2 + P^2)^{\frac{1}{2}}, \quad (4.5)$$

$$\frac{dD}{dr} = -2 \frac{\omega}{c} DP (Dr^2 + P^2)^{-1/2}. \quad (4.6)$$

Equation (4.2) is equivalent to the constancy of the value of metrically invariant angular velocity vector [19] and that is equivalent to the constancy of the value of the speed of rotation in the co-moving tetrads [14].

The relativistic  $\omega$  and the classical rotational speeds  $\Omega$  are connected by the equation

$$\omega = \Omega \left(1 - \frac{\Omega^2 r^2}{c^2}\right)^{-1}. \quad (4.7)$$

There is a steady-state solution for (4.3). This solution is applicable in the whole area  $0 \leq r \leq \infty$  and it is realized in the Riemannian space – time.

The solution of the system (4.5), (4.6) in the quadratures is absent. Analysis showed that at  $\omega r/c = 1$  the metric (4.3) coincides with (4.1). The centripetal acceleration in the revolving NRF has the form

$$a = c^2 A^1 = - \frac{\omega c P}{\sqrt{Dr^2 + P^2}}, \quad (4.8)$$

at small  $r$  it passes into the classical one and at  $r \rightarrow \infty$  gives  $a = -\omega c$ . After simplifications the system (4.5-4.6) is represented in the form

$$\frac{dv}{dx} + \frac{v}{x}(1 - v^2) = (2 - v^2)(1 - v^2), \quad (4.9)$$

$$D = \exp(-2 \int v dx), \quad v = \frac{U}{\sqrt{1+U^2}}, \quad U = \frac{P}{r\sqrt{D}}, \\ x = \frac{\omega r}{c}. \quad (4.10)$$

The  $v(x)$  function is the dimensionless linear disk velocity. For small velocities

$$D = \exp(-2 \int v dx) = \exp(-x^2) = 1 - x^2, \quad (4.11)$$

that is equivalent to the classical equation. It follows from (4.9) that for  $x \rightarrow \infty$  the equation has the solution  $v = 1$ . This solution is differed from the classical rigid disk, where the field of velocities at infinity is indefinitely great. The numerical solution diagram (4.9) is similar to the hyperbolic tangent diagram for  $x > 0$ .

It is known [4], that on a revolving disk at all points the clocks can not be identically synchronized. So synchronizing along a closed circuit and returning to the reference point, we obtain that the time differs from the original one by the value

$$\Delta t = - \frac{1}{c} \oint \frac{g_{02}}{g_{00}} d\varphi = \frac{\Omega r^2 2\pi}{c^2 (1 - \beta^2)}. \quad (4.12)$$

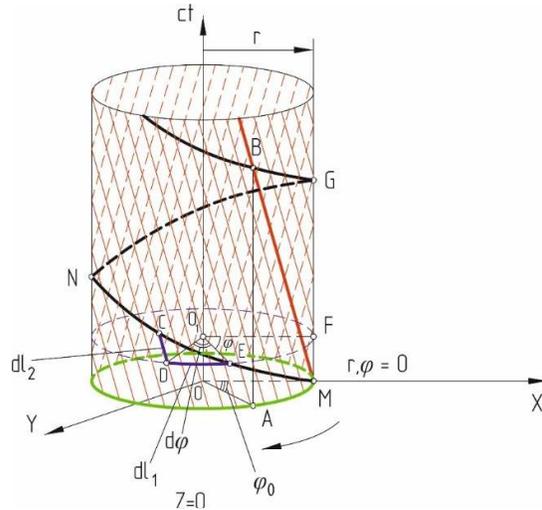
From our point of view this opinion is erroneous. The circuit in a physical space is unclosed. Let us divide the rotating thin disk into concentric thin hoops and consider particles located in one of them. World lines of this hoop's particles in the Minkowski space (that is true for small velocities) form the congruence of the helical lines on the cylinder with radius  $r$  and axis  $t$ , and the congruence of spacelike helical lines orthogonal to the congruence of world lines of hoop's particles will be a "physical space". This congruence is found from Pfaff's equation

$$V_0 dx^0 + V_\varphi d\varphi_0 = 0, \quad {}_t(r, \varphi) = \frac{\Omega r^2 \varphi}{c^2 (1 - \beta^2)} \quad (4.13)$$

Let at the law of motion (4.0)

$$\varphi_0 = \varphi + \Omega t,$$

for which the square of interval element (4.1) is obtained, the Euler coordinate  $\varphi_0$  coincides with the initial Lagrangian coordinate  $\varphi$ . That corresponds  $t = 0$ . From the Euler point  $(r, \varphi_0)$  at  $t = 0$  instant the world line of some hoop's particle starts. This line is located on the cylinder surface above the spatially similar line of a "physical space" (Fig. 1).



**Fig. 1. The spatio-temporal geometry of rotating hoop in a plane Z=0**

In the  $2\pi$  angle in the co-moving hoop's system the Lagrangian point  $\varphi$  in a "physical" space coincides with the world line of the hoop's particle with  $\varphi$  number. From (4.13) we have

$$t(r, 2\pi) = \frac{\Omega r^2 2\pi}{c^2(1-\beta^2)}. \quad (4.14)$$

Time interval  $t(r, \varphi)$  corresponds to the time distance along the element of cylinder from the plane  $t=0$  up to the "physical" spatially similar line  $t(r, \varphi)$  (Fig. 1). Thus, in spite of coincidence of formulae (4.14) and (4.12) they have quite distinct physical meaning.

Let us consider the light ray propagation relatively the source in IRF. The source is located at the origin of the Eulerian and Lagrangian coordinates. Light pulses are emitted in the opposite directions on the circle coinciding with one of the circles of the rotating disk. The

velocity of light in IRF is constant. But as the disk rotates, then the "catching up" pulse in a "physical" space spends more time up to reach the Lagrangian point  $2\pi$  on the  $\Delta t = t(r, 2\pi)$  value. The pulse propagating in the opposite direction to the linear velocity of the rotating disk spends less time on the same value. In accordance with the universal time the time difference to reach the Lagrangian point  $2\pi$  is equal to

$$\Delta t_0 = 2\Delta t = \frac{4\Omega r^2 \pi}{c^2(1-\beta^2)} = \frac{4S\Omega}{c^2(1-\beta^2)} = \frac{4S\omega}{c^2}, \quad (4.15)$$

where the classical angular velocity  $\Omega$  is substituted on the relativistic one  $\omega$  from (4.7), and  $S$  is the disk area. The expression obtained from the Sagnac experiment follows from (4.15). Let us consider the relativistic hoop. Using (4.10) we have

$$\Delta t = 2\Delta t = -\frac{2}{c} \int_0^{2\pi} \frac{g_{02}}{g_{00}} d\varphi = \frac{4\pi}{c} \frac{vr}{\sqrt{1-v^2}} \exp\left(\int v dx\right) \quad (4.16)$$

In nonrelativistic approximation for small disk velocities  $v \ll c$  and the formula passes into (4.15).

Instead of a cylinder we will consider a thin rotating hoop as the cylinder element. Fig. 1 shows that if one places along the hoop absolute identical clocks and at the initial instant of time sets on all of them the time  $t = t_1$  then on any hyperplane  $t = const$  lengths of world lines of all clocks will be identical, that means that all clocks on the hoop go synchronously. That's true from physical considerations as clocks at identical distances from the centre are in absolutely identical conditions. And statement [4] that clocks located on the rotating hoop can not be synchronized in all hoop's points is incorrect.

Let us consider the SRT paradox proposed by Erenfest [20].

"Erenfest considered not perfectly rigid cylinder with radius  $r$  and height  $H$ , which gradually began revolving on its axis and then it rotated with constant velocity. Let  $r'$  is the radius of this cylinder from the stationary observer viewpoint. Then from the Erenfest viewpoint the  $r'$  value should satisfy two requirements which contradict one another:"

- a) The perimeter of circle of the rotating cylinder as compared with the state of rest should be shorten:

$$2\pi r' < 2\pi r,$$

as each element of such circle moved along the tangent line with the instantaneous velocity  $\Omega r'$ ;

- b) The instantaneous velocity of each radius element was perpendicular to its direction. That meant that the radius elements did not subject the shrinkage as compared with the state of rest. (The elongation of the radius at the expense of centrifugal inertial forces was ignored.)

This implies that

$$r' = r''.$$

Let us consider the paradox.

Generally for the rotating cylinder one selects the rest-frame in which the cylindrical Eulerian coordinates  $r_0, \varphi_0, z_0, t_0 = t$  are introduced and passes to the rotating Lagrangian co-moving frame by the standard method using the IRF time  $t$  and the Lagrangian coordinates  $r, \varphi, z, t$ .

Firstly we will consider the cylinder motion with constant angular acceleration  $\mathcal{E}$  up to  $t_1$  instant

$$\begin{aligned} \Omega(t) &= \mathcal{E}, \quad \varphi_0(t) = \varphi + \frac{\mathcal{E}t^2}{2}, \quad t < t_1, \\ \Omega(t_1) &= \Omega = \text{const}. \end{aligned} \quad (4.17a)$$

We select (4.0) as the law of motion after the acceleration when angular velocity  $\Omega$  became constant.

The transformation formulae (4.0) give

$$\begin{aligned} r_0 &= r, \quad \varphi_0 = \varphi + \Omega t, \\ z_0 &= z, \quad t_1 < t. \end{aligned} \quad (4.17)$$

Let us consider the difference

$$(\varphi_{02} - \varphi_{01})r = (\varphi_2 - \varphi_1)r = \Delta l = \text{const}. \quad (4.18)$$

This means that in the Euler variables (in IRF) the length of an arbitral arc of the rotating cylinder is equal to the arc length in the Lagrangian coordinates (in NRF) at the initial time. If one selects the perimeter of circle instead the arc then the result will not change. It will be equal to  $2\pi r$ . If instead the  $\Omega t$  value one can

select an arbitrary function  $f(t)$ , derivative of which in the initial time is equal to zero, then we will obtain the same result. Zero derivative  $df/dt$  in the initial time means that the disk is at rest. (4.17a) satisfies the condition. Thus, the acceleration in speeding-up does not influence on the perimeter of circle both in the co-moving NRF and in the initial IRF. There are no any Lorentz contractions. The initial perimeter of circle of the cylinder being at rest in IRF is equal to the circumference of the rotating in this system cylinder. We don't take into account that in the cylinder speed-up the radius increase in NRF occurs because of a centrifugal inertial force.

Solutions (3.5) and (3.6) have identical properties. It follows from these solutions that at planes  $t = \text{const}$ ,  $\tau = \text{const}$  the distances between the world lines remain constant both in IRF and in NRF and no Lorentz contractions occur. Relativistic solutions (3.5) and (3.6) represent classical rigid motions of charged dust in uniform electrostatic field at zero initial velocity. However these dust motions do not satisfy to the relativistic Born's stiffness criterion. One can check that the law of motion (4.17) satisfy both the Born's stiffness criterion and the classical stiffness criterion. For that one needs calculate the relativistic strain velocity tensor  $\Sigma_{\mu\nu}$  (2.3) and the classical strain tensor from (3.1) and ascertain that both these tensors are equal to zero [21]. The lack of the law of motion (4.17) is the presence of the horizon which is absent in the proposed relativistic uniformly rotating NRF.

Let us determine the "physical space" in the Minkowski space for arbitrary moving body in the Euler variables. The "physical space" with the specified 4-velocity field  $V^\mu$  should belong to the hypersurface orthogonal to the world lines. The metric tensor of the hypersurface in the Euler variables has the form

$$\gamma_{\mu\nu} = -g_{\mu\nu} + V_\mu V_\nu. \quad (4.19)$$

Here  $\gamma_{\mu\nu}$  is the projection operator orthogonal to the 4-velocity  $V^\mu$ . Let the law of continuum motion at an arbitrary field of force in the Minkowski space is determined by the equation

$$x^\mu = \Psi^\mu(y^k, \xi^0), \quad (4.20)$$

where  $x^\mu$  are the Eulerian coordinates and  $y^k$  are the Lagrangian coordinates, which are constant along each fixed world line,  $(1/c)\xi^0$  is the same time parameter, for example, a proper time. As in the co-moving NRF the obvious correlations are valid

$$V^k=0, \quad V^0=\frac{1}{\sqrt{g_{00}}}, \quad V_0=\sqrt{g_{00}},$$

$$V_k=V_\mu \frac{\partial \Psi^\mu}{\partial y^k}=g_{k\alpha}V^\alpha=g_{k0}V^0=\frac{g_{k0}}{\sqrt{g_{00}}}, \quad (4.21)$$

then the element of the spatial interval in the Lagrangian co-moving NRF has the form

$$dL^2=\gamma_{kn}dy^kdy^n=\left(\frac{g_{n0}g_{ko}}{g_{00}}-g_{nk}\right)dy^ndy^k. \quad (4.22)$$

The element of the interval (4.22) coincides with the well-known relation [4] obtained by means of the radar method. From (4.1), (4.22) we find in the rotating disk that the circumference is equal to

$$L=\frac{2\pi r}{\sqrt{1-\frac{\Omega^2 r^2}{c^2}}}. \quad (4.23)$$

Let us ascertain the physical meaning of the formula (4.23). Let us consider the Euler coordinate system and the hoop being at rest. In Fig. 1 the uniformly accelerated hoop motion (4.17a) is not mapped, but the law of motion (4.17) is represented. The world lines of the particles of the hoop being at rest represent the collection of the elements of time-space cylinder with the radius  $r$ , and the congruence of the spacelike lines orthogonal to the world lines of the hoop particles is the collection of the circles parallel to the hoop circle. The curved line  $MNGB$  is the part of the helical world line of the particle being at the instant  $t=0$  at  $M$  point. At the same point one of the spacelike curved lines orthogonal to the world lines of the hoop particles begins. The  $B$  point is the intersection point of the curved helical lines  $MB$  and  $MNGB$ .

One can see from the Fig. 1 that the length of the "physical" spacelike line  $MNGB$  orthogonal to the world lines of the hoop points is equal to the length of the helical line from the intersection

point  $(\varphi=\varphi_0=0, r, t=0)$  up to the intersection point of this line with the same world line of the hoop

$$\left(\varphi=0, r, t(r, \varphi=2\pi)=\frac{\Omega r^2 2\pi}{c^2(1-\beta^2)}\right). \quad (4.24)$$

Let us find the length of the helical "physical" spacelike line beginning at the  $M$  point and finishing at the  $B$  point.  $B$  and  $M$  points belong to the world helical line of the same hoop particle along that the Lagrangian number of the particle  $\varphi=0$  is kept. From the viewpoint of the Lagrangian observer during the time (4.24) the spacelike "physical" line will again intersect with the world line of the hoop particle in  $2\pi$ . Previously these lines were intersect at the  $M$  point. We point out that the "physical" lines always are orthogonal to the world lines of the hoop points. Therefore a right angle in the Euclidean space is distorted in the pseudo-Euclidean space. Let us consider the infinitesimal curvilinear triangle  $EDC$ . The vertex angle  $C$  corresponds to the right angle.  $DE$  hypotenuse has a negative length as this line is a spacelike one.  $CE=dL_1$  also has a negative length and  $DC=dL_2$  is the element of the world line of some hoop point and it is time-similar with the positive length. The Pythagorean theorem for the pseudo-Euclidean space gives

$$(DC)^2-(CE)^2=-(DE)^2, dL_1^2=dL_2^2+dL_3^2$$

or

$$dL_1^2=r^2d\varphi^2+(1-\beta^2)c^2dt^2=\frac{r^2d\varphi^2}{1-\beta^2}. \quad (4.25)$$

In deriving we have used (4.13) from the Pfaff equation. We have taken into account that in the curve  $ED$  according to (4.18)

$$ED \rightarrow (rd\varphi=r\Omega dt=rd\varphi), dt=0. \quad (4.26)$$

Integrating (4.25) we obtain

$$L_1=\frac{2\pi r}{\sqrt{1-\frac{\Omega^2 r^2}{c^2}}}. \quad (4.27)$$

For relativistic hoop from (4.3), (4.9), (4.10) we have

$$dL_1^2 = r^2 d\varphi^2 + Dc^2 dt^2 = \frac{r^2 d\varphi^2}{1-v^2} = r^2 d\varphi^2 \left(1 + \frac{P^2}{r^2 D}\right) \quad (4.28)$$

$$L_1 = \frac{2\pi r}{\sqrt{1-v^2}} = 2\pi r \sqrt{\left(1 + \frac{P^2}{r^2 D}\right)}. \quad (4.29)$$

The result for nonrelativistic hoop coincides with well-known one [4]. It should be noted the identity of formulae (4.23) and (4.27). Expression (4.23) was calculated in the hoop NRF, and expression (4.27) was calculated in the IRF. That solves the Erenfest's paradox. Both for "physical" lengths and for usual ones in the intersection of the hypersurface  $t=const$  with the surface of the spatio-temporal cylinder (4.18) there are no any Lorentz contractions. The Lorentz contraction appears in transiting (passing) from "physical" invariant line lengths orthogonal to world hoop point lines to unphysical ones (line lengths). Lengths of "physical" and unphysical lines are connected by the relation

$$L_2 = \frac{2\pi r}{\sqrt{1 - \frac{\Omega^2 r^2}{c^2}}} \sqrt{1 - \frac{\Omega^2 r^2}{c^2}} = 2\pi r. \quad (4.30)$$

It is such Lorentz length contraction Erenfest pointed out. However such Lorentz contractions do not cause deformations and tensions in bodies. Only changes of invariant "physical" lengths result in tensions [21,22]. We point out in conclusion that from our viewpoint the standard transition from the IRF to the rotating NRF (4.0), (4.1) is a mixture of the nonrelativistic and relativistic approach and it should be clarified. That is shown at this part.

## 5. RIGID IRROTATIONAL SPHERICALLY SYMMETRICAL QUASI-NEWTONIAN AND QUASI-EINSTEIN NRF

Let us consider in the Minkowski space a centrosymmetrical continuum motion which occurs from some point. The origin of coordinates is located in that point. Obviously for observer in the Lagrangian co-moving reference frame the distance between adjacent medium elements on any sphere will vary with time i.e. such a system is not a rigid one. As all medium points located at the identical distance from the centre have identical velocities and accelerations

then such a medium moves without rotations. Thus, for a such motion the tensor of rotational velocity is equal to zero, and the strain velocity tensor and the field of the first curvature vectors are nonzero. If for the medium concerned one demands the fulfillment of rigidity condition then it follows from the analysis of the structure equation (2.7) that in the Minkowski space the spherically symmetrical NRF having nonzero radial acceleration and zero strain velocity tensor does not exist. It is accepted that for weak fields the Newton's and the Einstein's theories coincided. However it is not quite so. In the Newton's theory a body being at rest on the surface of the gravitating body has zero absolute acceleration. A vector sum of a gravity force and a supporting force is equal to zero, that causes absolute zero acceleration. Freely falling body in the Newton's theory has nonzero absolute acceleration. In the Einstein's theory the situation is opposite. A mass point being at rest on the surface of a gravitating body has nonzero absolute acceleration that is numerically equal to the gravitational acceleration on the body surface and directed upwards perpendicularly to the surface. And in the Schwarzschild field a geodetic line corresponds to the particle, i.e. zero absolute acceleration. Thus, the quasi-Einstein NRF corresponds to the static field in which particles being in equilibrium in this field are studied.

We obtain the metric of the spherically symmetrical Lagrangian co-moving NRF by analogy with GRT in the form

$$dS^2 = \exp(v)(dy^0)^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - \exp(\lambda)(dr)^2, \quad (5.1)$$

where  $v$  and  $\lambda$  depend only on  $r$ .

The metric (5.1) is rigid one, as the metric coefficients do not depend on the time. Zero components  $g_{0k}$  mean that rotations are absent. The system (2.1) taking into account mentioned demands and the fulfillment of the co-moving conditions has the form

$$V^k = V_k = 0, \quad V^0 = (g_{00})^{-1/2}, \quad V_0 = (g_{00})^{1/2}, \\ A^1 = A(r), \quad A^0 = A^2 = A^3 = 0.$$

That may be reduced to the equation

$$A^1 = \frac{1}{2} \frac{dv}{dr} \exp(-\lambda). \quad (5.2)$$

Structure equations (2.7) satisfy (5.2) without additional demands for  $\nu(r)$  and  $\lambda(r)$  functions. Hence, according to the specified field of the first curvature vectors  $A^1$  it is impossible uniquely to find the metric (5.1) without additional conditions.

As stated above, from the physical encyclopaedia [1], "reference frames (RF) are the collections of the coordinate and clock system connected with the body relatively to which the motion (or the equilibrium) of any other mass points or bodies is studied"... . So to investigate the motion (equilibrium) of other bodies an analytical specification of the body properties (the basis of RF itself) is needed. And what means the RF in vacuum? The problem is not included in the physical encyclopaedia. In the Schwarzschild field vacuum presents outside the body. According to GRT in vacuum in the static field (as well as the alternating one) we understand that RF "... is the collection of the infinite number of the bodies filling all space like some medium" [4]. Let us examine some possibilities.

a) Let the observers be positioned at the earth's surface. They measure the gravitational field with accelerometers. The earth's rotation is not taken into account, its density is constant, and the earth has a spherical shape. They will detect that the acceleration field is directed along the radius from the centre perpendicular to the surface. To measure the field far from the surface in vacuum we use the set of radial weightless rigid rods. The accelerometer system is installed along these rods. The set of rods and accelerometers is a basis of the radially accelerated rigid reference frame. The acceleration field will decrease with the distance from the earth according to the Newton's gravitation law (in zero approximation). If the observers consider that their space is plane and the gravitation law is exact, the metric (5.1) has the form [6,15,16,17,18].

$$dS^2 = \exp(-r_g/r)(dy^0)^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - (dr)^2, \quad (5.3)$$

where  $r_g = 2GM/c^2$  is called "the gravitational radius",  $M$  is a body mass,  $G$  is the gravitation constant,  $c$  is the velocity of light in free space. When deriving (5.3) we took into account that by the definition of the flat space  $\lambda = 0$  and  $\nu$  was found from (5.2) and the Newton's gravitation law. We call metric (5.3) quasi-Newtonian one.

$$A^1 = \frac{1}{2} \frac{dv}{dr} = \frac{GM}{c^2 r^2}.$$

In spite of the space metric being flat, the space-time metric (5.3) is the Riemannian one. That contradicts the Newton's theory where not only space is flat but also space-time.

One can show [23] that the calculation of the pericentre displacement over one rotation according to the formula (5.3) is one-third of the one in the Schwarzschild metric. The change of the light ray direction when passing nearby the central body according to (5.3) is half as large as the Schwarzschild's one.

b) When deriving (5.3) one assumes  $\lambda = 0$  that corresponds to the flat space model. The system of the rigid non-deformable rods was selected as the reference frame outside the earth. The sound spreads on these rods with infinitely large velocity (that contradicts to the finite velocity of the interaction spreading). We assume that the interaction propagation velocity is finite and we permit that the basis structure of the radial accelerated NRF outside the earth is equivalent to some elastic medium subjected to deformations (and tensions), and the strain velocity tensor is equal to zero.

It is convenient to define the connection between the deformation and stress tensors in the Lagrange co-moving NRF considering the elastic medium for which the Hooke law [21,24] is valid

$$P^{ij} = \tilde{\lambda} I_1 \gamma^{ij} + 2\mu \gamma^{ik} \gamma^{jl} \varepsilon_{kl}, \quad I_1(\varepsilon) = \gamma^{kl} \varepsilon_{kl} = \frac{1}{2} (1 - \exp(-\lambda)), \quad (5.4)$$

where  $I_1$  is the first invariant of the deformation tensor,  $\tilde{\lambda}, \mu$  are the Lamé factors,  $\gamma^{ij} = -g^{ij}$  is the metric of the spatial section (5.4).

$$\varepsilon_{ij} = \frac{1}{2} (\gamma_{ij} - \gamma'_{ij}),$$

$\gamma'_{ij}$  is the metric tensor of the flat space in spherical coordinates.

The elastic medium has to satisfy the continuity equation

$$\nabla_\mu (\rho V^\mu) = 0.$$

The solution of the continuity equation results in the correlation

$$\rho = \rho_0 \exp(-\lambda/2), \quad (5.5)$$

where  $\rho_0$  is the density of the “medium” in the unstrained state.

The equations of the “motion” of the elastic medium in the Lagrange NRF have a form analogous to the equilibrium condition of the elastic medium in the classic Newtonian gravitational field

$$\nabla_j P^{ij} = -\rho a^j, \quad (5.6)$$

where  $a^j$  are the “unphysical” (affine) acceleration components, and the raising and the lowering of tensor indices and the calculation of the covariant derivative is realized by means of the spatial metric  $\gamma_{ij}$ . The metric (5.1) is orthogonal, and to construct the tetrad field one can combine vectors of ortho reference mark  $\vec{e}_\alpha$  with vectors of the affine reference mark. One can write the tetrad field in the form of the Lamé calibration [25,26]

$$e_{(\alpha)}^\mu = \frac{\delta_\alpha^\mu}{\sqrt{|g_{\alpha\alpha}|}}, \quad e_\mu^{(\alpha)} = \delta_\mu^\alpha \sqrt{|g_{\alpha\alpha}|},$$

where the summation of  $\alpha$  is absent. The tetrad tensor components coincide with the “physical” ones. Assuming that the tetrad 4-acceleration components correspond to (as in the case a). The Newtonian value in the flat space, from (5.5) and (5.6) we have the expression in the spherical coordinates

$$\exp(-\lambda) \frac{d\lambda}{dr} = -2 \frac{\rho_0 k M}{(\tilde{\lambda} + 2\mu)r^2}, \quad (5.7)$$

the integration of which results in the relation (provided that at infinity the space is flat  $\lambda = 0$ )

$$\exp(-\lambda) = \left(1 - \frac{2kM}{c_0^2 r}\right), \quad c_0^2 = \frac{\tilde{\lambda} + 2\mu}{\rho_0}, \quad (5.8)$$

where  $c_0$  is the longitudinal velocity of a sound. The tetrad or “physical” component of the first curvature vector of the body world line being in equilibrium in the quasi-Newtonian gravitational field is equal to the rod reaction force or the attracting force with opposite sign.

$$A^{(1)} = e_1^{(1)} A^1 = \delta_1^1 \sqrt{|g_{11}|} A^1 = GM / (cr)^2. \quad (5.9)$$

Whence the affine component of the first curvature vector  $A^1$  has the form

$$\begin{aligned} A^1 &= c^{-2} a^1 = (\gamma_{11})^{-\frac{1}{2}} GM / (cr)^2 \\ &= \exp(-\lambda/2) GM / (cr)^2 \end{aligned} \quad (5.10)$$

Applying (5.2) and (5.8) we find the equation for  $v$ .

$$\begin{aligned} \frac{dv}{dr} &= 2A^1 \exp \lambda = \exp\left(\frac{\lambda}{2}\right) 2GM / (cr)^2 \\ &= \frac{r_g}{r^2 \sqrt{1 - \alpha/r}}, \\ \alpha &= \frac{2GM}{c_0^2}, \quad r_g = \frac{2GM}{c^2}. \end{aligned} \quad (5.11)$$

The integration of (5.11) gives (provided that at infinity  $v = 0$ )

$$v = 2 \left(\frac{c_0}{c}\right)^2 \left( \sqrt{1 - \frac{2GM}{c_0^2 r}} - 1 \right). \quad (5.12)$$

The limit of the expressions (5.8) and (5.12) when  $c_0 \rightarrow \infty$  results in the metric (5.3) that corresponds to the Newtonian perfectly rigid body model. We refer to such a body as a relativistic rigid body in which the longitudinal velocity of sound is equal to the light velocity in vacuum [21]. The expression (5.8) coincides with the  $\gamma_{11}$  component of the Schwarzschild metric in the standard form. The  $g_{00}$  component of this metrics is obtained from (5.12) if one expands  $\exp(v)$  into a series and keeps only the first infinitesimal order on  $(r_g/r)$ .

We represent the final output in the form

$$\begin{aligned} dS^2 &= \exp\left\{2 \sqrt{1 - \frac{2GM}{c_0^2 r}} - 2\right\} (dy^0)^2 - r^2 (d\theta^2 + \\ &\sin^2 \theta d\phi^2) - \frac{dr^2}{1 - \frac{2GM}{c_0^2 r}} \end{aligned} \quad (5.13)$$

The calculation of the known GRT effects according to the metric (5.13) differs only slightly from the calculation in accordance with the Schwarzschild’s metric. Using [23] we find that the difference is in the calculation of the pericenter shift which is equal to 5/6 from the Schwarzschild’s one. The change of the direction of a light beam when passing close by the central body coincides with the Schwarzschild’s

one. Therefore, we refer to such metric (5.13) as a quasi-Einstein one. Time-series identification of the physical reference frame as a reference body with specified physical properties resulted in a new approach to Newton's and Einstein's gravitation theories. The physical properties of the reference frame are similar to the introduction of the quantum-mechanical complementary principle into the Newton gravitation theory. The space-time geometry depends on facilities by means of which it is observed. The atomic systems are not described independently of observation capability. It is clear that the reference frame has to possess properties which provide minimal distortion of the initial field.

## 6. BELL'S PROBLEM SOLUTION

Let us consider the Bell's problem [18,27-35]. Assume we have two space ships which are being at rest relatively to some inertial reference frame (IRF). These space ships are connected with a tight string. At the time zero in accordance with the IRF clock both space ships begin to accelerate with constant proper acceleration  $g$  measured with the accelerometers located on each shipboard. The problem is: whether the string will be broken or whether the distance between these space ships will be increased?

As accelerometers on two ships show the identical acceleration  $g$ , then the ship motion is equivalent to the motion of the charged dust in uniform force field with the law of motion (3.5). It follows from (3.5) that from the IRF observer viewpoint for any particle pair the equality holds

$$x_1^1(y_1^1, t) - x_2^1(y_2^1, t) = y_1^1 - y_2^1 = const.$$

The difference between the current Euler coordinates and the original Lagrangian coordinates at any instant of time  $t = const$  is constant and no Lorentz contractions in IRF occur. However according to (3.9) a "physical distance" between the rockets increases that from the SRT and the relativistic elasticity theory viewpoint have to result in the string rupture.

From our viewpoint the string will break if one strictly adheres to the SRT approach as in such a motion the relativistic (Born's) rigidity of the string is disturbed. Deformations and tensions in the medium occur when the medium does not move in a Born rigid way.

From physical viewpoint that situation is absurd. Two identical Bell's rockets with similar driving forces (similar accelerometer readings) move differently from the astronaut viewpoint. The second rocket decelerates from the first one although all physical conditions are identical. In order to the second rocket does not retard (from the astronaut viewpoint) it is necessary that it will move with the greater acceleration than the first one (3.12a).

In SRT the Bell's paradox is not solved as according to the proved lemma in the Minkowski space conditions of relativistic rigidity and global relativistic uniform acceleration are not simultaneously satisfied. To solve the paradox one must admit that it is impossible to realize the transition into NRF by means of the transformation of coordinates containing nonlinear time. Such transformations can not result in nonzero space-time curvature [36].

As both rockets are absolutely identical and have identical accelerations, in the rocket system, they must be at rest relative to each other. Therefore after the relaxation period both the Born rigidity and the relativistic uniformly acceleration in the co-moving reference frame are simultaneously realized for the string.

The obtained metric (3.18) and formulae (3.13), (3.19) solve the Bell's problem. These formulae correspond to the relativistic rigidity and the global uniformly acceleration.

In accordance with that the string will not break. But we exit out of the Minkowski space into the Riemannian one.

In [30,31] the original formula was obtained

$$L(t) = \frac{c^2}{a_0} \ln \left( \cosh \left( \frac{a_0 L_0}{c^2} \right) + \sinh \left( \frac{a_0 L_0}{c^2} \right) \sqrt{1 + \beta^2} \right). \quad (6.1)$$

This formula is based on the calculation of the "physical" spatial string length  $L$  as compared with its initial length  $L_0$ .

Comparison of (6.1) and a similar formula of the Lorentz shrinkage

$$L(t) = L_0 \sqrt{1 + a_0^2 t^2 / c^2} \quad (6.2)$$

resulted in a great difference when calculating electron bunch deformations in modern linear colliders in co-moving reference frames. A

standard calculation (see formula (6.2)) increased the bunch length at the output of the collider in a 40000 times, and the calculation according to the formula (6.1) increased this length in 1.003 times. Detailed results are presented in [30,31]. Although the formula (6.1) is correct both for large and low accelerations it does not solve the Bell paradox in principle.

“It is easy to join words into the expression “the coordinate system of the accelerated observer” however it is more difficult to search for the conception to which that might correspond to. By careful consideration this expression proved to be contradictory” [37,38].

The detailed critical analysis of the extended body mechanics in SRT is presented in [9]. G. C. McVITTIE wrote that the “satisfied relativistic form of the dynamics of a rigid body is still off the beam”.

## 7. CONCLUSION

It is proved that the translational globally uniformly accelerated and Born’s rigid continuum motion is impossible in the Minkowski space. If one imposes supplementary conditions for the rigidity or continuum rotations besides the continuum motion equations, these conditions “remove” the moving medium from the flat space-time.

The metric of the Born rigid globally uniformly accelerated continuous medium is realized in the Riemannian space-time. This metric combines the Möller’s metric properties (the Born rigidity) and the Logunov’s metric properties (the global uniformly acceleration).

It should be noted that the proper time obtained by Einstein [39], which was called the exact time, can be obtained from the metric (3.18) for the fixed Lagrangian particle.

$$\tau_s = \exp\left(\frac{a_0 y^1}{c^2}\right) \tau,$$

where  $\tau_s$  is the proper time for the given space point,  $\tau$  is the universal time. But Einstein dismissed the exact expression for the approximate (Möller) one.

The relativistic Born rigid uniformly revolving NRF without the restriction of the radius value

and having at infinity the linear velocity which is equal to the light velocity and finite acceleration, and realized in the Riemannian space time, is obtained. The Sagnac’s and Ehrenfest’s effects are explained.

The spherically symmetrical rigid NRF, having no analog in the Minkowski space, which is equivalent to the balance of gravitational forces to elastic ones, is created. If in the elastic medium the longitudinal velocity of sound coincides with the light velocity in free space, this body is the relativistic rigid one, and the equilibrium solution obtained is described with the metric close to the Schwarzschild’s one. For a classical solid the sound velocity goes to infinity but the equilibrium space-time metric remains the Riemannian one with the flat space. It turns out that the connection between Newton’s and Einstein’s theories is much closer than commonly thought.

A solution of the Bell’s problem is proposed.

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## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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