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Open String under the Modified Born-Infeld Field

Aeran Kim[1*]

[1] *A Private Mathematics Academy, 1-#406, 23, Maebong 5-gil, Deokjin-gu, Jeonju-si, Jeollabuk-do, 54921, Republic of Korea.*

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

In this article we consider the two end-points of the string to be attached to D-brane with the different Born-Infeld field strength $\mathcal{F}$ and calculate the total momenta for the special case.

Keywords: Bloch vector.

1 INTRODUCTION

We consider a string ending on a Dp-brane, the bosonic part of the action is

$$S_B = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \Big[g^{\alpha\beta}G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu + \epsilon^{\alpha\beta}B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\Big] + \frac{1}{2\pi\alpha'}\oint_{\partial\Sigma} d\tau A_i(X)\partial_\tau X^i,$$

where A_i $(i = 0, 1, \cdots, p)$, is the $U(1)$ gauge field living on the Dp-brane [1]; [2]; [3]. The string background is

$$G_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi = \text{constant}, \quad H = dB = 0.$$

Here we use the boundary condition of the action S_B so that we can get more specific equations of motion for a free field and the canonical momentum.

*Corresponding author: E-mail: ae_ran_kim@hotmail.com;

2 EQUATIONS OF MOTION AND THE CANONICAL MOMENTUM

Variation of the action yields the equations of motion for a free field

$$\left(\partial_\tau^2 - \partial_\sigma^2\right) X^\mu = 0 \tag{2.1}$$

and the following boundary conditions at $\sigma = 0$:

$$\begin{aligned} &\partial_\sigma X^i + \partial_\tau X^j \mathcal{F}_j^i = 0, \quad i,j = 0,1,\cdots,p, \\ &X^a = x_0^a, \qquad\qquad\qquad a = p+1,\cdots,9, \end{aligned} \tag{2.2}$$

and at $\sigma = \pi$:

$$\partial_\sigma X^i + \partial_\tau X^j {\mathcal{F}'}^i_j = 0, \qquad i,j = 0,1,\cdots,p. \tag{2.3}$$

Here

$$\mathcal{F} = B - F \qquad \text{and} \qquad \mathcal{F}' = B' - F'$$

are the modified Born-Infeld field strength and x_0^a, x_0^b are the location of the D-branes. Indices are raised and lowered by $\eta_{ij} = (-,+,\cdots,+)$.

The general solution of X^k to the equations of motion in (2.1) is [1]

$$\begin{aligned} X^k = x_0^k &+ \left(a_0^k \tau + b_0^k \sigma\right) + c_0^k \sigma\tau \\ &+ d_0^k \left(\tau^2 + \sigma^2\right) \\ &+ \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \left(i a_n^k \cos n\sigma + b_n^k \sin n\sigma\right) \end{aligned} \tag{2.4}$$

and

$$X^a = x_0^a \ + b^a \sigma + \sum_{n\neq 0} \frac{e^{-in\tau}}{n} a_n^a \sin n\sigma,$$
$$\text{for } a = p+1,\cdots,9,$$

where $x_0^a \ + \pi b^a$ is the location of the D-brane to which the other end-point of the open string is attached.

Lemma 2.1. *The coefficients c_0^k and d_0^k in Eq. (2.4) are*

(a)

$$\begin{aligned} c_0^k = \sum_{n\in\mathbb{Z}} (-1)^n \Bigg(& in(b_n^l + a_n^j {\mathcal{F}'}^l_j) \\ &+ \frac{1}{\pi}(b_n^j + a_n^k {\mathcal{F}'}^j_k){\mathcal{F}'}^l_j \Bigg) (M'^{-1})_l^k, \end{aligned}$$

(b)

$$\begin{aligned} d_0^k = \frac{1}{2}\sum_{n\in\mathbb{Z}} (-1)^{n-1} \Bigg(& \frac{1}{\pi}(b_n^l + a_n^j {\mathcal{F}'}^l_j) \\ &+ in(b_n^j + a_n^k {\mathcal{F}'}^j_k){\mathcal{F}'}^l_j \Bigg) (M'^{-1})_l^k, \end{aligned}$$

where $M'_{ij} = \eta_{ij} - {\mathcal{F}'}^k_i {\mathcal{F}'}_{kj}$.

Proof. By (2.3) and (2.4) we have

$$\begin{aligned} 0 &= \partial_\sigma X^k + \partial_\tau X^j {\mathcal{F}'}^k_j \\ &= \partial_\sigma \Bigg(x_0^k + (a_0^k\tau + b_0^k\sigma) + c_0^k\sigma\tau \\ &\qquad + d_0^k(\tau^2+\sigma^2) \\ &\qquad + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^k\cos n\sigma + b_n^k \sin n\sigma)\Bigg) \\ &\quad + \partial_\tau \Bigg(x_0^j + (a_0^j\tau + b_0^j\sigma) + c_0^j\sigma\tau \\ &\qquad + d_0^j\left(\tau^2+\sigma^2\right) \\ &\qquad + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^j\cos n\sigma + b_n^j \sin n\sigma)\Bigg){\mathcal{F}'}^k_j \\ &= b_0^k + c_0^k\tau + 2d_0^k\sigma \\ &\qquad + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(-ina_n^k \sin n\sigma + nb_n^k\cos n\sigma\right) \\ &\qquad + \Bigg(a_0^j + c_0^j\sigma + 2d_0^j\tau + \sum_{n\neq 0}\frac{-ine^{-in\tau}}{n} \\ &\qquad \times \left(ia_n^j \cos n\sigma + b_n^j \sin n\sigma\right)\Bigg){\mathcal{F}'}^k_j \\ &= b_0^k + a_0^j{\mathcal{F}'}^k_j + (c_0^k + 2d_0^j{\mathcal{F}'}^k_j)\tau \\ &\qquad + (2d_0^k + c_0^j{\mathcal{F}'}^k_j)\sigma \\ &\qquad - \sum_{n\neq 0} e^{-in\tau}\Big(i\sin n\sigma(a_n^k + b_n^j{\mathcal{F}'}^k_j) \\ &\qquad - \cos n\sigma(b_n^k + a_n^j{\mathcal{F}'}^k_j)\Big) \\ &= (c_0^k + 2d_0^j{\mathcal{F}'}^k_j)\tau + (2d_0^k + c_0^j{\mathcal{F}'}^k_j)\sigma \\ &\qquad - \sum_{n\in\mathbb{Z}} e^{-in\tau}\Big(i\sin n\sigma(a_n^k + b_n^j{\mathcal{F}'}^k_j) \\ &\qquad - \cos n\sigma(b_n^k + a_n^j{\mathcal{F}'}^k_j)\Big) \end{aligned}$$

then, now since $\sigma = \pi$ and using the Taylor series, this identity can be written as

$$\begin{aligned}
&(c_0^k + 2d_0^j {\mathcal{F}'}^k_j)\tau + (2d_0^k + c_0^j {\mathcal{F}'}^k_j)\pi \\
&\quad + \sum_{n\in\mathbb{Z}} e^{-in\tau}(-1)^n (b_n^k + a_n^j {\mathcal{F}'}^k_j) \\
&= (c_0^k + 2d_0^j {\mathcal{F}'}^k_j)\tau + (2d_0^k + c_0^j {\mathcal{F}'}^k_j)\pi \\
&\quad + \sum_{n\in\mathbb{Z}} \left(\sum_{m=0}^{\infty} \frac{(-in\tau)^m}{m!}\right)(-1)^n (b_n^k + a_n^j {\mathcal{F}'}^k_j) \\
&= \left((2d_0^k + c_0^j {\mathcal{F}'}^k_j)\pi + \sum_{n\in\mathbb{Z}}(-1)^n (b_n^k + a_n^j {\mathcal{F}'}^k_j)\right) \\
&\quad + \Bigg(c_0^k + 2d_0^j {\mathcal{F}'}^k_j \\
&\quad - i\sum_{n\in\mathbb{Z}}(-1)^n n (b_n^k + a_n^j {\mathcal{F}'}^k_j)\Bigg)\tau \\
&\quad + \sum_{n\in\mathbb{Z}} \left(\sum_{m=2}^{\infty} \frac{(-in)^m}{m!}\right)(-1)^n (b_n^k + a_n^j {\mathcal{F}'}^k_j)\tau^m \\
&= 0.
\end{aligned}$$

Thus the above identical equation about τ shows that

$$(2d_0^k + c_0^j {\mathcal{F}'}^k_j)\pi + \sum_{n\in\mathbb{Z}}(-1)^n (b_n^k + a_n^j {\mathcal{F}'}^k_j) = 0, \tag{2.5}$$

$$c_0^k + 2d_0^j {\mathcal{F}'}^k_j - i\sum_{n\in\mathbb{Z}}(-1)^n n (b_n^k + a_n^j {\mathcal{F}'}^k_j) = 0, \tag{2.6}$$

and

$$\sum_{n\in\mathbb{Z}} n^m (-1)^n (b_n^k + a_n^j {\mathcal{F}'}^k_j) = 0 \quad \text{for } m \geq 2.$$

(a) From (2.5) we can easily obtain

$$\begin{aligned}
&2d_0^j {\mathcal{F}'}^k_j + c_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&\quad + \frac{1}{\pi}\sum_{n\in\mathbb{Z}}(-1)^n (b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j = 0.
\end{aligned} \tag{2.7}$$

Subtracting (2.7) from (2.6) we get

$$\begin{aligned}
&c_0^k - i\sum_{n\in\mathbb{Z}}(-1)^n n (b_n^k + a_n^j {\mathcal{F}'}^k_j) \\
&\quad - c_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&\quad - \frac{1}{\pi}\sum_{n\in\mathbb{Z}}(-1)^n (b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j = 0
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{n\in\mathbb{Z}}(-1)^n \Bigg(in(b_n^k + a_n^j {\mathcal{F}'}^k_j) \\
&\qquad + \frac{1}{\pi}(b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j\Bigg) \\
&= c_0^k - c_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&= c_0^l \eta_l^k - c_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&= c_0^l (\eta_l^k - {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j) \\
&= c_0^l {M'}_l^k
\end{aligned}$$

so

$$\begin{aligned}
c_0^l = &\sum_{n\in\mathbb{Z}}(-1)^n \Bigg(in(b_n^k + a_n^j {\mathcal{F}'}^k_j) \\
&\quad + \frac{1}{\pi}(b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j\Bigg)({M'}^{-1})^l_k.
\end{aligned}$$

(b) In a similar manner, by (2.6) we have

$$\begin{aligned}
&c_0^j {\mathcal{F}'}^k_j + 2d_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&\quad - i\sum_{n\in\mathbb{Z}}(-1)^n n (b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j = 0.
\end{aligned} \tag{2.8}$$

After dividing (2.5) by π, we subtract (2.8) from (2.5) and obtain

$$\begin{aligned}
&2d_0^k + \frac{1}{\pi}\sum_{n\in\mathbb{Z}}(-1)^n (b_n^k + a_n^j {\mathcal{F}'}^k_j) \\
&\quad - 2d_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&\quad + i\sum_{n\in\mathbb{Z}}(-1)^n n (b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j = 0
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{2}\sum_{n\in\mathbb{Z}}(-1)^{n-1} \Bigg(\frac{1}{\pi}(b_n^k + a_n^j {\mathcal{F}'}^k_j) \\
&\qquad + in(b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j\Bigg) \\
&= d_0^k - d_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&= d_0^l \eta_l^k - d_0^l {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j \\
&= d_0^l (\eta_l^k - {\mathcal{F}'}^j_l {\mathcal{F}'}^k_j) \\
&= d_0^l {M'}_l^k
\end{aligned}$$

so

$$d_0^l = \frac{1}{2}\sum_{n\in\mathbb{Z}}(-1)^{n-1}\Bigg(\frac{1}{\pi}(b_n^k + a_n^j{\mathcal{F}'}_j^k) + in(b_n^j + a_n^l{\mathcal{F}'}_l^j){\mathcal{F}'}_j^k\Bigg)(M'^{-1})_k^l.$$

□

Remark 2.1. Let us consider the two end-points of the string to be attached to D-brane with the same $\mathcal{F}$ field. Then we can see that

$$b_n^k + a_n^j\mathcal{F}_j^k = 0, \qquad \text{for all } n$$

in [1]. Applying this fact to Lemma 2.1, we simply have $c_0^k = d_0^k = 0$, which equates the result obtained in [1].

Now the canonical momentum is given by

$$2\pi\alpha' P^k(\tau,\sigma) = \partial_\tau X^k + \partial_\sigma X^j\left(\frac{\mathcal{F}_j^k + {\mathcal{F}'}_j^k}{2}\right)$$

So by (2.4), we note that

$$\begin{aligned}
&2\pi\alpha' P^k(\tau,\sigma)\\
&= \partial_\tau\Big(x_0^k + a_0^k\tau + b_0^k\sigma + c_0^k\sigma\tau + d_0^k(\tau^2+\sigma^2)\\
&\quad + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^k\cos n\sigma + b_n^k\sin n\sigma)\Big)\\
&\quad + \partial_\sigma\Big(x_0^j + a_0^j\tau + b_0^j\sigma + c_0^j\sigma\tau + d_0^j(\tau^2+\sigma^2)\\
&\quad + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^j\cos n\sigma + b_n^j\sin n\sigma)\Big)\\
&\quad \times\Big(\frac{\mathcal{F}_j^k + {\mathcal{F}'}_j^k}{2}\Big)\\
&= a_0^k + c_0^k\sigma + 2d_0^k\tau\\
&\quad - i\sum_{n\neq 0}e^{-in\tau}(ia_n^k\cos n\sigma + b_n^k\sin n\sigma)\\
&\quad + \Big(b_0^j + c_0^j\tau + 2d_0^j\sigma - \sum_{n\neq 0}e^{-in\tau}(ia_n^j\sin n\sigma\\
&\quad - b_n^j\cos n\sigma)\Big)\Big(\frac{\mathcal{F}_j^k + {\mathcal{F}'}_j^k}{2}\Big)\\
&= \Big(a_0^k + \frac{b_0^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)}{2}\Big)\\
&\quad + \Big(c_0^k + d_0^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)\Big)\sigma\\
&\quad + \Big(2d_0^k + \frac{c_0^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)}{2}\Big)\tau\\
&\quad - \sum_{n\neq 0}e^{-in\tau}\Big\{i\Big(b_n^k + \frac{a_n^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)}{2}\Big)\sin n\sigma\\
&\quad - \Big(a_n^k + \frac{b_n^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)}{2}\Big)\cos n\sigma\Big\}\\
&= \Big(c_0^k + d_0^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)\Big)\sigma\\
&\quad + \Big(2d_0^k + \frac{c_0^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)}{2}\Big)\tau\\
&\quad - \sum_{n\in\mathbb{Z}}e^{-in\tau}\Big\{i\Big(b_n^k + \frac{a_n^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)}{2}\Big)\sin n\sigma\\
&\quad - \Big(a_n^k + \frac{b_n^j(\mathcal{F}_j^k + {\mathcal{F}'}_j^k)}{2}\Big)\cos n\sigma\Big\}.
\end{aligned} \tag{2.9}$$

Theorem 2.2. *If $\mathcal{F}' = -\mathcal{F}$, the total momenta*

$$P_{tot}^k(\tau) = \frac{\pi}{4\alpha'}c_0^k + \frac{1}{\alpha'}d_0^k\tau + \frac{1}{2\alpha'}a_0^k + \frac{1}{2\pi\alpha'}\sum_{n\neq 0}\frac{ie^{-in\tau}}{n}\left((-1)^n - 1\right)b_n^k,$$

where

$$\begin{aligned}
c_0^k &= \frac{i}{2}\sum_{n\in\mathbb{Z}}n\left((1+(-1)^n)b_n^k + (1-(-1)^n)a_n^j\mathcal{F}_j^k\right),\\
d_0^k &= \frac{i}{4}\sum_{n\in\mathbb{Z}}n\left((1+(-1)^n)b_n^j\mathcal{F}_j^k + (1-(-1)^n)a_n^i\mathcal{F}_i^j\mathcal{F}_j^k\right)\\
&\quad - \frac{1}{2\pi}\sum_{n\in\mathbb{Z}}(-1)^n\left(b_n^k - a_n^j\mathcal{F}_j^k\right).
\end{aligned}$$

Proof. By the condition $\mathcal{F}' = -\mathcal{F}$ and (2.9), we have

$$\begin{aligned}
&2\pi\alpha' P^k(\tau,\sigma)\\
&= c_0^k\sigma + 2d_0^k\tau\\
&\quad - \sum_{n\in\mathbb{Z}}e^{-in\tau}\left(ib_n^k\sin n\sigma - a_n^k\cos n\sigma\right)
\end{aligned}$$

and so

$$
\begin{aligned}
&P^k_{tot}(\tau)\\
&=\int_0^\pi d\sigma P^k(\tau,\sigma)\\
&=\frac{1}{2\pi\alpha'}\int_0^\pi d\sigma\Bigg(c^k_0\sigma+2d^k_0\tau\\
&\qquad-\sum_{n\in\mathbb{Z}}e^{-in\tau}\left(ib^k_n\sin n\sigma-a^k_n\cos n\sigma\right)\Bigg)\\
&=\frac{1}{2\pi\alpha'}\Bigg(\frac{\pi^2}{2}c^k_0+2\pi d^k_0\tau+a^k_0\pi\\
&\qquad+\sum_{n\neq 0}\frac{ie^{-in\tau}}{n}\left((-1)^n-1\right)b^k_n\Bigg)\\
&=\frac{\pi}{4\alpha'}c^k_0+\frac{1}{\alpha'}d^k_0\tau+\frac{1}{2\alpha'}a^k_0\\
&\qquad+\frac{1}{2\pi\alpha'}\sum_{n\neq 0}\frac{ie^{-in\tau}}{n}\left((-1)^n-1\right)b^k_n.
\end{aligned}
$$

And using the boundary condition (2.2) and Taylor series for τ we obtain

$$\sum_{n\in\mathbb{Z}}(b^k_n+a^j_n\mathcal{F}^k_j)=0, \tag{2.10}$$

$$c^k_0+2d^j_0\mathcal{F}^k_j-i\sum_{n\in\mathbb{Z}}n(b^k_n+a^j_n\mathcal{F}^k_j)=0, \tag{2.11}$$

and

$$\sum_{n\in\mathbb{Z}}n^m(b^k_n+a^j_n\mathcal{F}^k_j)=0 \quad \text{for } m\geq 2.$$

Also applying the assumption $\mathcal{F}'=-\mathcal{F}$ to Eqs. (2.5) and (2.6), we have

$$(2d^k_0-c^j_0\mathcal{F}^k_j)\pi+\sum_{n\in\mathbb{Z}}(-1)^n(b^k_n-a^j_n\mathcal{F}^k_j)=0, \tag{2.12}$$

$$c^k_0-2d^j_0\mathcal{F}^k_j-i\sum_{n\in\mathbb{Z}}(-1)^n n(b^k_n-a^j_n\mathcal{F}^k_j)=0. \tag{2.13}$$

Then by (2.11) and (2.13) we deduce that

$$
\begin{aligned}
2c^k_0-i\sum_{n\in\mathbb{Z}}n(b^k_n+a^j_n\mathcal{F}^k_j)&\\
-i\sum_{n\in\mathbb{Z}}(-1)^n n(b^k_n-a^j_n\mathcal{F}^k_j)&=0
\end{aligned}
$$

so

$$
\begin{aligned}
c^k_0=\frac{i}{2}\sum_{n\in\mathbb{Z}}n\Big((1+(-1)^n)b^k_n&\\
+(1-(-1)^n)a^j_n\mathcal{F}^k_j\Big)&.
\end{aligned}
$$

Finally, substituting the above c^k_0 into (2.12) we complete the proof. □

3 CONCLUSION

Here we focus on the coefficients c^k_0 and d^k_0 existing in the general solution of X^k explaining the equations of brane motion given by [1], i.e.,

$$
\begin{aligned}
X^k=x^k_0&+\left(a^k_0\tau+b^k_0\sigma\right)+c^k_0\sigma\tau\\
&+d^k_0\left(\tau^2+\sigma^2\right)\\
&+\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(ia^k_n\cos n\sigma+b^k_n\sin n\sigma\right)
\end{aligned}
$$

and obtain coefficients value.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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