



Effects of Some Flow Parameters on Unsteady MHD Fluid Flow Past a Moving Vertical Plate Embedded in Porous Medium in the Presence of Hall Current and Rotating System

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Authors' contributions

This work was carried out in collaboration among all authors. Author MOD designed the study. Author KAJ performed the numerical study and wrote the first draft of the manuscript. Authors MOD, KAJ and FOO managed the analyses of the study and the literature searches. All authors read and approved the final manuscript.

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Abstract

This paper is on the numerical study of the effects of some flow parameters like Hall current, rotation, thermal diffusion (Soret) and diffusion thermo (Dufour) on unsteady magnetohydrodynamic natural convective heat and mass transfer of a viscous, rotating, electrically conducting and incompressible fluid flow past an impulsively moving vertical plate embedded in porous medium. The fundamental governing dimensionless coupled boundary layer partial differential equations are solved by the method of lines (MOL). Computations are then performed to determine the effects of the governing flow parameters. The results show that an increase in Soret number, Dufour number and Hall current parameter, causes an increase in the primary and secondary velocities of the fluid flow. As rotating parameter increases, the primary velocity of the flow decreases. Similarly, as Dufour and Soret numbers increase, the temperature and concentration profiles of the fluid flow increase. The effects of the flow parameters on primary and secondary velocity, temperature and concentration fields for externally cooling of the plate are shown graphically.

Keywords: MHD flow; hall current; rotating system; method of lines (MOL).

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1 Introduction

Heat and mass transfer (double diffusion) finds applications in a variety of engineering processes. Natural processes such as reduction of toxic waste in water bodies, vaporization of mist and fog, photosynthesis, drying of porous solids, transpiration, sea-wind formation where upward convection is modified by Coriolis forces, and formation of ocean currents, occur due to thermal and solutal buoyancy forces developed as a result of difference in temperature or concentration or a combination of these two, Bejan [1].

Considering the importance of fluid flow problems, extensive researches have been carried out by many authors Yih [2], Chamkha et al. [3], Ganesan and Palami [4]. Chen [5] analyzed combined heat and mass transfer in MHD free convection flow from a vertical surface with Ohmic heating and viscous dissipation. Ibrahim et al. [6] considered unsteady MHD micropolar fluid flow and heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusions with a constant heat source.

In this paper, we study the effects of Hall current and rotation on unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting and optically thick radiating fluid past an impulsively moving vertical plate embedded in a fluid saturated porous medium considering the effects of thermal and mass diffusions when temperature of the plate has a ramped profile. Natural convection heat and mass transfer flow resulting from such ramped temperature profile of a plate and thermal radiation prevalent, has significant effects in designing of electromagnetic devices, high temperature aerodynamics, plasma physics, cosmic flight, nuclear power reactors etc. where initial temperature profiles are of much significance and thermal radiation is highly prevalent.

2 Mathematical Model and Analysis

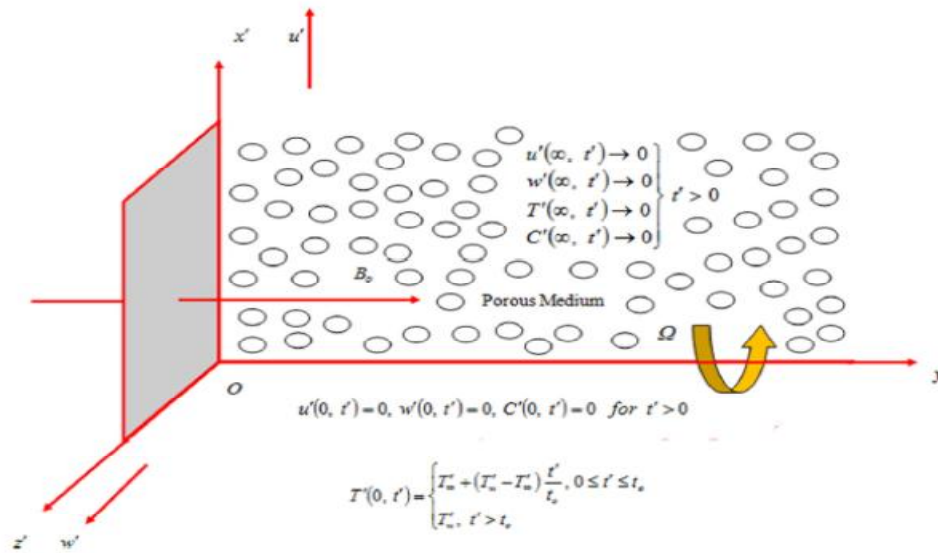


Fig. 1. The geometry of the problem (Source: Jithender et al. [9])

We consider an unsteady MHD natural convection flow with heat and mass transfer of an optically thick radiating, incompressible and electrically conducting viscous fluid past an infinite vertical plate, embedded in a uniform porous medium with a rotating system, taking Hall Current into account. The Hall effect is the production of a voltage difference across an electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current. It was discovered by Edwin Hall in

1879 while he was working on his doctoral degree at Johns Hopkins University in Baltimore, Maryland. Eighteen years before the electron was discovered, his measurements of the tiny effect produced in the apparatus he used were an experimental tour de force, Edwin Hall [7].

Also, we consider x' -axis along the plate in upward direction and y' -axis normal to plane of the plate in the fluid. A uniform transverse magnetic field B_0 is applied in a direction which is parallel to y' -axis. The fluid and the plate rotate with uniform angular velocity Ω' about the y' -axis. Initially i.e. at time $t' \leq 0$, both the fluid and plate are in rest and these are maintained at a uniform temperature T'_∞ . Also, species concentration is at the surface of the plate as well as at every point within the fluid and it is maintained at uniform concentration C'_∞ . At time $t' > 0$, plate starts moving in x' -direction with uniform velocity U_0 in its own plane. The temperature of the plate is raised or lowered to $T'_\infty + \frac{(T'_w - T'_\infty)t'}{t_0}$ when $0 < t' \leq t_0$ and it is maintained at uniform temperature T'_w when $t' > t_0$. Also, at time $t' > 0$, species concentration is at the surface of the plate, and it is raised to uniform species concentration C'_w and it is maintained thereafter. Since the plate is an infinite extent in x' and z' directions and it is electrically non-conducting, all physical quantities except pressure depends on y' and t' only. Also, no applied or polarized voltages are assumed to exist, so that the effect of polarization of fluid is negligible. The induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications. Cramer and Pai [8]. Fig. 1 shows the geometry of the problem.

Keeping in view of these assumptions and under the Boussinesq's approximation, the governing equations are given by Jithender et al. [9]:

Momentum equation along x' -axis

$$\begin{aligned} \frac{\partial u'}{\partial t'} + 2\Omega'w' &= \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(u' + mw') \\ -\frac{\nu u'}{K_1} + g\beta'(T' - T'_\infty) &+ g\beta^*(C' - C'_\infty) \end{aligned} \quad (1)$$

Momentum equation along z' -axis

$$\frac{\partial w'}{\partial t'} - 2\Omega'u' = \nu \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(mu' - w') - \frac{\nu w'}{K_1} \quad (2)$$

Energy equation

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} + \frac{D_m K_T \partial^2 C'}{c_s c_p \partial y'^2} \quad (3)$$

Concentration equation

$$\frac{\partial C'}{\partial t'} = D_m \frac{\partial^2 C'}{\partial y'^2} + \frac{D_m K_T \partial^2 T'}{T_m \partial y'^2} \quad (4)$$

Subject to the boundary conditions:

$$\text{For } t' \leq 0: u' = w' = 0, T' = T'_\infty, C' = C'_\infty \text{ for } y' \geq 0 \quad (5)$$

$$\text{For } t' > 0: u' = U_0, w' = 0, C' = C'_w \text{ at } y' = 0 \quad (6)$$

$$T' = T'_\infty + \frac{(T'_w - T'_\infty)t'}{t_0} \text{ at } y' = 0 \text{ for } 0 < t' \leq t_0 \quad (7)$$

$$\text{For } t' > t_0: T' = T'_w, \text{ at } y' = 0 \quad (8)$$

$$\text{For } t' > 0, u' = w' = 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ at } y' \rightarrow \infty \quad (9)$$

where B_0 -uniform applied magnetic field (T), $u'(ms^{-1})$ -fluid velocity along the x' -axis, $w'(ms^{-1})$ -fluid velocity along the z' -axis, t_0 - characteristic time (s), c_p - specific heat at constant pressure ($Jkg^{-1}K$), g - acceleration due to gravity (ms^{-2}), K_1 -permeability parameter, K_T - thermal diffusion ratio, T_m -mean fluid temperature (K), C_s -Concentration susceptibility (mol^{-1}), p -fluid pressure (Nm^{-2}), q_r -radiative flux, m - hall current parameter, N - radiation parameter, C' -species concentration (kgm^{-3}), C'_∞ -species concentration of the fluid far away from the plate (kgm^{-3}), C'_w -species concentration at the plate (kgm^{-3}), D_m - molecular mass diffusivity (m^2s^{-1}), D_T -molecular diffusivity (m^2s^{-1}), T'_w -temperature at the plate (K), T'_∞ -temperature of the fluid far away from the plate (K), t' -time (s), T' -Fluid temperature (K), U_0 -plate velocity (ms^{-1}), T - non-dimensional temperature (K), C -non-dimensional species concentration (kgm^{-3}), ρ -fluid density (kgm^{-3}), κ -thermal conductivity ($Wm^{-1}K^{-1}$), σ -electrical conductivity (Sm^{-1}), ν -kinematic viscosity (m^2s^{-1}), β' - coefficient of volume expansion for heat transfer (K^{-1}), Ω -rotation parameter (degrees), Ω' -uniform angular, velocity (degrees), β^* -coefficient of volume expansion for mass transfer (m^3kg^{-1}).

In momentum equation (1), the terms $\frac{\sigma B_0^2}{\rho(1+m^2)}(u' + mw')$, $g\beta'(T' - T'_\infty)$, $g\beta^*(C' - C'_\infty)$ represents the magnetohydrodynamic effect due to Lorentz force, thermal buoyancy effects and concentration buoyancy effects respectively. In momentum equation (2), the term $\frac{\sigma B_0^2}{\rho(1+m^2)}(mu' - w')$ represents the magnetohydrodynamic effect due to Lorentz force. In energy equation (3), the terms $\frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'}$, $\frac{D_m K_T \partial^2 C'}{C_s c_p \partial y'^2}$ represent the radiation and diffusion thermal (Dufour) effect. In concentration equation (4), the terms $D_m \frac{\partial^2 C'}{\partial y'^2}$, $\frac{D_m K_T \partial^2 T'}{T_m \partial y'^2}$ represent the molecular diffusivity and thermal diffusion effect (Soret). The radiative heat flux term q_r in energy equation (3), by using Rosseland approximation, Sparrow and Cess [10] is given by:

$$q_r = -\frac{4\sigma^* \partial T'^4}{3k^* \partial y'} \quad (10)$$

where k^* - mean absorption coefficient (m^{-1}), σ^* - Stefan-Boltzmann constant ($Wm^{-2}K^{-4}$). Here, by using the Rosseland approximation, the analysis is limited to optically thick fluid. If the temperature differences within the flow are sufficiently very small, equation (10) can be linearized by expanding T' into the Taylor series about T'_∞ , which after neglecting higher order terms take the form:

$$T'^4 \approx 4T'^3_\infty - 3T'^4_\infty \quad (11)$$

Substituting equations (10) and (11) into equation (3) gives:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{16\sigma^* T'^3_\infty}{3k^*} \frac{\partial^2 T'}{\partial y'^2} + \frac{D_m K_T \partial^2 C'}{C_s c_p \partial y'^2} \quad (12)$$

To transform the governing equations and boundary conditions into dimensionless form, the following non-dimensional quantities are introduced, Jithender et al. [9]:

$$u = \frac{u'}{U_0}, \quad w = \frac{w'}{U_0}, \quad y = \frac{y' U_0}{\nu}, \quad t = \frac{u_0^2 t'}{\nu},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty} \quad (13)$$

$$M^2 = \frac{\sigma B_0^2 v}{\rho(U_0^2)}, \Omega = \frac{v \Omega'}{U_0^2}, K_1 = \frac{K_1' U_0^2}{v^2},$$

$$G_r = \frac{g\beta v(T_w' - T_\infty')}{U_0^3}, P_r = \frac{v\rho c_p}{k} = \frac{v}{\alpha}, \quad (14)$$

$$S_c = \frac{v}{D}, G_m = \frac{g\beta^* v(C_w' - C_\infty')}{U_0^3}, N = \frac{16\sigma^* \partial T_\infty'^3}{3k^* k} \quad (15)$$

$$S_r = \frac{D_m K_T(T_w' - T_\infty')}{v T_m(C_w' - C_\infty')}, D_r = \frac{D_m K_T(C_w' - C_\infty')}{v T_m(T_w' - T_\infty')} \quad (16)$$

$$k_r^2 = \frac{k_r'^2 v}{v_0^2}, R = \frac{16\sigma_s T_\infty'^3}{3k_e k}, M = \frac{\sigma B_0^2 v}{\rho v_0^2} \quad (17)$$

In view of equations (13) – (17), the equations (1), (2), (4) and (12) reduce to dimensionless forms:

$$\frac{\partial u}{\partial t} + 2\Omega w = \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{(1+m^2)}(u + mw) - \frac{u}{K_1} + G_r T + G_m C \quad (18)$$

$$\frac{\partial w}{\partial t} - 2\Omega u = \frac{\partial^2 w}{\partial y^2} - \frac{M^2}{(1+m^2)}(mw - u) - \frac{w}{K_1} \quad (19)$$

$$\frac{\partial T}{\partial t} = \frac{1+N}{P_r} \frac{\partial^2 T}{\partial y^2} + D_r \frac{\partial^2 C}{\partial y^2} \quad (20)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} + S_r \frac{\partial^2 T}{\partial y^2} \quad (21)$$

where u and w are the non-dimensional fluid velocities along the x' and z' axes respectively, T -non-dimensional temperature, C - non-dimensional specie concentration, Ω -rotating parameter, M^2 – magnetic parameter, m -hall current parameter, K_1 - Permeability parameter, G_r -grashof number for heat transfer, G_m -grashof number for mass transfer, N – radiation parameter, P_r - Prandtl number, D_r -dufour number, S_c – Schmidt number, S_r – Soret number.

Similarly, the boundary conditions in equations (5) – (9) reduce to the dimensionless forms:

$$\text{For } t \leq 0: u = w = 0, T = 0, C = 0 \text{ for } y \geq 0 \quad (22)$$

$$\text{For } t > 0: u = 1, w = 0, C = 1 \text{ at } y = 0 \quad (23)$$

$$\text{For } 0 < t' \leq 1: T = t \text{ at } y = 0 \quad (24)$$

$$\text{For } t > 1: T = 1, \text{ at } y = 0 \quad (25)$$

$$\text{For } t > 0, u \rightarrow 0, w \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ at } y \rightarrow \infty \quad (26)$$

3 Method of Lines (MOL)

The basic idea of the MOL is to replace the spatial (boundary value) derivatives in the PDE with algebraic approximations, Biazar and Nomidi [11], Shiesser [12], Knapp [13]. Once this is done, only the initial value variable, typically time in a physical problem, remains. Then, with only one remaining independent variable, we have a system of ODEs that approximates the original PDE. Any suitable integration algorithm for initial value ODEs can now be used to compute an approximate numerical solution to the PDE.

For computation and linearization purpose, and to explicitly decouple equations (18) – (21), we adopt the following approximations: w, C, T in equation (18), u in equation (19), $\frac{\partial^2 C}{\partial y^2}$ in equation (20), and $\frac{\partial^2 T}{\partial y^2}$ in equation (21) to be unity (constant) i.e. $w = 1, C = 1, T = 1$ in equation (18), $u = 1$ in eq. (19), $\frac{\partial^2 C}{\partial y^2} = 1$ in equation (20), and $\frac{\partial^2 T}{\partial y^2} = 1$ in equation (21). Chung [14].

Rewriting equations (18) – (21), with approximation adopted above, we have:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+m^2}(u + m) - \frac{u}{K_1} + G_r + G_m - 2\Omega \tag{27}$$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} - \frac{M^2}{1+m^2}(m - w) - \frac{w}{K_1} + 2\Omega \tag{28}$$

$$\frac{\partial T}{\partial t} = \frac{1+N}{P_r} \frac{\partial^2 T}{\partial y^2} + D_r \tag{29}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} + S_r \tag{30}$$

Then, we solve equations (27) – (30) subject to the transformed boundary conditions (22) – (26) by method of lines (MOL).

Discretizing equation (17) in space variable y while leaving time variable t continuous, we have the system of ODEs:

$$\left(\frac{du}{dt}\right)_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} - \frac{M^2}{1+m^2}(u_i + m) - \frac{u_i}{k_1} + G_r + G_m - 2\Omega \tag{31}$$

$$\begin{aligned} &= \frac{u_{i+1}}{h^2} - \frac{2u_i}{h^2} + \frac{u_{i-1}}{h^2} - \frac{M^2}{1+m^2}u_i - \frac{M^2 m}{1+m^2} - \frac{u_i}{k_1} + G_r + G_m - 2\Omega \\ &= \frac{1}{h^2}u_{i-1} - \left(\frac{2}{h^2} + \frac{M^2}{1+m^2} + \frac{1}{k_1}\right)u_i + \frac{1}{h^2}u_{i+1} - \left(\frac{M^2 m}{1+m^2} - G_r - G_m + 2\Omega\right) \\ &= \alpha_1 u_{i-1} - \alpha_2 u_i + \alpha_3 u_{i+1} - \alpha_4 \end{aligned} \tag{32}$$

$$\text{where } \alpha_1 = \frac{1}{h^2}, \alpha_2 = \frac{2}{h^2} + \frac{M^2}{1+m^2} + \frac{1}{k_1}, \alpha_3 = \frac{1}{h^2}, \alpha_4 = \frac{M^2 m}{1+m^2} - G_r - G_m + 2\Omega \tag{33}$$

Now, equations (32) – (33) with conditions $u(0, t) = u_0(y, t) = 1$ and $u(\infty, t) \approx u(N + 1, t) = 0$ can be solved iteratively. For $i = 1, 2, \dots, N$, $u(0, t) = u_0(y, t) = 1$ and $u(\infty, t) \approx u(N + 1, t) = 0$, equation (32) can be written in matrix form:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \vdots \\ \dot{u}_{N-1} \\ \dot{u}_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{bmatrix} + \begin{bmatrix} \alpha_4 \\ \alpha_4 \\ \alpha_4 \\ \vdots \\ \alpha_4 \end{bmatrix} \tag{34}$$

where the coefficients $\alpha_1, \alpha_2, \alpha_3$ and α_4

are given by eq. (33) and $\dot{u}_i = \left(\frac{du}{dt}\right)_i$

In a similar way, equation (28) becomes:

$$\left(\frac{dw}{dt}\right)_i = \frac{w_{i+1}-2w_i+w_{i-1}}{h^2} + \frac{M^2}{1+m^2}(m-w_i) - \frac{w_i}{k_1} + 2\Omega \tag{35}$$

$$\begin{aligned} &= \frac{1}{h^2}w_{i-1} - \left(\frac{2}{h^2} + \frac{M^2}{1+m^2} + \frac{1}{k_1}\right)w_i + \frac{1}{h^2}w_{i+1} + \frac{M^2m}{1+m^2} + 2\Omega \\ &= \beta_1w_{i-1} - \beta_2w_i + \beta_3w_{i+1} + \beta_4 \end{aligned} \tag{36}$$

$$\text{Where } \beta_1 = \beta_3 = \frac{1}{h^2}, \beta_2 = \frac{2}{h^2} + \frac{M^2}{1+m^2} + \frac{1}{k_1}, \beta_4 = \frac{M^2m}{1+m^2} + 2\Omega \tag{37}$$

Now, equations (36) – (37) with conditions $w(0, t) = w_0(y, t) = 1$ and $w(\infty, t) \approx w(N + 1, t) = 0$ can be solved iteratively. For $i = 1, 2, \dots, N$, $w(0, t) = w_0(y, t) = 0$ and $w(\infty, t) \approx w(N + 1, t) = 0$, eq. (35) can be written in matrix form:

$$\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \vdots \\ \dot{w}_{N-1} \\ \dot{w}_N \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 0 \\ w_1 \\ w_2 \\ \vdots \\ w_{N-1} \\ w_N \end{bmatrix} + \begin{bmatrix} \beta_4 \\ \beta_4 \\ \beta_4 \\ \vdots \\ \beta_4 \end{bmatrix} \tag{38}$$

where the coefficients $\beta_1, \beta_2, \beta_3$ and β_4 are given by equation (37) and $\dot{w}_i = \left(\frac{dw}{dt}\right)_i$

Also, equation (29) becomes:

$$\left(\frac{dT}{dt}\right)_i = \frac{1+N}{P_r} \left(\frac{T_{i+1}-2T_i+T_{i-1}}{h^2}\right) + D_r \tag{39}$$

$$\begin{aligned} &= \frac{1+N}{h^2P_r}T_{i-1} - 2\left(\frac{1+N}{h^2P_r}\right)T_i + \frac{1+N}{h^2P_r}T_{i+1} + D_r \\ &= c_1T_{i-1} - c_2T_i + c_3T_{i+1} + c_4 \end{aligned} \tag{40}$$

$$\text{where } c_1 = c_3 = \frac{1+N}{h^2}, c_2 = 2\left(\frac{1+N}{h^2P_r}\right), c_4 = D_r \tag{41}$$

Now, equation (40) – (41) with conditions $T(0, t) = T_0(y, t) = 1$ and $T(\infty, t) \approx T(N + 1, t) = 0$ can be solved iteratively.

For $i = 1, 2, \dots, N$, $T(0, t) = T_0(y, t) = 1$ and $T(\infty, t) \approx T(N + 1, t) = 0$, equation (40) can be written in matrix form:

$$\begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \vdots \\ \dot{T}_{N-1} \\ \dot{T}_N \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \gamma_1 & \gamma_2 & \gamma_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} 1 \\ T_1 \\ T_2 \\ \vdots \\ T_{N-1} \\ T_N \end{bmatrix} + \begin{bmatrix} \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \vdots \\ \gamma_4 \end{bmatrix} \tag{42}$$

where the coefficients $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are given by equation (41) and $\dot{T}_i = \left(\frac{dT}{dt}\right)_i$

In the same way, equation (30) becomes:

$$\left(\frac{dC}{dt}\right)_i = \frac{1}{S_c} \left(\frac{C_{i+1}-2C_i+C_{i-1}}{h^2}\right) + S_r \tag{43}$$

$$\begin{aligned} &= \frac{1}{h^2S_c}C_{i-1} - \frac{2}{h^2S_c}C_i + \frac{1}{S_ch^2}C_{i+1} + S_r \\ &= \delta_1C_{i-1} - \delta_2C_i + \delta_3C_{i+1} + \delta_4 \end{aligned} \tag{44}$$

$$\text{where } \delta_1 = \delta_3 = \frac{1}{h^2S_c}, \delta_2 = \frac{2}{h^2S_c}, \delta_4 = S_r \tag{45}$$

Now, equation (44) – (45) with conditions $C(0, t) = C_0(y, t) = 1$ and $C(\infty, t) \approx C(N + 1, t) = 0$ can be solved iteratively. For $i = 1, 2, \dots, N$, $C(0, t) = C_0(y, t) = 1$ and $C(\infty, t) \approx C(N + 1, t) = 0$, eq. (43) can be written in matrix form:

$$\begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \\ \vdots \\ \dot{C}_{N-1} \\ \dot{C}_N \end{bmatrix} = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \delta_1 & \delta_2 & \delta_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \delta_1 & \delta_2 & \delta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \delta_1 & \delta_2 \end{bmatrix} \begin{bmatrix} 1 \\ C_1 \\ C_2 \\ \vdots \\ C_{N-1} \\ C_N \end{bmatrix} + \begin{bmatrix} \delta_4 \\ \delta_4 \\ \delta_4 \\ \vdots \\ \delta_4 \end{bmatrix} \quad (46)$$

where the coefficients $\delta_1, \delta_2, \delta_3$ and δ_4 are given by eq (45) and $\dot{C}_i = \left(\frac{dC}{dt}\right)_i$

The skin friction due to primary velocity at the wall along x' -axis in dimensionless form is given by

$$\tau_x = \left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (47)$$

The skin friction due to secondary velocity at the wall along z' -axis in dimensionless form is given by

$$\tau_z = \left(\frac{\partial w}{\partial y}\right)_{y=0} \quad (48)$$

The rate of heat transfer (Nusselt number) due to temperature profiles in dimensionless form is given by

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (49)$$

The rate of mass transfer (Sherwood number) due to temperature profiles in dimensionless form is given by:

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (50)$$

Jithender et al. [9].

4 Results and Discussion

In this paper, the effects of hall current and rotation on an unsteady radiative MHD free convective heat and mass transfer of an optically thick radiating, incompressible, electrically conducting and viscous fluid past an impulsively moving vertical porous plate with ramped temperature were considered taking into account the thermal diffusion and diffusion thermo. Method of lines (MOL) is used to solve the governing equations of the flow model. Effectes of governing flow physical parameters: Hall current, Rotation, Soret and Dufour on the primary and secondary velocity, temperature and concentration fields for ramped temperature of the plate for externally cooling ($Gr > 0$) case are illustrated graphically. For the analysis of the result, the values of the flow parameters $G_r = 6, G_m = 5, m = 0.5, M^2 = 0.5, h = 0.1, P_r = 0.71, D_r = 1, S_c = 0.6, S_r = 1, k_1 = 0.5, \Omega = N = 5$ had been used. Also the MATLAB code is used in obtaining solutions of systems of ODEs in equations (34), (38), (42) and (46) for $i = 1, 2, 3$. and to simulate the graphs.

Figs. 2 – 4, show the effects of Soret number, (S_r), Dufour number (D_r), Hall current parameter (m) on primary velocity profile and it can be seen that as parameters increase, the primary velocity of the flow increases. Fig. 5 shows the effect of Rotating parameter (Ω) on primary velocity profile and as it increases, the velocity of the flow decreases. Figs. 6 – 8 show the effect of Soret number (S_r), Dufour number (D_r), Hall current parameter (m) on secondary velocity profile and as they increase, the secondary velocity of the flow increases. Fig. 9 shows the effect of Rotating parameter (Ω) on secondary velocity profile and as it increases, the secondary velocity of the flow increases.

Fig. 10 shows the influence of Dufour number, D_r , on the temperature profile and as the Dufour number increases, the temperature profile also increases. Figure 11 shows the influence of Soret number, S_r , on the concentration profile and as the Soret number increases, the concentration profile also increases.

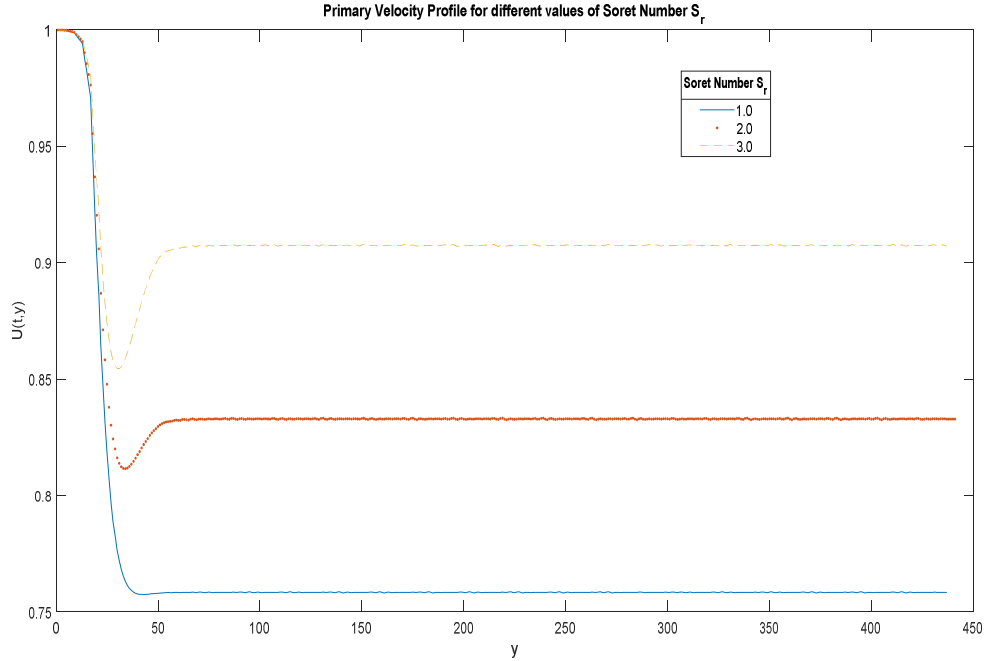


Fig. 2. Primary velocity profile with variations in Soret Number (S_r)

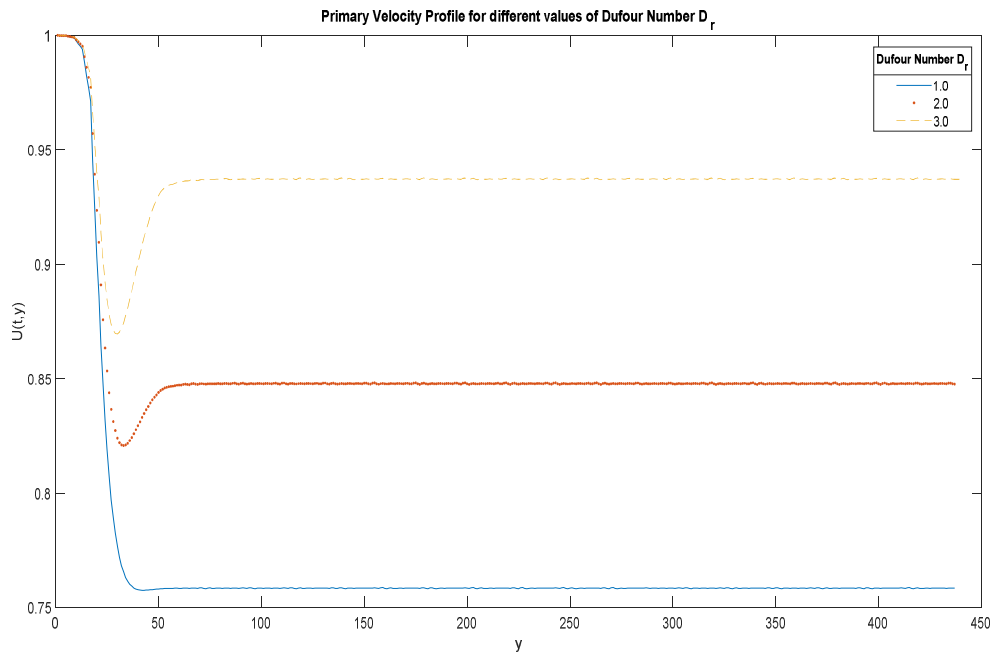


Fig. 3. Primary velocity profile with variations in Dufour Number (D_r)

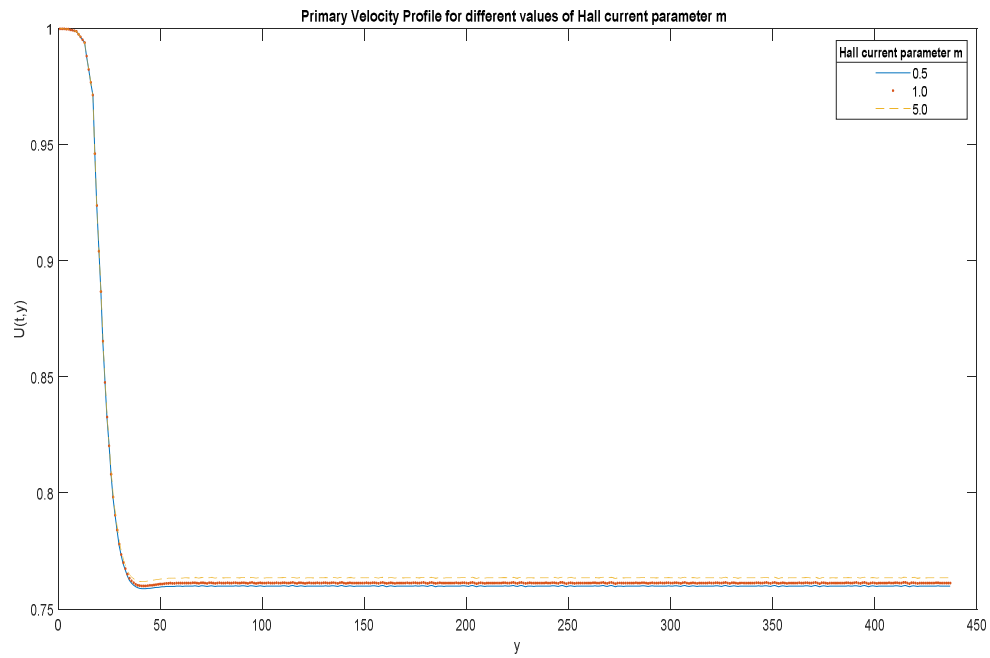


Fig. 4. Primary velocity profile with variations in Hall current parameter (m)

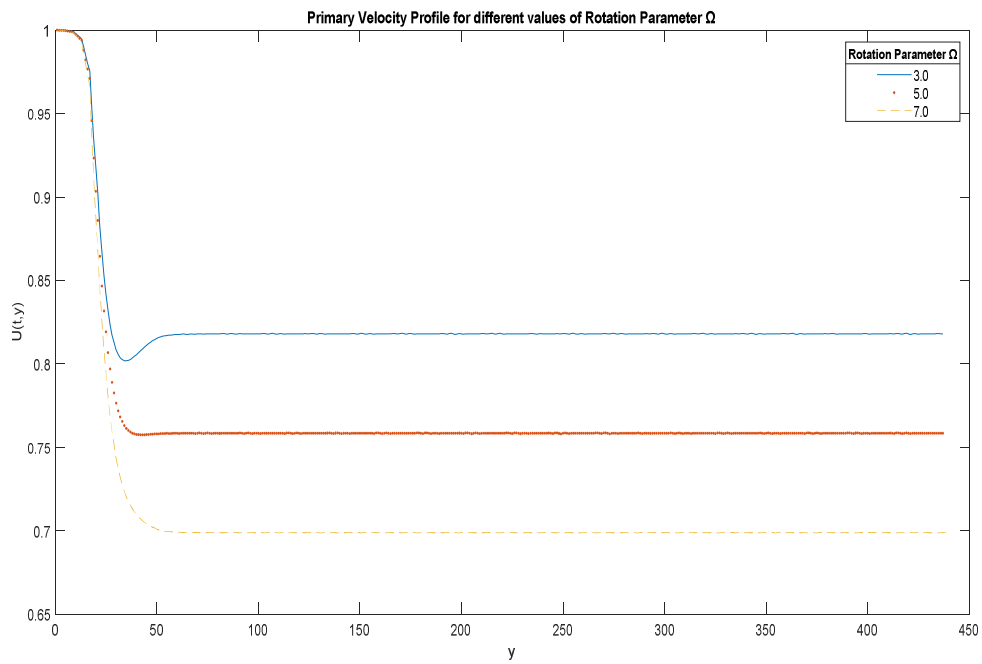


Fig. 5. Primary velocity profile with variations in Rotating Parameter (Ω)

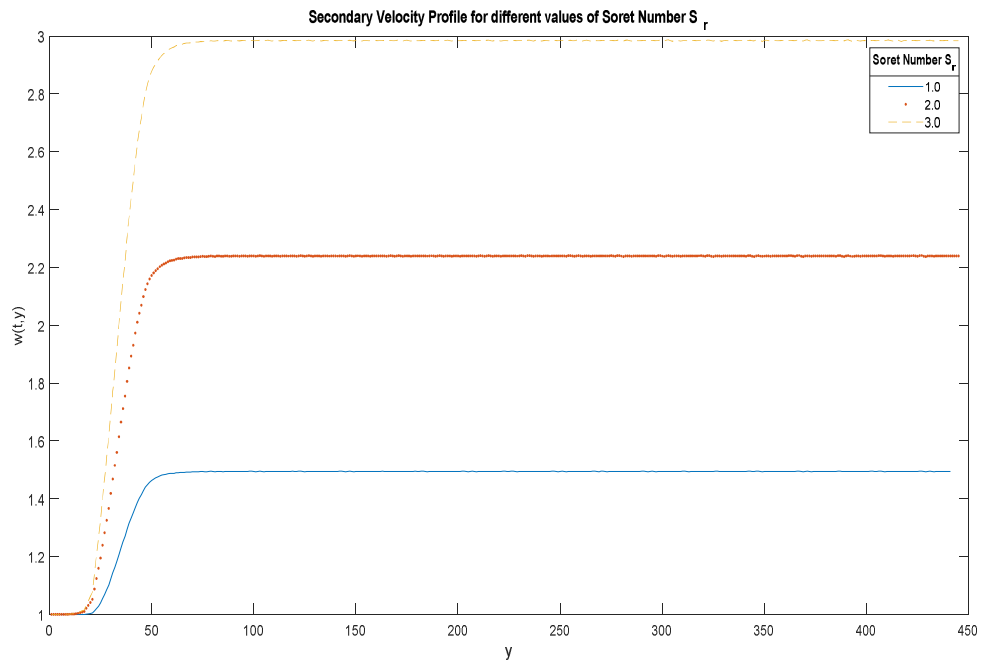


Fig. 6. Secondary velocity profile with variations in Soret Number (S_r)

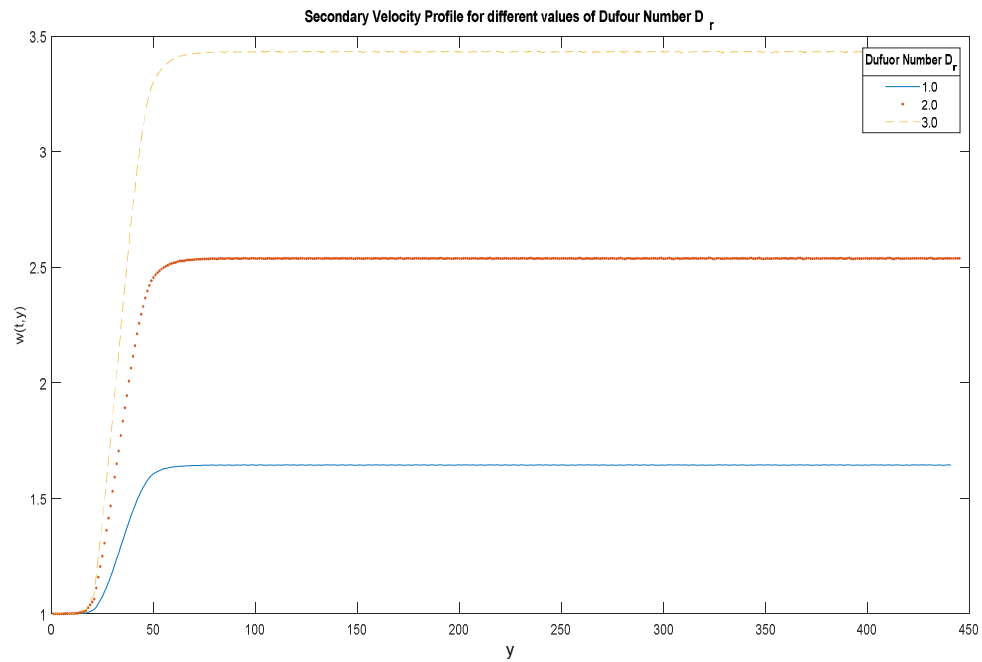


Fig. 7. Secondary velocity profile with variations in Dufour Number (D_r)

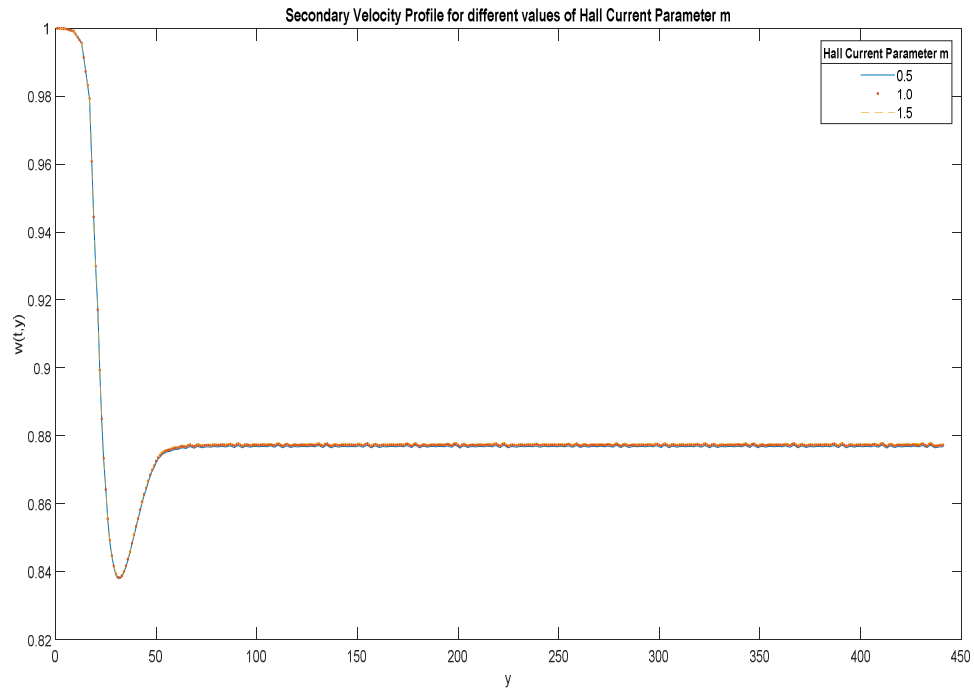


Fig. 8. Secondary velocity profile with variations in Hall Current Parameter (m)

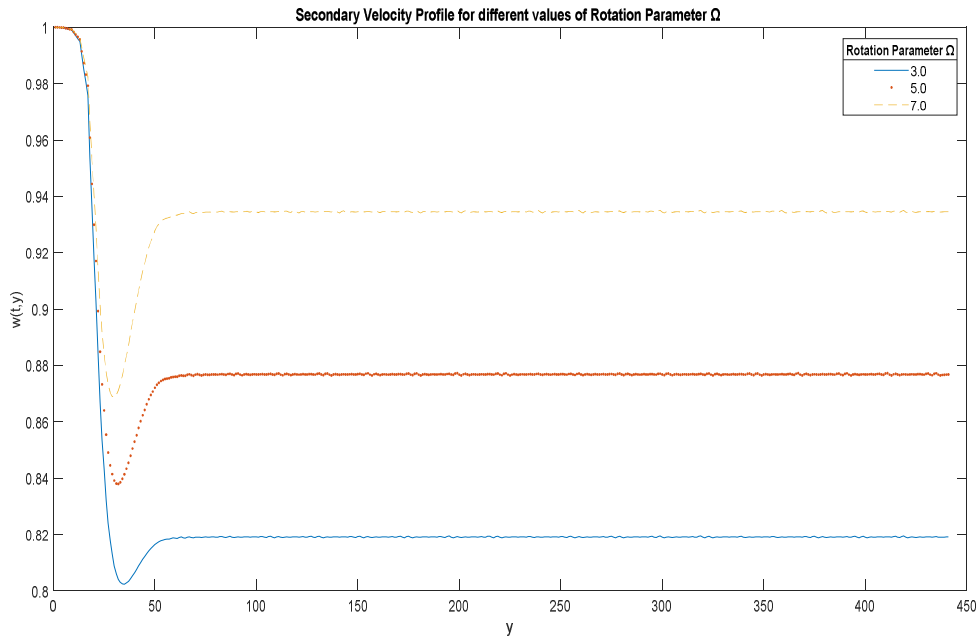


Fig. 9. Secondary velocity profile with variations in Rotation Parameter (Ω)

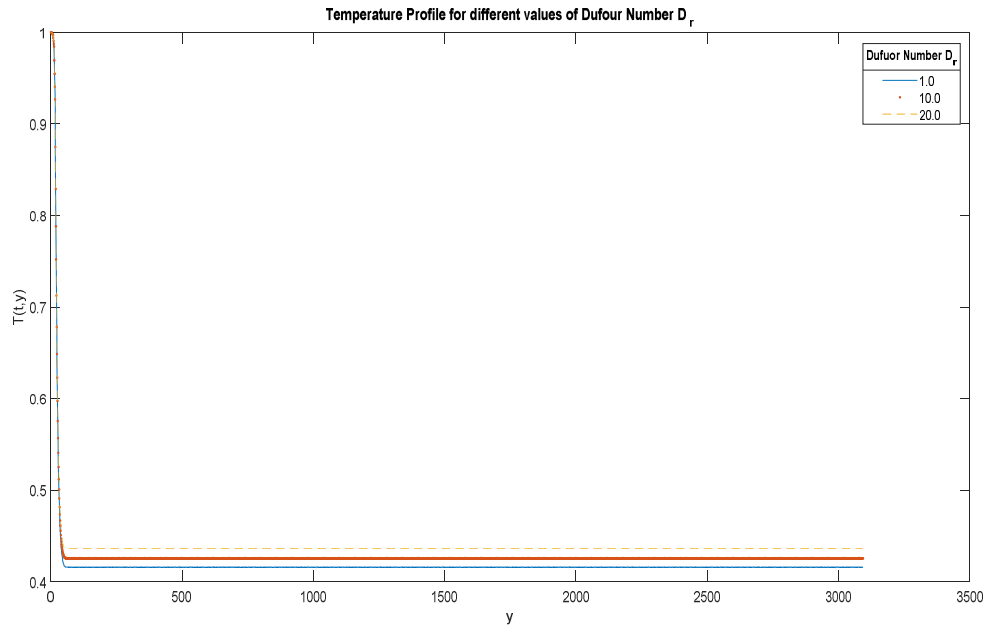


Fig. 10. Temperature profile with variations in Dufour Number (D_r)

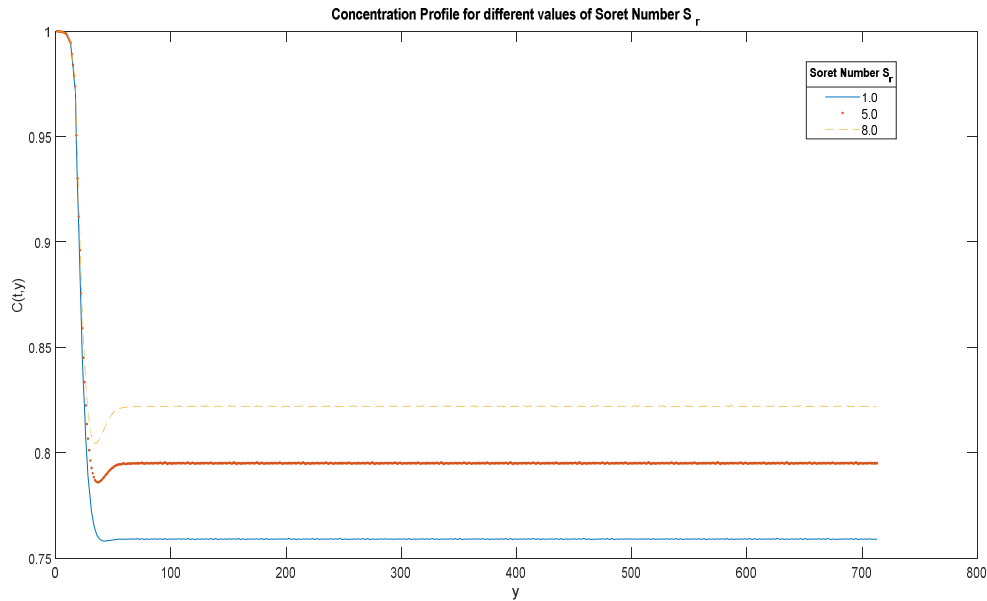


Fig. 11. Concentration profile with variations in Soret Number (S_r)

5 Conclusions

In this paper, we have used method of lines (MOL) in solving coupled differential equations of the flow model. The effects of hall current, rotation, Soret, Dufour on flow variables primary and secondary

velocities, temperature and concentrations are discussed. Then, the following conclusions are drawn from the study:

1. The primary velocity increases with increase in Soret S_r and Dufour D_r , hall current (m) while it decreases with increase in rotation parameter, Ω .
2. The secondary velocity increases with increase in Soret S_r and Dufour D_r , hall current (m) and rotation parameter, Ω .
3. The temperature increases with increase in Dufour D_r .
4. The concentration increases with increase in Soret S_r .

Competing Interests

Authors have declared that no competing interests exist.

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