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Reformulating Special Relativity on a Two-World Background II

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

The exposition of the two-world background of the special theory of relativity started in the first part of this paper is continued in this second part. The negative sign of mass in the negative universe is derived from the generalized mass expression in special relativity (SR) in the two-world picture. Four-dimensional inversion is shown to be a special Lorentz transformation in the two-world picture. Also by starting with the negativity of spacetime dimensions (that is, negativity of distances in space and of intervals of time) in the negative universe, derived in part one of this paper, and requiring the symmetry of natural laws between the positive and negative universes, the signs of mass and other physical parameters and physical constants in the negative universe are derived and tabulated. The invariance of natural laws, including the fundamental interactions, in the negative universe is demonstrated. The derived negative sign of mass in the negative universe is a conclusion of a century and a score years of efforts toward the incorporation of the concept of negative mass into physics. It is shown that the anti-particles observed in our universe originate from the negative

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universe, and conversely, but how a particle can make transition across the event horizon separating the universes without hitting singularity in the Lorentz transformation is as yet unexplained. Experimental test of the two-world picture depends on the possibility of exchange of particles between the two universes without hitting the singularity in LT at the point of making transition across the universes.

Keywords: Negative mass in negative universe; Four-dimensional inversion as special Lorentz transformation in two-world; signs of physical parameters and physical constants in negative universe; invariance of natural laws in negative universe; anti-particles originate from negative universe.

1 DERIVING THE SIGN OF MASS IN THE NEGATIVE UNIVERSE FROM THE GENERALIZED MASS EXPRESSION IN SPECIAL RELATIVITY IN THE TWO-WORLD PICTURE

This second part of this paper is a direct continuation of the first part [1]. This section should have been section 7 of the first part. The literature review done under the Introduction of the first part covers this second part.

Now the particle's primed intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$ contains the intrinsic rest mass $\varnothing m_0$ of the particle at rest relative to it and the particle's primed affine frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ contains the rest mass m_0 of the particle at rest relative to it, in the positive universe in Figs. 8a and 8b and their inverses, Figs. 9a and 9b, of the first part of this paper [1]. However only Fig. 8a of that paper is required to be reproduced as Fig. 1 for the purpose of discussions in this paper.

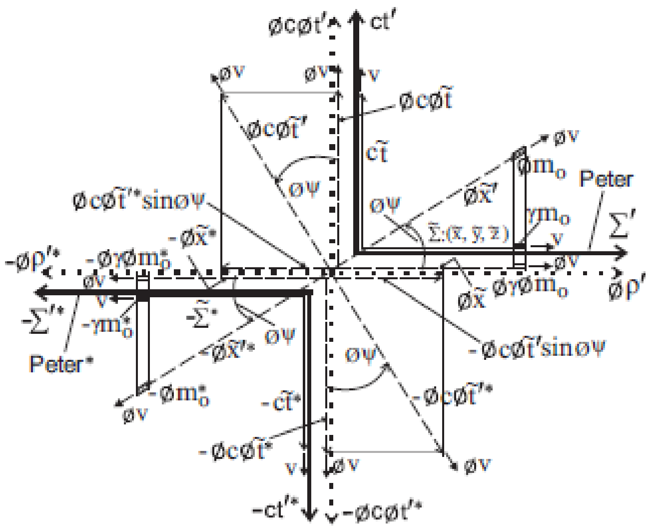

Fig. 1. The diagram (Fig. 8a of [1]) used to derive partial intrinsic Lorentz transformations and partial Lorentz transformations with respect to 3-observers in the Euclidean 3-spaces in the positive and negative universes in [1].

The question arises: what are the signs of the intrinsic rest mass and rest mass of the symmetry-partner particle contained in the symmetry-partner particle's primed intrinsic affine frame $(-\varnothing\tilde{x}'^{*}, -\varnothing c\varnothing\tilde{t}'^{*})$ and symmetry-partner particle's primed affine frame $(-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}, c\tilde{t}'^{*})$ respectively in the negative universe? The negative sign already attached to the mass and intrinsic mass of the symmetry-partner particle in the negative universe in Fig. 1, in an ad-hoc manner, as discussed the in the sixth paragraph before Fig. 8a of [1], must be disregarded at this point.

The answer to the question in the preceding paragraph shall be sought from the the generalized intrinsic mass expression in the context of the intrinsic special theory of relativity (∅SR) and from the corresponding generalized mass expression in the context of the special theory of relativity (SR) in the two-world picture in this section.

The forms of intrinsic Lorentz transformation (the intrinsic Lorentz boost) (∅LT) and its inverse in which they can be applied in all the four quadrants of the spacetime hyperplane formed by the combined spacetimes of our (or positive) universe and the negative universe and the four quadrants of the underlying intrinsic spacetime hyperplane formed by the combined intrinsic spacetimes of our (or positive) universe and the negative universe, in Fig. 7 of [1], reproduced as Fig. 2 of this article, have been derived and presented as systems (42) and (43) of that article. They shall be reproduced here as follows

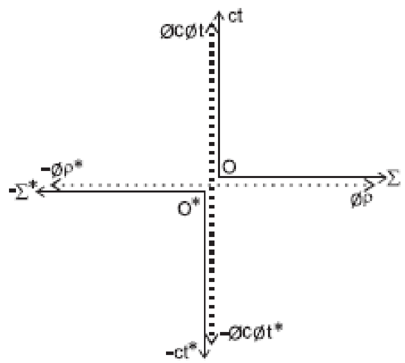

Fig. 2. Combined flat four-dimensional spacetimes and underlying combined flat two-dimensional intrinsic spacetimes of the positive and negative universes (Fig. 7 of [1]).

$$\begin{aligned} \varnothing c\varnothing\tilde{t}' &= \sec\varnothing\psi(\varnothing c\varnothing\tilde{t} - \varnothing\tilde{x}\sin\varnothing\psi)\ ; \\ \varnothing\tilde{x}' &= \sec\varnothing\psi(\varnothing\tilde{x} - \varnothing c\varnothing\tilde{t}\sin\varnothing\psi) \end{aligned} \tag{1}$$

and

$$\begin{aligned} \varnothing c\varnothing\tilde{t} &= \sec\varnothing\psi(\varnothing c\varnothing\tilde{t}' + \varnothing\tilde{x}'\sin\varnothing\psi)\ ; \\ \varnothing\tilde{x} &= \sec\varnothing\psi(\varnothing\tilde{x}' + \varnothing c\varnothing\tilde{t}'\sin\varnothing\psi)\ ; \end{aligned} \tag{2}$$

for $\varnothing\psi$ in the concurrent open intervals, $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$.

The 3-observers in the proper Euclidean 3-space Σ' of the positive universe can formulate intrinsic special relativity (∅SR) and, hence special relativity (SR), and indeed observe SR for intrinsic angles $\varnothing\psi$ in the range $(-\varnothing\pi/2, \varnothing\pi/2)$. However as Fig. 10a of [1], reproduced as Fig. 3a of this article shows, 3-observers in Σ' in the positive universe can construct ∅SR and, hence SR, relative to themselves for all intrinsic angles $\varnothing\psi$ in the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$, by using the generalized ∅LT and its inverse of systems (1) and (2) and obtaining the LT and its inverse as the outward manifestations on flat four-dimensional spacetime of the ∅LT on flat two-dimensional intrinsic spacetime, as done in [1], although they can observe special relativity for intrinsic angles $\varnothing\psi$ in $(-\varnothing\pi/2, \varnothing\pi/2)$ in Fig. 3a of this article only. The forms of systems (1) and (2) in the negative universe are also presented as systems (44) and (45) in [1]. Figures 10a and 10b of [1] are reproduced as Figs. 3a and 3b of this article.

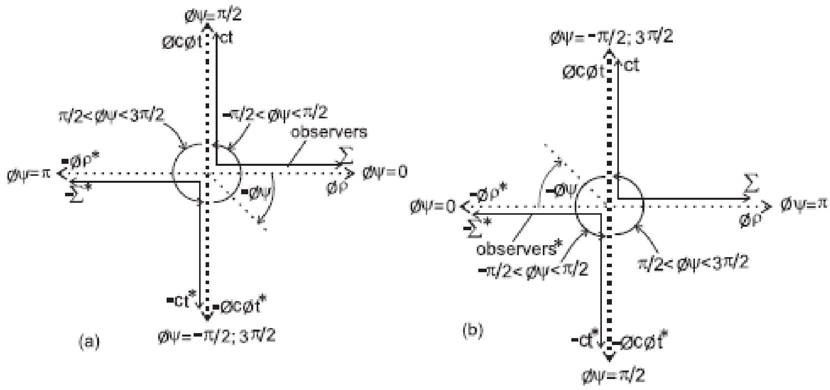

Fig. 3. The concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ within which the intrinsic angle $\varnothing\psi$ can take on values: (a) with respect to 3-observers in 3-space in the positive universe and (b) with respect to 3-observers in 3-space in the negative universe (Figs. 10a and 10b of [1]).

There is the corresponding intrinsic mass expression in the form in which it can be applied in all the four quadrants of the intrinsic spacetime hyperplane formed by the combined intrinsic spacetimes of our universe and the negative universe, that is, for intrinsic angle $\varnothing\psi$ in the concurrent open intervals, $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$, by 3-observers in the proper Euclidean 3-spaces Σ' of our universe and $-\Sigma'^*$ of the negative universe, in Figs. 3a and 3b. It is deduced below.

The mass expression on the flat four-dimensional spacetime in the context of SR in the existing one-world picture, in Scheme I of Table I of [1], is equally valid in the two-world picture in Scheme II of Table I of that paper. It is the following

$$m = m_0\left(1 - v^2/c^2\right)^{-1/2} \; . \qquad (3)$$

The corresponding intrinsic mass expression on the flat two-dimensional intrinsic spacetime in the context of the intrinsic special theory of relativity (∅SR), to be obtained by simply introducing the symbol ∅ used to denote intrinsic coordinates and intrinsic parameters in the present two-world picture in Eq. (3) is

$$\varnothing m = \varnothing m_0\left(1 - \varnothing v^2/\varnothing c^2\right)^{-1/2} \; . \qquad (4)$$

The masses, m_0 and m, in the three-dimensional proper Euclidean space Σ' are the outward manifestations of the lines of intrinsic masses, $\varnothing m_0$ and $\varnothing m$, respectively, in the one-dimensional proper intrinsic metric space $\varnothing\rho'$, as illustrated in Fig. 6(a) of [1]. Figures 6a and 6b of [1] are reproduced as Figs. 4a and 4b of this article.

Using the relations, $\sec\varnothing\psi = (1 - \varnothing v^2/\varnothing c^2)^{-1/2}$ and $\sec\psi = (1 - v^2/c^2)^{-1/2}$, derived and presented as Eqs. (17) and (30) respectively of [1], Eqs. (3) and (4) can be written respectively as

$$m = m_0 \sec\psi \qquad (5)$$

and

$$\varnothing m = \varnothing m_0 \sec\varnothing\psi \; . \qquad (6)$$

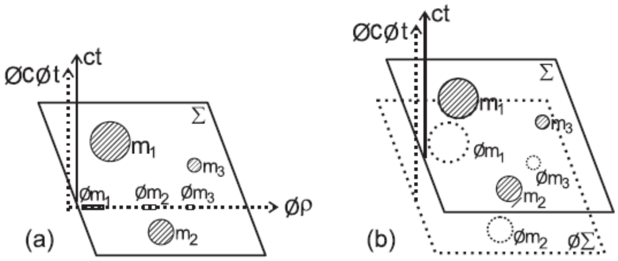

Fig. 4. (a) The flat 4-dimensional spacetime and its underlying flat 2-dimensional intrinsic spacetime with the inertial masses of three objects scattered in the Euclidean 3-space and their intrinsic inertial masses aligned along the one-dimensional isotropic intrinsic space with respect to observers in spacetime. (b) The flat 2-dimensional intrinsic spacetime with respect to observers in spacetime in (a) is a flat four-dimensional intrinsic spacetime containing intrinsic inertial masses of particles and objects in 3-dimensional intrinsic space with respect to hypothetical intrinsic-mass-observers in intrinsic spacetime (Figs. 6a and 6b of [1]).

Equations (5) and (6) are the generalized forms of the mass expression and the intrinsic mass expression in the contexts of SR and ∅SR respectively, in the two-world picture. They can be applied for all intrinsic angles $\varnothing\psi$ in the range $[0, 2\varnothing\pi]$, except that $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\pi/2$ must be avoided.

1.1 Showing that Four-dimensional Iinversion of Affine Spacetime and Two-dimensional Inversion of Intrinsic Affine Spacetime are Special Lorentz Transformation and Special Intrinsic Lorentz Transformation

We shall assume that the particle's primed intrinsic affine frame $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ in the positive universe is continuously accelerated relative to the 'stationary' observer Peter in the proper Euclidean 3-space Σ', such that its intrinsic affine coordinates, $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$, are continuously rotated by increasing intrinsic angle $\varnothing\psi$ relative to their projective unprimed intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, (of the particle's unprimed intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$) in Fig. 1 of this article (in the sense of positive rotation by $\varnothing\psi$ in Fig. 3a of this article). We shall assume that $\varnothing\psi$ increases from zero to $\varnothing\pi/2 - \varnothing\epsilon$, dodges $\varnothing\psi = \varnothing\pi/2$ and then increases from $\varnothing\psi = \varnothing\pi/2 + \varnothing\epsilon$ to $\varnothing\psi = \varnothing\pi$. In other words, we shall allow $\varnothing\psi$ to increase from zero to $\varnothing\pi$ while avoiding $\varnothing\pi/2$. We shall not be concerned with how this can be accomplished in this paper.

Then by virtue of the prescribed perfect symmetry of state between the positive and negative universes in [1], the symmetry-partner particle's primed intrinsic affine frame $(-\varnothing\tilde{x}\,'^*, -\varnothing c\varnothing\tilde{t}\,'^*)$ in the negative universe is identically accelerated simultaneously relative to the symmetry-partner 'stationary' observer* Peter* in $-\Sigma'^*$, such that the intrinsic affine coordinates, $-\varnothing\tilde{x}\,'^*$ and $-\varnothing c\varnothing\tilde{t}\,'^*$, of the symmetry-partner particle's primed intrinsic affine frame $(-\varnothing\tilde{x}\,', -\varnothing c\varnothing\tilde{t}\,'^*)$, are continuously rotated in Fig. 1 (in the sense of positive rotation by $\varnothing\psi$ in Fig. 3b), relative to their projective unprimed intrinsic affine coordinates, $-\varnothing\tilde{x}\,^*$ and $-\varnothing c\varnothing\tilde{t}\,^*$, of the symmetry-partner particle's unprimed intrinsic affine frame $(-\varnothing\tilde{x}\,^*, -\varnothing c\varnothing\tilde{t}\,^*)$ in the negative universe, by increasing intrinsic angle $\varnothing\psi$ from $\varnothing\psi = 0$ to $\varnothing\psi = \varnothing\pi$, while avoiding $\varnothing\psi = \varnothing\pi/2$.

In deriving intrinsic Lorentz transformation with respect to the 'stationary' observer Peter in Σ' in the positive universe for the scenario described in the penultimate paragraph, we must apply the generalized intrinsic Lorentz transformation of system (1). We must let $\varnothing\psi = \varnothing\pi$ in that system to have

$$\varnothing c\varnothing\tilde{t}\,' = -\varnothing c\varnothing\tilde{t} \equiv -\varnothing c\varnothing\tilde{t}^{\,*}\,;\ \varnothing\tilde{x}\,' = -\varnothing\tilde{x} \equiv -\varnothing\tilde{x}^{\,*} \quad (7a).$$

However $\varnothing\psi = \varnothing\pi$ implies, $\sin\varnothing\psi = \varnothing v/\varnothing c = 0$, hence $\varnothing v = 0$. This means that the primed and unprimed intrinsic affine frame are the same at $\varnothing\psi = \varnothing\pi$. We must therefore replace the unprimed intrinsic affine coordinates, $-\varnothing c\varnothing\tilde{t}^{\,*}$ and $-\varnothing\tilde{x}^{\,*}$, with the primed intrinsic affine coordinates, $-\varnothing c\varnothing\tilde{t}\,'^{*}$ and $-\varnothing\tilde{x}\,'^{*}$, respectively in system (7a) to have

$$\varnothing c\varnothing\tilde{t}\,' = -\varnothing c\varnothing\tilde{t}\,'^{*} \ \text{ and } \ \varnothing\tilde{x}\,' = -\varnothing\tilde{x}\,'^{*}\,. \quad (7b)$$

The implication of system (7b) is that rotation by $\varnothing\psi = \varnothing\pi$ transforms the intrinsic affine coordinates, $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$, of the particle's primed intrinsic affine frame at rest relative to the observer in the positive universe, into the intrinsic affine coordinates, $-\varnothing\tilde{x}\,'^{*}$ and $-\varnothing c\varnothing\tilde{t}\,'^{*}$, of the particle's primed intrinsic affine frame at rest relative to the symmetry-partner observer in the negative universe. In other words,

$$(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}') \xrightarrow{\text{rot. by } \varnothing\psi=\varnothing\pi} (-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})\,. \quad \text{(7c)}$$

Once $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ has transformed into $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$ by virtue of intrinsic rotation by $\varnothing\psi = \varnothing\pi$, according to the transformation scheme (7c), then the particle's primed intrinsic affine frame $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$ formed in the negative universe will be made manifested in the particle's primed affine frame $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$ in the negative universe. Thus, although the affine coordinates, $\tilde{x}\,', \tilde{y}\,', \tilde{z}\,'$ and $c\tilde{t}\,'$, are not rotated along with the intrinsic affine coordinates, $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$, in the positive universe, the transformation (7c) in intrinsic spacetime will automatically be made manifested in the following in spacetime

$$(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}') \xrightarrow{\text{int. rotation by } \psi=\pi} (-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}, -c\tilde{t}'^{*}) \quad \text{(7d)}$$

In obtaining the correspondence of system (7d) in the negative universe, we must write the $\varnothing$LT (1) in terms of the negative intrinsic affine coordinates, $-\varnothing\tilde{x}\,'^{*}$, $-\varnothing c\varnothing\tilde{t}\,'^{*}$, $-\varnothing\tilde{x}^{\,*}$ and $-\varnothing c\varnothing\tilde{t}^{\,*}$, of the negative universe and let $\varnothing\psi = \varnothing\pi$ in the resulting $\varnothing$LT in the negative universe to have

$$-\varnothing c\varnothing\tilde{t}\,'^{*} = \varnothing c\varnothing\tilde{t}^{\,*} \equiv \varnothing c\varnothing\tilde{t}\,;\ -\varnothing\tilde{x}\,'^{*} = \varnothing\tilde{x}^{\,*} \equiv \varnothing\tilde{x} \quad (8a)\,.$$

Again, $\varnothing\psi = \varnothing\pi$ implies $\sin\varnothing\psi = \varnothing v/\varnothing c = 0$, hence $\varnothing v = 0$. This means that the primed intrinsic affine frame and the unprimed intrinsic affine frame are the same at $\varnothing\psi = \varnothing\pi$. We must replace the unprimed intrinsic affine coordinates, $\varnothing c\varnothing\tilde{t}$ and $\varnothing\tilde{x}$, with the primed intrinsic affine coordinates, $\varnothing c\varnothing\tilde{t}\,'$ and $\varnothing\tilde{x}\,'$, respectively in the last displayed system to have

$$-\varnothing c\varnothing\tilde{t}\,'^{*} = \varnothing c\varnothing\tilde{t}\,' \ \text{ and } \ -\varnothing\tilde{x}\,'^{*} = \varnothing\tilde{x}\,'\,, \quad (8b)$$

or

$$(-\varnothing\tilde{x}'^{*}, -\varnothing c\varnothing\tilde{t}'^{*}) \xrightarrow{\text{rot. by } \varnothing\psi=\varnothing\pi} (\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}') \quad \text{(8c)}$$

The transformation scheme (8c) states that the primed intrinsic affine frame $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$ at rest relative to the 'stationary' observer* Peter* in $-\Sigma\,'^{*}$ in the negative universe, forms the primed intrinsic affine frame $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ at rest relative to the symmetry-partner 'stationary' observer Peter in $\Sigma\,'$ in our universe, upon rotating its intrinsic affine coordinates, $-\varnothing\tilde{x}\,'^{*}$ and $-\varnothing c\varnothing\tilde{t}\,'^{*}$, by $\varnothing\psi = \varnothing\pi$ relative to Peter*.

The transformation scheme (8c) is then made manifested outwardly in the following in our universe

$$(-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}, -c\tilde{t}'^{*}) \xrightarrow{\text{intr. rot. by } \psi=\pi} (\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}') \quad \text{(8d)}$$

What must be concluded from Eq. (7b) and the implied transformation schemes (7c) and (7d) in the positive universe and the corresponding Eq. (8b) and the implied transformation schemes (8c) and (8d) in the negative universe is that, the intrinsic rotations of the intrinsic affine coordinates, $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$, by intrinsic angle, $\varnothing\psi = \varnothing\pi$ (assuming that rotation by $\varnothing\psi = \varnothing\pi/2$ can be avoided), in the positive universe, transforms the primed particle's intrinsic affine frame $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ and the primed particle's affine frame $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ in the positive universe, into the symmetry-partner primed particle's intrinsic affine frame $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$ and symmetry-partner primed particle's affine frame $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$ respectively, in the negative universe.

The simultaneous intrinsic rotations of the intrinsic affine coordinates, $-\varnothing\tilde{x}\,'^{*}$ and $-\varnothing c\varnothing\tilde{t}\,'^{*}$, of the symmetry-partner particle's intrinsic affine frame $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$ by intrinsic angle, $\varnothing\psi = \varnothing\pi$, while avoiding $\varnothing\psi = \varnothing\pi/2$, in the negative universe, transforms the symmetry-partner primed particle's intrinsic affine frame $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$ and the symmetry-partner primed particle's affine frame $(-\tilde{x}'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$ in the negative universe, into the primed particle's intrinsic affine frame $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ and primed particle's affine frame $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ respectively, in the positive universe.

Now the transformation schemes (7c) and (7d) are intrinsic affine spacetime inversion and affine spacetime inversion in our (or positive) universe, likewise the corresponding transformation schemes (8c) and (8d) in the negative universe. What must be concluded from the preceding two paragraphs and the derivations that lead to them is that, four-dimensional affine spacetime inversion and two-dimensional intrinsic affine spacetime inversion, are special Lorentz transformation and special intrinsic Lorentz transformation respectively, in each of our universe and the negative universe.

It is to be noted however that the conclusion reached in the preceding paragraph does not apply to the intrinsic metric spacetimes, $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$, and the metric spacetimes, (Σ', ct') and $(-\Sigma'^{*}, -ct'^{*})$, of our universe and the negative universe. This is so, because the intrinsic metric spacetime coordinates are not rotated in the context of $\varnothing$SR.

It has been concluded in the context of the existing one-world background of the special theory of relativity (or in the one-world picture in Scheme I of Table I of [1]) that, four-dimensional inversion (7d) is impossible as actual transformation of the coordinates of a frame of reference. This, as discussed on page 39 of [2], for example, is due to the fact that four-dimensional inversion carries the (affine) time coordinate from the future light cone into the past light cone, which is impossible without going through regions of spacelike geodesics that requires the introduction of imaginary (affine) spacetime coordinates.

The light cone concept does not exist in the two-world picture, as deduced in sub-section 4.7 of [1]. Consequently continuous rotation of intrinsic affine spacetime coordinates of the particle's primed intrinsic affine frame $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ relative to the intrinsic affine spacetime coordinates of the particle's unprimed intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$, in the generalized intrinsic Lorentz transformation (1), through all intrinsic angles $\varnothing\psi$ in the range $[0, 2\varnothing\pi]$, while avoiding $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$, is possible (granting that how $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$ can be avoided shall be explained), without going into regions of spacelike geodesics in the two-world picture. Thus the intrinsic two-dimensional inversion (7c), obtained by letting $\varnothing\psi = \varnothing\pi$ in the generalized intrinsic Lorentz transformation (1), is a special intrinsic Lorentz transformation in the two-world picture.

The four-dimensional inversion (7d), which does not involve actual rotation of the primed affine spacetime coordinates of the primed affine frame $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ relative to the unprimed affine spacetime coordinates of the unprimed affine frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$, is the outward manifestation in the four-dimensional spacetime of intrinsic two-dimensional inversion that involves actual rotation of intrinsic affine spacetime coordinates. It is consequently a special Lorentz transformation in the two-world picture.

1.2 Signs of Mass and Intrinsic Mass in the Negative Universe

After the relevant digression in the preceding subsection, let us resume the development of the topic of the section. Now letting $\varnothing\psi = \varnothing\pi$ in the generalized intrinsic mass expression (6) in the context of ∅SR in the positive universe gives

$$\varnothing m = -\varnothing m_0 \equiv -\varnothing m_0^* \,. \tag{9a}$$

But $\varnothing\psi = \varnothing\pi$ implies $\sin\varnothing\psi = \varnothing v/\varnothing c = 0$, hence $\varnothing v = 0$. This means that the primed intrinsic affine frame containing $\varnothing m_0$ and the unprimed intrinsic affine frame containing, $\varnothing m = \varnothing\gamma\varnothing m_0$, are the same at $\varnothing\psi = \varnothing\pi$, as mentioned earlier. We must therefore replace $\varnothing m$ by $\varnothing m_0$ in Eq. (9a) to have

$$\varnothing m_0 = -\varnothing m_0^* \,. \tag{9b}$$

The outward manifestation in spacetime of Eq. (9b) in intrinsic spacetime, which also follows by letting $\psi = \pi$ in the mass expression (5) is

$$m_0 = -m_0^* \,. \tag{9c}$$

Equations (7c) and (9b) state that the rotations of the intrinsic affine coordinates, $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$, of the particle's primed intrinsic affine frame $(\varnothing\tilde{x}\,',\ \varnothing c\varnothing\tilde{t}\,')$ along with the line of intrinsic rest mass $\varnothing m_0$ of the particle in $\varnothing\tilde{x}'$, to be denoted by $(\varnothing\tilde{x}\,',\ \varnothing c\varnothing\tilde{t}\,'; \varnothing m_0)$, in the positive universe, by intrinsic angle, $\varnothing\psi = \varnothing\pi$ (assuming that rotation by $\varnothing\psi = \varnothing\pi/2$ can be avoided), transforms $(\varnothing\tilde{x}\,',\ \varnothing c\varnothing\tilde{t}\,'; \varnothing m_0)$ in the positive universe into $(-\varnothing\tilde{x}\,'^*, -\varnothing c\varnothing\tilde{t}\,'^*;\ -\varnothing m_0^*)$ in the negative universe, according to the following transformation scheme,

$$(\varnothing\tilde{x}',\ \varnothing c\varnothing\tilde{t}'; \varnothing m_0) \xrightarrow{\text{rot. by } \varnothing\psi=\varnothing\pi} (-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*; -\varnothing m_0^*) \tag{10a}$$

The transformation scheme (10a) in intrinsic spacetime will be automatically made manifested in the following in spacetime

$$(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0) \xrightarrow{\text{intr. rot. by } \psi=\pi} (-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*; -m_0^*) \tag{10b}$$

The transformation scheme (10b) is also implied by the transformation scheme (7d) along with letting $\psi = \pi$ in Eq. (5). Although the affine coordinates, $\tilde{x}\,'$, $\tilde{y}\,'$, $\tilde{z}\,'$ and $c\tilde{t}\,'$, of the affine frame $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$, containing the rest mass m_0 are not rotated, the transformation scheme (10a) implied by actual rotations of $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ by $\varnothing\psi = \varnothing\pi$, is automatically made manifested in the transformation scheme (10b) in the two-world picture.

The correspondences in the negative universe of the intrinsic transformation schem (10a) and the transformation scheme (10b) in the positive universe, obtained by simply reversing the direction of the arrows in the transformation schemes (10a) and (10b) are the following

$$(-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*; -\varnothing m_0^*) \xrightarrow{\text{rot. by } \varnothing\psi=\varnothing\pi} (\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,'; \varnothing m_0) \tag{11a}$$

and

$$(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*; -m_0^*) \xrightarrow{\text{intr. rot. by } \psi=\pi} (\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0) \tag{11b}$$

What can be concluded from the intrinsic transformations of primed intrinsic affine coordinates and intrinsic rest mass implied by (10a) in the positive universe and the simultaneous corresponding transformations implied by (11a) in the negative universe is that, the positive intrinsic rest mass $\varnothing m_0$ of a particle or body in the primed intrinsic affine frame $(\varnothing\tilde{x}\,',\ \varnothing c\varnothing\tilde{t}\,')$ (or $(\varnothing\tilde{x}\,',\ \varnothing c\varnothing\tilde{t}\,'; \varnothing m_0)$) in the positive universe, corresponds to the negative intrinsic rest mass $-\varnothing m_0^*$ of the symmetry-partner particle or body in the symmetry-partner primed intrinsic affine frame $(-\varnothing\tilde{x}\,'^*, -\varnothing c\varnothing\tilde{t}\,'^*)$ (or $(-\varnothing\tilde{x}\,'^*, -\varnothing c\varnothing\tilde{t}\,'^*; -\varnothing m_0^*)$) in the negative universe.

And what can be concluded from the transformations of primed affine coordinates and rest mass implied by (10b) in the positive universe and the corresponding transformation implied by (11b) in the negative universe is that, the positive rest mass m_0 of a particle or body in the primed affine frame $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t})$ (or $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,'; m_0)$) in the positive universe, corresponds to negative rest mass $-m_0^*$ of the symmetry-partner particle or body in the symmetry-partner primed affine frame $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$ (or $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*}; -m_0^*)$) in the negative universe.

Now letting $\varnothing m_0 \to -\varnothing m_0^*$ implies letting $\varnothing m\,(= \varnothing\gamma\,\varnothing m_0) \to -\varnothing m^*\,(= -\varnothing\gamma\,\varnothing m_0^*)$ and letting $m_0 \to -m_0^*$ implies $m\,(= \gamma\, m_0) \to -m^*\,(= -\gamma\, m_0^*)$, with transition from the positive to the negative universe. Thus the negation of intrinsic rest mass and rest mass in the negative universe concluded in the preceding two paragraphs is equally true of the special-relativistic intrinsic mass $\varnothing m\,(= \varnothing\gamma\,\varnothing m_0)$ and special-relativistic mass $m\,(= \gamma\, m_0)$.

The negation of inertial mass m_i and intrinsic inertial mass $\varnothing m_i$ and the negation of passive gravitational mass m_p and intrinsic passive gravitational massive mass $\varnothing m_p$ in the negative universe, follow from the equivalences of the mass concepts: $m_0 = m_i = m_p$ and, consequently, $\varnothing m_0 = \varnothing m_i = \varnothing m_p$, in classical mechanics in our (or positive) universe [3]. These become $-m_0^* = -m_i^* = -m_p^*$ and $-\varnothing m_0^* = -\varnothing m_i^* = -\varnothing m_p^*$ in the negative universe. The situation with the active gravitational mass m_a and intrinsic active gravitational mass $\varnothing m_a$, shall be deferred until further development of the two-world picture elsewhere. We must have begun to glimpse the confirmation of Schuster's contemplation in 1898 of a universe with negative mass [3].

2 RE-DERIVATION OF THE SIGN MASS AND DERIVATION OF THE SIGNS OF OTHER PHYSICAL PARAMETERS AND PHYSICAL CONSTANTS IN THE NEGATIVE UNIVERSE BY APPLICATION OF SYMMETRY OF LAWS BETWEEN THE POSITIVE AND NEGATIVE UNIVERSES

Four-dimensional inversion is the transformation of the positive (affine) spacetime coordinates of a frame in the positive universe into the negative (affine) spacetime coordinates of the symmetry-partner frame in the negative universe, as systems (7d) in our universe shows, or conversely, as system (8d) shows. Thus the simultaneous negation of spacetime coordinates in the classical or special-relativistic form of a natural law in our universe, amounts to writing that law in the negative universe.

Now the prescribed perfect symmetry of state between the positive and negative universes discussed in sub-section 4.1 of [1], will be impossible unless there is also a perfect symmetry of laws between the two universes. That is, unless natural laws take on identical forms in the two universes. Perfect symmetry of laws between the positive and negative universes is immutable, as shall be demonstrated shortly in this article. It is to be recalled that Lorentz invariance in the negative universe (which is an important component of the invariance of laws in the negative universe), has been validated from the derived LT and its inverse in the negative universe of systems (36) and (37) of [1].

The simultaneous negation of space and time dimensions in a natural law in the positive universe, in the process of writing it in the negative universe, will change the form of that law in general unless physical parameters and physical constants, such as mass, electric charge, temperature, flux, specific heat capacity, thermal conductivity, etc, which also appear in the law (usually as differential coefficients in the local instantaneous differential laws), are given the appropriate signs. By combining the simultaneous negation of space and time dimensions with the invariance of laws, the signs of physical quantities and constants in the negative universe can be derived. The derivations of the signs of the fundamental quantities namely, mass, electric charge and absolute temperature in the negative universe shall be done below. The signs of all derived (or non-fundamental) physical quantities and physical constants can then be inferred from their dimensions, as shall be demonstrated.

Consider a body of constant mass m to be accelerated by a force $\vec{F}$ directed along the positive $X-$axis of the frame attached to it. In the positive universe, Newton's second law of motion for this body is the following

$$\vec{F} = m\frac{d^2x}{dt^2}\,\hat{\mathrm{i}} \qquad (12)$$

Since the dimensions of 3-space of the negative universe is inversion in the origin of the dimensions of 3-space of the positive universe, the dimensions, unit vector and force, $(x,\, y,\, z,\, t;\, \hat{\mathrm{i}};\, \vec{F}\,)$, in the positive universe correspond to $(-x^*,\, -y^*, -z^*,\, -t^*;\, -\hat{\mathrm{i}}^*;\, -\vec{F}^*)$ in the negative universe. Thus in the negative universe, we must let $x \to -x^*,\ t \to -t^*,\ \hat{\mathrm{i}} \to -\hat{\mathrm{i}}^*$ and $\vec{F} \to -\vec{F}^*$, while leaving m unchanged meanwhile in Eq. (12) to have

$$-\,\vec{F}^* = m\frac{d^2(-x^*)}{d(-t^*)^2}\,(-\hat{\mathrm{i}}^*) = m\frac{d^2x^*}{dt^{*2}}\,\hat{\mathrm{i}}^* \qquad (13)$$

While Eq. (12) states that a body pushed in the positive $x-$ direction by a force $\vec{F}$, moves along the positive $x-$direction (away from the force), in the positive universe, Eq. (13) states that a body pushed in the $-x^*-$ direction by a force $-\vec{F}^*$, moves in the $+x^*-$direction, with unit vector $+\hat{\mathrm{i}}^*$ (toward the force), in the negative universe. This implies that Newton's second law of motion is different in the negative universe, contrary to the required invariance of natural laws in that universe.

In order for Eq. (13) to retain the form of Eq. (12), so that Newton's second law of motion remains unchanged in the negative universe, we must let $m \to -\,m^*$ in it to have as follows

$$-\vec{F}^* = -m^*\frac{d^2{x^*}^2}{dt^*}\,(\hat{\mathrm{i}}^*) = m^*\frac{d^2{x^*}^2}{dt^*}\,(-\hat{\mathrm{i}}^*), \qquad (14)$$

which is of the form of Eq. (12) upon canceling the signs. The fact that we must let $m \to -\,m^*$ in Eq. (13) to arrive at Eq. (14) implies that mass is a negative quantity in the negative universe.

Newton second law in free space has been chosen because it involves spacetime coordinates and mass and no other physical quantity or constant. However the negation of mass in the negative universe does not depend on the natural law adopted; it follows from any chosen law once the signs in the negative universe of the other physical quantities and physical constants that appear in that law have been correctly substituted, in addition to the simultaneous negation of space and time dimensions in the law.

The negation of mass also follows from the invariance of the metric tensor with reflection of spacetime dimensions. For if we consider the Schwarzschild metric in empty space at the exterior of a spherically symmetric gravitational field source, for example, then the non-trivial components of the metric tensor are, $g_{00} = -g_{11}^{-1} = 1 - 2GM/rc^2$. By letting $r \to -r^*$, we must also let $M \to -M^*$ in order to preserve the metric tensor in the negative universe. It can be verified that this is true for all other metric tensors in general relativity. Thus negative mass derived from the generalized mass expression in the special theory of relativity in the two-world picture in section one of this paper, has again been re-derived from the requirement of symmetry of natural laws between the positive and negative universes.

For electric charge, the electrostatic field $\vec{E}$ emanating from a particle (assumed spherical in shape), with net electric charge q in the positive universe, is given at radial distance r from the center of the particle as

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_{\rm o}r^3}\ . \qquad (15)$$

The symmetry-partner electrostatic field emanating from the symmetry-partner particle in the negative universe is inversion in the origin of the electrostatic field in the positive universe. Hence the electrostatic field in the negative universe points in opposite direction in space as its symmetry-partner field $\vec{E}$ of Eq. (15) in the positive universe. This implies that the symmetry-partner electrostatic field in the negative universe is $-\vec{E}^*$. By letting $r \to -r^*,\ \vec{r} \to -\vec{r}^*$ and $\vec{E} \to -\vec{E}^*$ in Eq. (15), while retaining q and $\epsilon_{\rm o}$

meanwhile we have

$$-\vec{E}^* = \frac{q(-\vec{r}^*)}{4\pi\epsilon_o(-r*)^3} = \frac{q\vec{r}^*}{4\pi\epsilon_o r^{*3}} . \quad (16)$$

In order for Eq. (16) to retain the form of Eq. (15), so that Coulomb's law remains unchanged in the negative universe, we must let $q/\epsilon_o \to -(q^*/\epsilon_o^*)$ to have

$$-\vec{E}^* = -\frac{q^*\vec{r}^*}{4\pi\epsilon_o^* r^{*3}} , \quad (17)$$

which is of the form of Eq. (15) upon canceling the signs.

The negative sign of $-(q^*/\epsilon_o^*)$ is associated with the electric charge, while the electric permittivity of free space in our (or positive) universe, retains its positive sign in the negative universe. This can be ascertained from the relation for the divergence of electric field

$$\vec{\nabla} \cdot \vec{E} = \varrho/\epsilon_o . \quad (18)$$

In the negative universe, we must let $\vec{\nabla} \to -\vec{\nabla}^*$, $\vec{E} \to -\vec{E}^*$, $\varrho \to \varrho^*$ (since $\varrho = q/V \to -q^*/(-V^*) = q^*/V^* = \varrho^*$), while retaining ϵ_o meanwhile in (18) to have

$$-\vec{\nabla}^* \cdot (-\vec{E}^*) = \varrho^*/\epsilon_o . \quad (19)$$

In order for (19) to retain the form of Eq. (18), we must let $\epsilon_o \to \epsilon_o^*$, which confirms the positivity of the electric permittivity of free space in the negative universe. The conclusion then is that the electric charge of a particle in the negative universe has opposite sign as the electric charge of its symmetry-partner in the positive universe.

The magnetic permeability of empty space (or vacuum) is likewise positive in the negative universe, like in our universe. This follows from the relation, $c^2 = 1/\epsilon_o\mu_o$, with ϵ_o positive in the negative universe.

Let us consider the energy stored in a uniform electric field $\vec{E}$ in a parallel plate capacitor, say, and in a uniform magnetic field $\vec{B}$ in an electromagnet, say. These are given as follows in the positive universe

$$\varepsilon_E = \epsilon_o|\vec{E}|^2 V ; \quad (20a)$$
$$\varepsilon_B = |\vec{B}|^2 V/\mu_o , \quad (20b)$$

where V is the volume of empty space (or vacuum) occupied by the uniform electric field within the capacitor and by the uniform magnetic field within the electromagnet.

Volume of space (meter^3) is a negative quantity in the negative universe and electric field and magnetic field in the negative universe are inversions of their symmetry-partners in the positive universe. Consequently in writing Eqs. (20a) and (20b) in the negative universe, we must let $V \to -V^*$, $\vec{E} \to -\vec{E}^*$, $\vec{B} \to -\vec{B}^*$, $\epsilon_o \to \epsilon_o^*$ and $\mu_o \to \mu_o^*$, while leaving ε_E and ε_B, meanwhile to have

$$\varepsilon_E = \epsilon_o^*|-\vec{E}^*|^2(-V^*) = -\epsilon_o^*|\vec{E}^*|^2 V^* \quad (21a)$$
$$\varepsilon_B = |-\vec{B}^*|^2(-V^*)/\mu_o^* = -|\vec{B}^*|^2 V^*/\mu_o^* \quad (21b)$$

In order for Eqs. (21a) and (21b) to retain the forms of Eqs. (20a) and (20b), so that the expressions for energy stored in electric and magnetic fields are invariant in the negative universe, we must let $\varepsilon_E \to -\varepsilon_E^*$ and $\varepsilon_B \to -\varepsilon_B^*$ in Eqs. (21a) and (21b) to have

$$-\varepsilon_E^* = -\epsilon_o|\vec{E}^*|^2 V^* \quad (22a)$$
$$-\varepsilon_B^* = -|\vec{B}^*|^2 V^*/\mu_o \quad (22b)$$

Equations (22a) and (22b) are in the forms of Eqs. (20a) and (20b) upon canceling the signs. The transformations, $\varepsilon_E \to -\varepsilon_E^*$ and $\varepsilon_B \to -\varepsilon_B^*$, which converts Eqs. (21a) and (21b) to Eqs. (22a) and (22b), imply that energy stored in electric field and energy stored in magnetic field are negative in the negative universe. Indeed most energy are negative in the negative universe. For example kinetic energy, rest energy and total are negative by virtue of negative mass, but gravitational potential energy is positive in the negative universe.

We are now left to determine the sign in the negative universe of the last fundamental quantity namely, absolute temperature. It has been found impossible to determine the sign of absolute temperature in the negative universe in a unique manner from consideration of the equations of thermodynamics, kinetic theory of gas and transport phenomena. It has been necessary to make recourse to the more fundamental notions of the "arrow of entropy" and "arrow of time" in order to propagate. These notions have been made tangible by the works of [4].

We know that entropy always increases or always "flows" along the positive direction of the 'entropy axis' S in our (or positive) universe, even as time always increases or always "flows" into the future direction, that is, along the positive time axis ct in our universe. Thus the arrow of time and the arrow of entropy lie parallel to each other in our universe. Or in the words of Prigogine, "[positively directed] arrow of time is associated with [positively directed] arrow of entropy" [4]. Thus absolute entropy and change in entropy are positive quantities in our (or positive) universe, just as time and intervals of time are positive quantities in our (or positive) universe.

The arrow of time and the arrow of entropy should likewise lie parallel to each other in the negative universe. We then infer from this that 'entropy axis' is negatively directed and, hence that entropy is a negative quantity in the negative universe, since time is negatively directed and is hence a negative quantity in the negative universe.

Having determined the sign of absolute entropy in the negative universe from the above reasoning, it is now an easy matter to determine the sign of absolute temperature in the negative universe. For let us write the following fundamental relation for absolute entropy in our universe,

$$S = k \ln W \; , \tag{23}$$

where k is the Boltzmann constant and W is the number of micro-states in an ensemble in the quantum-mechanical formulation [5]. In the negative universe, we must let $S \to -S^*$ and $W \to W^*$ (since being a dimensionless number, W retains its positive sign in the negative universe, as shall be discussed for all dimensionless numbers shortly), while retaining k meanwhile to have

$$-S^* = k \ln W^* \; . \tag{24}$$

In order for Eq. (24) to retain the form of Eq. (23) we must let $k \to -k^*$ (in Eq. (24)) to have

$$-S^* = -k^* \ln W^* \; , \tag{25}$$

which is of the form of Eq. (23) upon canceling the signs. Thus the Boltzmann constant is a negative quantity in the negative universe.

The average energy ε of a molecule, for one degree-of-freedom motion of a diatomic molecule in a gas maintained at thermal equilibrium at temperature T is

$$\varepsilon = \frac{2}{3} kT \; , \tag{26}$$

where, again, k is the Boltzmann constant. In the negative universe, we must let $\varepsilon \to -\varepsilon^*$, (since the kinetic energy $\frac{1}{2}mv^2$ of molecules, like mass m, is a negative quantity in the negative universe), and also let $k \to -k^*$ in Eq. (26), while retaining T meanwhile to have

$$-\varepsilon^* = \frac{2}{3}(-k^*)T \; , \tag{27}$$

which is of the form of Eq. (26) upon canceling the signs. The transformation, $T \to T^*$, required to convert Eq. (27) to Eq. (26), implies that absolute temperature is a positive quantity in the negative universe.

In summary, the fundamental quantities namely, mass m, electric charge Q and absolute temperature T, transform between the positive and negative universes as, $m \to -m^*; Q \to -Q^*$ and $T \to T^*$. Thus by writing various natural laws in terms of negative spacetime dimensions, negative mass, negative electric charge and positive absolute temperature, and requiring the laws to retain their usual forms in the positive universe, the signs of other physical quantities and constants in the negative universe can be derived. However a faster way of deriving the signs in the negative universe of derived (or non-fundamental) physical quantities and constants is to check the signs of their dimensions in the negative universe, as demonstrated for a few quantities and constants below.

Let us consider the Boltzmann constant k and absolute entropy S, whose negative signs in the negative universe have been deduced above. They both have the unit, Joule/Kelvin, or dimension $\frac{ML^2}{T^2\Theta}$ in the positive universe, where M represents mass 'dimension', L represents length dimension, T represents time dimension and Θ represents absolute temperature 'dimension'. In the negative universe, we must let, $M \to -M^*, L \to -L^*, T \to -T^*$ and $\Theta \to \Theta^*$, to have the dimensions of Boltzmann constant and absolute entropy in the negative universe as, $\frac{-M^*(-L^*)^2}{(-T^*)^2\Theta^*} = -\frac{M^*L^{*2}}{T^{*2}\Theta^*}$.

The Boltzmann constant and absolute entropy are negative quantities in the negative universe, since their common dimension is negative in the negative universe.

The Planck constant has the unit Joule/second and dimension $\frac{ML^2}{T^3}$ in the positive universe. In the negative universe, it has dimension $\frac{-M^*(-L^*)^2}{(-T^*)^3}$, which is positive. Hence the Planck constant is a positive quantity in the negative universe.

The specific heat capacity has the unit $\frac{\text{Joule}}{\text{Kg.Kelvin}}$ and dimension $\frac{L^2}{T^2\Theta}$ in the positive universe. In the negative universe it has dimension $\frac{(-L^*)^2}{(-T^*)^2\Theta^*}$, which is positive. Hence specific heat capacity is a positive quantity in the negative universe.

The electric permittivity of vacuum ϵ_o has the unit of Joule·metre / Coulomb^2 and dimension $\frac{ML^3}{T^2C^2}$ in the positive universe, where C is used to represent the charge 'dimension'. In the negative universe, it has dimension $\frac{(-M^*)(-L^*)^3}{(-T^*)^2(-C^*)^2} = \frac{M^*L^{*3}}{T^{*2}C^{*2}}$, which is positive. Hence the electric permittivity of vacuum is a positive quantity in the negative universe. This fact has been derived earlier in the process of deriving the sign of electric charge in the negative universe. Likewise magnetic permeability of vacuum μ_o has dimension $\frac{ML}{C^2}$ in the positive universe and dimension, $\frac{-M^*(-L^*)}{(-C^*)^2} = \frac{M^*L^*}{C^{*2}}$, in the negative universe. It is hence a positive quantity in both the positive and negative universes, as has also been derived above.

An angular measure in space in the positive universe has the same sign as the symmetry-partner angular measure in the negative universe. This follows from the fact that an arc length, $s = r\theta$ (metre), in the positive universe corresponds to a negative arc length, $s^* = -(r^*\theta^*)$ ($-$metre*), in the negative universe. In other words, an arc length in the positive universe and its symmetry-partner in the negative universe transform as, $r\theta \to -(r^*\theta^*)$. But the radii of the symmetry-partner arcs transform as, $r \to -r^*$. It follows from these two transformations that an angular measure in space in the positive universe has the same sign as its symmetry-partner in the negative universe, that is, $\pm\theta \to \pm\theta^*$ and $\pm\varphi \to \pm\varphi^*$, etc.

Angular momentum $\vec{L}$ $(= m\vec{v} \times \vec{r})$ and intrinsic spin s have unit Joules·Second and dimension $\frac{ML^2}{T}$ in the positive universe. Their dimension is, $\frac{-M^*(-L^*)}{(-T^*)} = \frac{M^*L^{*2}}{T^*}$, in the negative universe. Hence angular momentum $\vec{L}^*$ and intrinsic spin s^* of a particle* in the negative universe have the same signs as the angular momentum $\vec{L}$ and intrinsic spin s of the symmetry-partner particle in the positive universe.

The magnetic moment, $\mu = qh/2mc$, of a particle in the positive universe corresponds to magnetic moment, $\mu^* = (-q)h/2(-m^*)c = qh/2m^*c$, of the symmetry-partner particle in the negative universe. Hence the signs of the magnetic moments of a particle in the positive universe and its symmetry-partner in the negative universe are the same.

Finally, a dimensionless quantity or constant in the positive universe necessarily has the same sign as its symmetry-partner in the negative universe, as follows from the above. Examples of dimensionless constants are the dielectric constants, ϵ_r and μ_r, and the number of micro-states in an ensemble W in Eq. (23).

Table 1. gives a summary of the signs of some physical quantities and physical constants, along with their intrinsic counterparts, in the positive and negative universes. The signs in the positive and negative universes of other physical quantities and constants that are not included in Table I can be easily determined from the signs of their dimensions in the negative universe. The appropriateness of the names positive universe and negative universe is made clearer by Table 1.

Table 1. The signs of spacetime/intrinsic spacetime dimensions, some physical parameters/intrinsic parameters and some physical constants/intrinsic constants in the positive and negative universes

Physical quantity and constant	Symbol	Intrinsic quantity and constant	Sign	
			positive universe	negative universe
Distance/dimension of space	$dx\,;\,x$	$d\varnothing x\,;\,\varnothing x$	$+$	$-$
Interval/dimenion of time	$dt\,;\,t$	$d\varnothing t\,;\,\varnothing t$	$+$	$-$
Mass	m	$\varnothing m$	$+$	$-$
Electric charge	Q	$\varnothing Q$	$+$ or $-$	$-$ or $+$
Absolute entropy	S	$\varnothing S$	$+$	$-$
Absolute temp.	T	$\varnothing T$	$+$	$+$
Energy				
(kinetic)	E	$\varnothing E$	$+$	$-$
(potential)	U	$\varnothing U$	$+$ or $-$	$-$ or $+$
Radiation energy	$h\nu$	$h\varnothing\nu$	$+$	$-$
Electrostatic pot.	Φ_E	$\varnothing\Phi_E$	$+$ or $-$	$+$ or $-$
Gravitational pot.	Φ_g	$\varnothing\Phi_g$	$-$	$-$
Gravi. field	$\vec{g}$	$\varnothing g$	$-$ or $+$	$+$ or $-$
Electric field	$\vec{E}$	$\varnothing E$	$+$ or $-$	$-$ or $+$
Magnetic field	$\vec{B}$	$\varnothing B$	$+$ or $-$	$-$ or $+$
Planck constant	h	h	$+$	$+$
Boltzmann constant	k	$\varnothing k$	$+$	$-$
Thermal conductivity	k	$\varnothing k$	$+$	$-$
Specific heat cap.	c_p	$\varnothing c_p$	$+$	$+$
speed	v	$\varnothing v$	$+$	$+$
Electric permittivity	ϵ_o	$\varnothing\epsilon_o$	$+$	$+$
Electric flux	Q/ϵ_o	$\varnothing Q/\varnothing\epsilon_o$	$+$ or $-$	$-$ or $+$
Magnetic permeability	μ_o	$\varnothing\mu_o$	$+$	$+$
Angular measure	θ, φ	$\varnothing\theta, \varnothing\varphi$	$+$ or $-$	$+$ or $-$
Parity	Π	$\varnothing\Pi$	$+$ or $-$	$-$ or $+$
Angular momentum	$\vec{L}$	$\varnothing L$	$+$ or $-$	$+$ or $-$
Intrinsic spin	s	$\varnothing s$	$+$ or $-$	$+$ or $-$
Magnetic moment	μ	$\varnothing\mu$	$+$ or $-$	$+$ or $-$
⋮	⋮	⋮	⋮	⋮

3 DEMONSTRATING THE INVARIANCE OF THE NATURAL LAWS IN THE NEGATIVE UNIVERSE

It shall be shown in this section that the simultaneous negations of spacetime dimensions and mass, along with simultaneous reversal of the sign of electric charge, retention of the positive sign of absolute temperature and substitution of the signs of other physical quantities and physical constants in the negative universe, summarized in column 5 of Table I in its complete form, render all natural laws unchanged. However only the invariance of a few laws in the negative universe namely, mechanics (classical and special-relativistic), quantum mechanics, electromagnetism and propagation of light, the theory of gravity, cosmology and fundamental interactions in elementary particle physics, shall be demonstrated for examples.

3.1 Invariance of Classical Mechanics, Classical Gravitation and Special Relativity in the Negative Universe

Demonstrating the invariance of classical mechanics in the negative universe consists essentially in showing that Newton's laws of motion for an object under an impressed force and due to interaction of an object with an external force field are invariant under the simultaneous operations of inversion of all coordinates (or dimensions) of 3-space (parity inversion), time reversal and negation of mass. The laws are given respectively as follows in the positive universe

$$\vec{F}_{\text{mech}} = m\frac{d^2r}{dt^2}\hat{r} \tag{28}$$

and

$$\vec{F}_{\text{field}} = m(-\nabla\Phi)\hat{k}\ , \tag{29}$$

where $\hat{r}$ and $\hat{k}$ are unit vectors in the directions of the forces $\vec{F}_{\rm mech}$ and $\vec{F}_{\rm field}$ respectively.

In the negative universe, we must let $\vec{F}_{\rm mech} \to -\vec{F}^*_{\rm mech}$, $\vec{F}_{\rm field} \to -\vec{F}^*_{\rm field}$, $m \to -m^*$, $r \to -r^*$, $t \to -t^*$, $\nabla \to -\nabla^*$, $\Phi \to \Phi^*$ (for gravitational and elastic potentials), $\hat{r} \to -\hat{r}^*$ and $\hat{k} \to -\hat{k}^*$, in Eqs. (28) and (29) to have

$$-\vec{F}^*_{\text{mech}} = -m^*\frac{d^2(-r^*)}{d(-t^*)^2}(-\hat{r}^*) = m^*\frac{d^2r^*}{dt^{*2}}(-\hat{r}^*) \tag{30}$$

and

$$\begin{aligned} -\vec{F}^*_{\text{field}} &= -m^*\,(-(-\nabla^*)(\Phi^*))\,(-\hat{k}^*) \\ &= m^*\,(\nabla^*\Phi^*)\,(-\hat{k}^*) \end{aligned} \tag{31}$$

Equations (30) and (31) are the same as Eqs. (28) and (29) respectively upon canceling the signs.

The invariance in the negative universe of the classical laws of motion (28) and (29) in the positive universe, imply that a body of negative mass $-m^*$ in the negative universe moves along a trajectory, when impressed upon by an external mechanical force $-\vec{F}^*_{\rm mech}$, or when it is moving within a force field with potential function Φ^* in the negative universe, which is identical to the trajectory followed by the symmetry-partner body of positive mass m in the positive universe, which is impressed upon by an external symmetry-partner mechanical force $\vec{F}_{\rm mech}$, or which is moving within a symmetry-partner force field with potential function Φ in the positive universe.

The invariance in the negative universe of the trajectories of a body implied by the invariance in the negative universe of the differential classical laws of motion (28) and (29) for the body, established above can be alternatively formulated as the invariance in the negative universe of the variational formula of Maupertuis. In the positive universe, this is given as follows

$$\delta\int_{p_1}^{P_2}\left(\frac{2}{m}\,(E-U)\right)^{1/2}dt = 0\ . \tag{32}$$

In the negative universe, we must let $m \to -m^*$, $E \to -E^*$, $U \to -U^*$ and $dt \to -dt^*$ in Eq. (32) to have

$$\delta\int_{p_1^*}^{p_2^*}\left(\frac{2}{-m^*}\,(-E^*-(-U^*))\right)^{1/2}(-dt^*) = 0$$

or

$$\delta \int_{p_1^*}^{p_2^*} \left(\frac{2}{m^*} (E^* - U^*) \right)^{1/2} dt^* = 0 \ . \tag{33}$$

The summary of the above is that, although inertial mass, kinetic energy, distances in space and periods of time are negative in the negative universe, material particles in the negative universe perform identical motions under impressed forces and external force fields as their symmetry-partners perform under symmetry-partner impressed forces and external force fields in the positive universe. Thus outward external forces lead to outward motions of bodies both in the positive and negative universes. Attractive gravitational field in the positive universe correspond to symmetry-partner repulsive gravitational field in the negative universe, but they both give rise to attractive motions of particles (toward the field sources) in both universes (as shown below). The transformation of classical mechanics in the positive universe into the negative universe does not give rise to strange motions and associated strange phenomena.

Demonstrating the invariance of classical gravitation (or classical gravitational interaction) in the negative universe, consists in showing the invariance in the negative universe of Newton law of gravity in differential form and the implied Newton law of universal gravity namely,

$$\vec{\nabla} \cdot \vec{g} = -4\pi G \varrho \ , \tag{34}$$

or

$$\nabla^2 \Phi = 4\pi G \varrho \tag{35}$$

and

$$\vec{F} = m\vec{g} = -\frac{GMm\vec{r}}{r^3} \ , \tag{36}$$

where

$$\varrho = m/V \ ; \ \text{(mass density)} \tag{37}$$

$$\Phi = -GM/r \ ; \tag{38}$$

$$\vec{g} = -GM\vec{r}/r^3 \ . \tag{39}$$

In writing Eqs. (37) – (39) in the negative universe, we must let $m \to -m^*$; $M \to -M^*$; $r \to -r^*$ and $V \to -V^*$ (volume of m) to have

$$m/V \to -m^*/(-V^*) = m^*/V^* \Rightarrow \varrho \to \varrho^* \ ; \tag{40}$$

$$-\frac{GM}{r} \to -\frac{G(-M^*)}{(-r^*)} = -\frac{GM^*}{r^*} \Rightarrow \Phi \to \Phi^* \tag{41}$$

and

$$-\frac{GM\vec{r}}{r^3} \quad \to \quad -\frac{G(-M)(-\vec{r}^*)}{(-r^*)^3} = \frac{GM^*\vec{r}^*}{r^{*3}} \Rightarrow \vec{g} \to -\vec{g}^* \ . \tag{42}$$

By using the transformations (40) – (42) along with $\vec{\nabla} \to -\vec{\nabla}^*$ in Eqs. (34) – (36) we have

$$\begin{aligned} (-\vec{\nabla}^*) \cdot (-\vec{g}^*) &= -4\pi G \varrho^* \ , \\ \vec{\nabla}^* \cdot \vec{g}^* &= -4\pi G \varrho^* \ , \end{aligned} \tag{43}$$

or

$$\begin{aligned} (-\nabla^*)^2 \Phi^* &= 4\pi G \varrho^* \ , \\ \nabla^{*2} \Phi^* &= 4\pi G \varrho^* \end{aligned} \tag{44}$$

and

$$\vec{F}^* = (-m^*)(-\vec{g}^{\,*}) = -\frac{G(-M^*)(-m^*)(-\vec{r}^{\,*})}{(-r^*)^3} \ ,$$

or

$$\vec{F}^* = m^*\vec{g}^* = -\,GM^*m^*\vec{r}^{\,*}/r^{*3} \ . \tag{45}$$

A comparison of Eqs. (34) – (36) in the positive universe with the corresponding Eqs, (43) – (45) in the negative universe, shows that Newton law of gravity in differential form and the implied Newton law of universal gravity are invariant in the negative universe. The invariance of classical gravitation (or classical gravitational interaction) in the negative universe has thus been demonstrated. This is true despite the fact that gravitational potential does not change sign, while gravitational field (or gravitational acceleration) changes sign in the negative universe, according to Eqs. (41) and (42).

Demonstrating the invariance of the special theory of relativity in the negative universe consists in demonstrating the invariance of Lorentz transformation, time dilation and length contraction formulas and the special-relativistic expressions for mass and other parameters in that universe. Now for motion at speed v of a particle of rest mass m_0 along the $x-$axis of the coordinate system attached to it relative to the observer in the positive universe, the Lorentz transformation of the primed affine spacetime coordinates, $(\tilde{x}\,', \, \tilde{y}\,', \, \tilde{z}\,', \, c\tilde{t}\,')$, of the particle's primed affine frame into the unprimed affine spacetime coordinates, $(\tilde{x}, \, \tilde{y}, \, \tilde{z}, \, c\tilde{t})$, of the particle's unprimed affine frame, has been written as system (31) in the first part of this paper [1]. The special-relativistic mass is given in the positive universe by the usual expression (5) of this paper.

In the negative universe, we must let, $(\tilde{x}\,', \, \tilde{y}\,', \, \tilde{z}\,', c\tilde{t}\,'; \, m_0) \to (-\tilde{x}\,'^*, -\tilde{y}\,'^*, -\tilde{z}\,'^*, -c\tilde{t}\,'^*; -m_0^*)$, and $(\tilde{x}, \, \tilde{y}, \, \tilde{z}, c\tilde{t}; m) \to (-\tilde{x}^{\,*}, \, -\tilde{y}^{\,*}, \, -\tilde{z}^{\,*}, \, -c\tilde{t}^{\,*}; \, -m^*)$. Consequently the Lorentz transformation of the affine spacetime coordinates of the frame of reference attached to the symmetry-partner particle in motion relative to the symmetry-partner 3-observer* in the negative universe, written as system (36) in [1], shall be re-written here as

$$\begin{aligned} -\tilde{t}\,'^* &= \gamma\left(-\tilde{t}^{\,*} - \frac{v}{c^2}(-\tilde{x}^{\,*})\right) \ ; \\ -\tilde{x}\,'^* &= \gamma\left(-\tilde{x}^{\,*} - v(-\tilde{t}^{\,*})\right) \ ; \\ -\tilde{y}\,'^* &= -\tilde{y}^{\,*} \ ; \ -\tilde{z}\,'^* = -\tilde{z}^{\,*} \ ; \end{aligned} \tag{46}$$

while the expression for special-relativistic mass in the negative universe becomes the following

$$-\,m^* = -\gamma m_0^* \ . \tag{47}$$

The expressions for time dilation and length contraction in the negative universe are similarly given respectively as

$$-\tilde{t}^{\,*} = \gamma(-\tilde{t}\,'^*) = (1 - v^2/c^2)^{-1/2}(-\tilde{t}\,'^*) \ ; \tag{48}$$

$$-\tilde{x}^{\,*} = \gamma^{-1}(-\tilde{x}\,'^*) = (1 - v^2/c^2)^{1/2}(-\tilde{x}\,'^*) \ . \tag{49}$$

Although the negative signs must be retained in (46), (47), (48) and (49) in the negative universe, mathematically they cancel, thereby making Lorentz transformation and the other equations of special relativity written above, as well as others not written, in the negative universe, to retain their usual forms in the positive universe. Thus Lorentz invariance obtains in the negative universe.

3.2 Invariance of Quantum Mechanics in the Negative Universe

The time-dependent Schrödinger wave equation is the following in the positive universe

$$H(\vec{r}, \, t, \, m, \, q)|\Psi(\vec{r}, \, t, \, m, \, q)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi(\vec{r}, \, t, \, m, \, q)\rangle \tag{50}$$

Writing Eq. (50) in the negative universe, while leaving Ψ unchanged meanwhile gives

$$-H^*(-\vec{r}^{\,*},\,-t^*,\,-m^*,\,-q^*)|\Psi(\vec{r},\,t,\,m,\,q)\rangle = i\hbar^*\frac{\partial}{\partial(-t^*)}|\Psi(\vec{r},\,t,\,m,\,q)\rangle\;, \tag{51}$$

where the fact that the Boltzmann constant transforms as $\hbar \to \hbar^*$ between the positive and negative universes in Table I has been used.

Now the wave function should transform between the positive and negative universes either as

$$\Psi(\vec{r},\,t,\,m,\,q) \to \Psi^*(-\vec{r}^{\,*},\,-t^*,\,-m^*,\,-q^*) = \Psi^*(\vec{r}^{\,*},\,t^*,\,m^*,\,q^*)\;, \tag{52}$$

or

$$\Psi(\vec{r},\,t,\,m,\,q) \to -\Psi^*(-\vec{r}^{\,*},\,-t^*,\,-m^*,\,-q^*) = -\Psi^*(\vec{r}^{\,*},\,t^*,\,m^*,\,q^*)\;. \tag{53}$$

The parity of the wave function is conserved in Eq. (52) but inverted in Eq. (53). The phase of the wave function, being a dimensionless number, remains unchanged in the negative universe.

Let us consider the following wave function in the positive universe,

$$\Psi(\vec{r},\,t) = A\sin\left(\vec{k}\cdot\vec{r} - \omega t\right)\;. \tag{54}$$

The symmetry-partner wave function in the negative universe is obtained by letting $\vec{r} \to -\vec{r}^{\,*}$, $\vec{k} \to -\vec{k}^*$, $\omega \to -\omega^*$, $t \to -t^*$ and $A \to -A^*$ in Eq. (54) yielding

$$\Psi^*(\vec{r},\,t) = -A^*\sin\left(-\vec{k}^*\cdot(-\vec{r}^{\,*}) - (-\omega^*)(-t^*)\right) = -A^*\sin(\vec{k}^*\cdot\vec{r}^{\,*} - \omega^* t^*)\;. \tag{55}$$

The transformation $A \to -A^*$ is necessary, since inversion in the origin of the coordinates of a Euclidean 3-space inverts the amplitude of a wave in that space. On the other hand, the phase of a wave function, being a dimensionless number, does not change sign in the negative universe, as mentioned above.

Thus the transformation (53), and not (52), is the correct transformation of the wave function between the positive and negative universes. This is obviously so since Eq. (53) is a parity inversion situation, which is in agreement with the natural parity inversion $(x \to -x^*, y \to -y^*, z \to -z^*)$ (or $\Pi \to -\Pi$), between the positive and negative universes included in Table I. By incorporating the transformation (53) into Eq. (51) we have the following

$$-H^*(-\vec{r}^{\,*},-t^*,-m^*,-q^*)|-\Psi^*(-\vec{r}^{\,*},-t^*,-m^*,-q^*)\rangle \quad = -i\hbar^*\frac{\partial}{\partial t^*}|-\Psi^*(-\vec{r}^{\,*},-t^*,-m^*,-q^*)\rangle\;,$$

or

$$H^*(\vec{r}^{\,*},t^*,m^*,q^*)|\Psi^*(\vec{r}^{\,*},t^*,m^*,q^*)\rangle = i\hbar^*\frac{\partial}{\partial t^*}|\Psi^*(\vec{r}^{\,*},t^*,m^*,q^*)\rangle\;. \tag{56}$$

This is of the form of Eq. (50).

The invariance of the Schrödinger wave equation in the negative universe has thus been established. It is straight forward to similarly demonstrate the invariance in the negative universe of the Dirac's equation for the electron and of the Gordon's equation for bosons.

3.3 Invariance of Maxwell Equations in the Negative Universe

The Maxwell equations in a medium with the presence of electric charge density ϱ and electric current density $\vec{J}$, as sources, are given in the positive universe as

$$\begin{aligned}\vec{\nabla}\cdot\vec{E} &= \varrho/\epsilon\;;\;\vec{\nabla}\cdot\vec{B} = 0\;;\\ \vec{\nabla}\times\vec{B} &= \mu\vec{J} + \epsilon\mu\frac{\partial\vec{E}}{\partial t}\;;\;\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\;.\end{aligned} \tag{57}$$

Now, $\varrho = \frac{charge}{volume}$, is the electric charge density of the medium in the positive universe. The charge density of the symmetry-partner medium in the negative universe is the positive quantity, $\frac{-charge^*}{-volume^*} = \frac{charge^*}{volume^*} = \varrho^*$. The magnitude of an electric current is, $I = \frac{charge}{time}$, or $I = \varrho v A$, in the positive universe and the magnitude of its symmetry-partner in the negative universe is the positive quantity, $\frac{-charge^*}{-time^*} = \frac{charge^*}{time^*} = I^*$, or $\varrho^* v A^* = I^*$, since speed v, charge density ϱ and area A, do not change sign in the negative universe. Similarly the magnitude of an electric current density of a medium in the positive universe is, $J = \frac{current}{area}$, and the magnitude of the current density of the symmetry-partner medium in the negative universe is, $\frac{current^*}{area^*} = J^*$.

Thus in obtaining the Maxwell equations in the negative universe, we must let, $\vec{E} \to -\vec{E}^*$, $\vec{B} \to -\vec{B}^*$, $\varrho \to \varrho^*$, $\vec{J} \to \vec{J}^*$, $\vec{\nabla} \to -\vec{\nabla}^*$, $\epsilon \to \epsilon^*, \mu \to \mu^*$ and $t \to -t^*$, in system (57) to have

$$\begin{aligned} -\vec{\nabla}^* \cdot (-\vec{E}^*) &= \varrho^*/\epsilon^* \ ; \ -\vec{\nabla}^* \cdot (-\vec{B}^*) = 0 \ ; \\ -\vec{\nabla}^* \times (-\vec{B}^*) &= \mu^* \vec{J}^* + \epsilon^* \mu^* \frac{\partial(-\vec{E}^*)}{\partial(-t^*)} \ ; \\ -\vec{\nabla}^* \times (-\vec{E}^*) &= -\frac{\partial(-\vec{B}^*)}{\partial(-t^*)} \ . \end{aligned} \tag{58}$$

System (58) with the negative signs is the form in which the Maxwell equations are written by physicists* in the negative universe. The signs cancel mathematically thereby making system (58) to retain the form of system (57) and thereby establishing the invariance of Maxwell equations in the negative universe.

The law of propagation of electromagnetic waves derived from Maxwell equations remain invariant in the negative universe as a consequence of the above. The equations are given with the assumption of spatially uniform sources, ϱ and $\vec{J}$, time-independent $\vec{J}$ and space-independent and time-independent μ and ϵ in the medium, in the positive universe as

$$\nabla^2 \vec{E} = \frac{1}{V^2} \frac{\partial^2 \vec{E}}{\partial t^2} \ ; \ \nabla^2 \vec{B} = \frac{1}{V^2} \frac{\partial^2 \vec{B}}{\partial t^2} \ , \tag{59}$$

where, $V = 1/\sqrt{\mu\epsilon}$, is the speed of light in the medium. While in the negative universe, the electromagnetic wave equations are

$$(-\nabla^*)^2 (-\vec{E}^*) = \frac{1}{V^2} \frac{\partial^2(-\vec{E}^*)}{\partial(-t^*)^2} \ ; \ (-\nabla^*)^2 (-\vec{B}^*) = \frac{1}{V^2} \frac{\partial^2(-\vec{B}^*)}{\partial(-t^*)^2} \ . \tag{60}$$

Thus as electric field and magnetic field, $\vec{E}$ and $\vec{B}$, propagate as electromagnetic wave at a speed V through a medium in the positive universe, the symmetry-partner fields, $-\vec{E}^*$ and $-\vec{B}^*$, propagate as the identical symmetry-partner electromagnetic wave at the speed V through the symmetry-partner medium in the negative universe. Consequently, although electric charge and electric and magnetic fields change signs in the negative universe, the law of propagation of electric and magnetic fields (electromagnetism) and the law of propagation of electromagnetic waves remain invariant in the negative universe.

3.4 Invariance of General Relativity and Cosmology in the Negative Universe

Since system of coordinates does not enter the covariant tensor formulation of Einstein's field equations, the equations are equally valid for the negative dimensions of the negative universe. The most general form of Einstein field equations in the positive universe is the following

$$R_\mu^\nu - \frac{1}{2} R g_\mu^\nu + \Lambda g_\mu^\nu = -\frac{8\pi G}{c^2} T_\mu^\nu \ , \tag{61}$$

where the energy-momentum tensor T_μ^ν is

$$T_\mu^\nu = (p + \varrho)\, u^\nu u_\mu - p g_\mu^\nu \;, \tag{62}$$

Λ is the cosmological constant, p and ϱ are the pressure and mass-density of the universe respectively, while the other quantities in Eqs. (61) and (62) are as defined in the theory. Λ is usually set to zero when considering local gravitational problems in general relativity, but retained in cosmological problems.

For the static exterior field of a spherical body, we must let $\Lambda = T_\mu^\nu = 0$ in Eq. (61) and require the vanishing of the Ricci tensor to have

$$R_{\mu\nu} = 0 \;. \tag{63}$$

Adopting a metric with signature $(+ \; - \; - -)$, the solution to the Einstein free space field equations (63) first derived by K. Schwarzschild (in 1916) is

$$ds^2 \;\; = \;\; c^2 dt^2 \left(1 - 2GM/rc^2\right) - \frac{dr^2}{(1 - 2GM/rc^2)} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{64}$$

Letting $t \to -t^*,\; r \to -r^*,\; \theta \to \theta^*,\; \varphi \to \varphi^*$ and $M \to -M^*$ in Eq. (64), we find that the Schwarzschild line element or metric tensor remains invariant in the negative universe. Other forms of exterior line elements or metric tensors in general relativity, such as the Kerr's line element and metric tensor, the interior line elements and metric tensors, etc, remain invariant in the negative universe as well. This is so because ds^2 is quadratic in intervals, $cdt,\; dr,\; rd\theta$ and $r\sin\theta d\varphi$, and the components of the metric tensor are dimensionless. This concludes the invariance of general relativity in the negative universe.

Now the metric of spatially homogeneous universe in co-moving coordinates is the Robertson-Walker metric associated with the line element

$$ds^2 = c^2 dt^2 - R(t)^2 \left(\frac{du^2 + u^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)}{\left(1 + (k/4)u^2\right)^2}\right) \;, \tag{65}$$

where, $u = r/r_0$ and the constant k is -1, 0 or $+1$, corresponding to spherical universal 3-space, Euclidean universal 3-space or pseudo-spherical universal 3-space respectively, in pages 400 –406 of [6], for instance. Assuming that the universe is filled with perfect fluid, the field equation (61) along with the energy-momentum tensor (62), have been cast in the following forms from which various cosmological models have been derived in general relativity, as can be found in [6] and other standard texts on general relativity,

$$\frac{8\pi G\varrho}{c^2} \;\; = \;\; -\Lambda + \left[\frac{3k}{R(t)^2} + \frac{3\dot{R}(t)^2}{c^2 R(t)^2}\right] \;; \tag{66}$$

$$\frac{8\pi G}{c^2}\left(\frac{p}{c^2}\right) \;\; = \;\; \Lambda - \left[\frac{k}{R(t)^2} + \frac{\dot{R}(t)^2}{c^2 R(t)^2} + \frac{2\ddot{R}(t)}{c^2 R(t)}\right] \;; \tag{67}$$

$$R(t) \;\; = \;\; R_0 \exp(Ht),\; R_0 = R(t=0) \;, \tag{68}$$

where $R(t)$ is the "radius" of the universe, H is the Hubble constant given by

$$H = \frac{\dot{R}(t)}{R(t)} = \frac{1}{R(t)}\frac{dR(t)}{dt} \;, \tag{69}$$

and the cosmological constant Λ is related to Hubble constant H as

$$\Lambda = 3H^2/c^2 \;. \tag{70}$$

The parameters that appear in cosmological model, that is, in Eqs. (66) through (68), are the global time t, the "radius" of the universe $R(t)$, the mass-density of the universe ϱ, the pressure of the universe p, the Hubble constant H and the cosmological constant Λ. Also the rate of expansion $\dot{R}(t)$, as well as the acceleration $\ddot{R}(t)$, of the expanding universe, enter into the equations. In the negative universe, we must let $t \to -t^*$, $R(t) \to -R^*(-t^*)$, $\varrho \to \varrho^*$, $p \to p^*$, $H \to -H^*$, $\Lambda \to \Lambda^*$, $\dot{R}(t) \to \dot{R}^*(-t^*)$ and $\ddot{R}(t) \to -\ddot{R}^*(-t^*)$, in Eqs. (66) – (68). Doing this, we find that the equations remain unchanged, so that physicists* in the negative universe formulate identical cosmological models as those formulated by physicists in the positive universe. Consequently observers* (or peoples*) in the negative universe make observation of that universe, which are identical to the observation made of the positive universe by observers (or peoples) in the positive universe at all times.

It is easy and straight forward to demonstrate the invariance of the kinetic theory of gas, the laws of propagation of heat (conduction, convection and radiation) in continuous media, transport phenomena and the other macroscopic laws of physics, by following the procedure used to demonstrate the invariance of some macroscopic natural laws above, with the aid of the complete form of Table I.

3.5 Invariance of Fundamental Interactions in the Negative Universe

In a formal sense, the invariance in the negative universe of quantum chromodynamics, quantum electrodynamics, the electro-weak theory and quantum gravity, must be demonstrated with the aid of the complete form of Table I, in order to show the invariance in the negative universe of strong, electromagnetic, weak and gravitational interactions among elementary particles, as has been done for the macroscopic natural laws in this section. However we shall not attempt this. Rather we shall make recourse to the CPT theorem to demonstrate the invariance of the strong, electromagnetic and weak interactions in this section.

The CPT theorem, in a simplified form on page 712 of [7], for instance, states that any hermitian interaction relativistically invariant, commutes with all products of the three operators C (charge conjugation), P (parity inversion), and T (time reversal) in any order. Even if an interaction is not invariant under one or two of the three operations, it is invariant under CPT. The invariance of strong, weak and electromagnetic interactions under CPT is a well established fact in elementary particle physics [7].

Now the spacetime dimensions, $-x^*, -y^*, -z^*$ and $-ct^*$ (in the Cartesian system of the dimensions of 3-space), of the third quadrant (or of the negative universe), are the products of natural parity inversion operation (P) and time reversal operation (T) (or of natural operation PT), on the spacetime dimensions, x, y, z and ct of the first quadrant (or of the positive universe), in Fig. 2. These imply, for instance, that the parity of a Schrodinger wave in the negative universe is natural inversion of the parity of the symmetry-partner Schrodinger wave in the positive universe, as found earlier in sub-section 3.2.

It is premature and, indeed, impossible at this point to formally incorporate intrinsic parity into the present picture in which the flat four-dimensional spacetimes of our universe and the negative universe containing the masses of symmetry-partner material particles and bodies, are underlay by flat two-dimensional intrinsic spacetimes containing the intrinsic masses of the symmetry-partner particles and bodies, in the two-world picture. It shall simply be noted that, while parity in quantum mechanics pertains to the physical Euclidean 3-spaces Σ' and $-\Sigma'^*$, intrinsic parity in intrinsic quantum pertains to the one-dimensional intrinsic spaces $\varnothing\rho'$ and $-\varnothing\rho'^*$ in the two universes. Natural parity inversion, $x' \to -x'^*$, $y' \to -y'^*$, $z' \to -z'^*$, corresponds to natural intrinsic parity inversion, $\varnothing x' \to -\varnothing x'^*$, between the positive and negative universes.

The natural parity inversion of classical quantum-mechanical waves between the positive and negative universes shown in sub-section 3.2, equally applies to intrinsic parities of intrinsic quantum mechanical waves in of intrinsic quantum mechanics in intrinsic spacetime.

These shall, by the principle of sufficient reason, be considered to be equally valid in relativistic quantum mechanics and quantum field theories.

As also derived earlier in this article and included in Table I, the electric charge Q of a particle in the positive universe, corresponds to an electric charge of equal magnitude but of opposite sign $-Q^*$, of the symmetry-partner particle in the negative universe. Thus the electric charge of a particle in the negative universe is the product of natural charge conjugation operation (C) on the electric charge of its symmetry-partner particle in the positive universe.

It follows from the foregoing three paragraphs that strong, weak and electromagnetic interactions among elementary particles in the negative universe are the products of natural operations of parity inversion P, time reversal T and charge conjugation C, in any order (or of natural operation CPT), on strong, weak and electromagnetic interactions among elementary particles in the positive universe. The invariance of strong, weak and electromagnetic interactions among elementary particles in the negative universe follow from this and the CPT theorem.

The invariance of classical gravitation and the general theory of relativity (or of gravitational interaction), at the macroscopic level in the negative universe has been demonstrated earlier in this section. The invariance of macroscopic electromagnetic interaction between the positive and negative universes established in sub-section 3.3 and the invariance of electromagnetic interaction among elementary particles between the positive and negative universes, established by CPT theorem in this sub-section, should obtain for the gravitational interaction counterparts. The invariance in the negative universe of gravitational interaction among elementary particles shall be considered to follows from this.

This section shall be ended with a remark that, all natural laws, including the fundamental interactions among elementary particles, take on the same forms in the positive and negative universes, and this is perfect symmetry of (natural) laws between the positive and negative universes.

4 ON THE CONCEPT OF NEGATIVE MASS IN PHYSICS

The concept of negative mass is not new in physics. The earliest speculations include the elaborate theory of negative mass by Föppl in 1897 and Schuster's contemplation of a universe with negative mass in 1898 [3] . However, as mentioned in [3], the fundamental modern paper on negative mass can be deemed to begin with [8]. As also stated in [3], Bondi pointed out that the mass in classical mechanics actually consists of three concepts namely, inertial mass, m_i, passive gravitational mass m_p, and active gravitational mass m_a. In the Newtonian mechanics (including Newton theory of gravity), $m_i = m_p = m_a$. Also in the general theory of relativity, the principle of equivalence requires that $m_i = m_p$, but m_a may be different [3]. Although all three mass concepts are usually taken to be positive in physics, the theories do not compel this, as noted in [3].

Several papers on negative mass listed in [3] have appeared after [8]. As noted in [3], most of those papers investigate the interaction and possible co-existence of particles with masses of both signs. The paper [3] stands as a reappraisal of the concept of negative mass in the more recent time.

In his analysis, Bonnor starts with the assumption $m_i, m_p > 0,\ m_a < 0$. He arrives at the result that either $m_i < 0, m_p < 0$ and $m_a < 0$ for all particles and bodies or $m_i > 0, m_p > 0$ and $m_a > 0$ for all particles and bodies. He then chooses to work with the former case, that is, all three mass concepts are negative in an hypothetical universe. He substitutes negative mass, including negative rest mass, into mechanics, relativity, gravitation and cosmology and finds that observers located in the hypothetical universe would observe strange phenomena, such as pebbles or sand falling on a stretched membrane producing tension and not compression of the membrane, and a push on a trolley causing it to accelerate toward the person who pushed it, etc. It is certain that this universe of ours is not the hypothetical universe containing negative mass in [3].

The hypothetical universe containing negative mass in [3] is not the negative universe isolated in the two parts of this paper either. This is so, because only mass is made negative, while space and time dimensions, as well as other physical parameters and physical constants are inherently assumed to retain their signs (in our universe) in the hypothetical universe of [3]. This inherent proviso leads to the deduced observation of strange phenomena in the hypothetical universe.

On the other hand, the negative universe being isolated in the first part of this paper and this second part, contains negative mass (all mass concepts), along with the negation of space and time dimensions, as well as the signs of other physical parameters and physical constants summarized in column 5 of Table I. (It is noted that the active gravitational mass may be negative in our universe and positive in the hypothetical universe in [3].) As demonstrated in the preceding section, the natural laws retain their usual forms in the negative universe, and observers located in the negative universe observe phenomena in their universe that are identical to the phenomena observed in our (or positive) universe. There are consequently no strange phenomena in the negative universe of the two parts of this paper.

This section is perhaps the conclusion of about a century and a score years of efforts to incorporate the concept of negative mass into physics. Schuster's speculation in 1898 of a universe containing negative mass must have now been realized.

5 Showing how antiparticles in our universe can originate from the negative universe and conversely

As developed in section 4 of [1], the intrinsic affine spacetime coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, of the proper (or primed) particle's intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$ attached to the particle's line of intrinsic rest mass $\varnothing m_0$ in our universe, are simultaneously rotated anticlockwise by equal intrinsic angle $\varnothing\psi$ relative to their projective relativistic (or unprimed) intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, of the particle's unprimed intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$, containing the relativistic intrinsic mass $\varnothing\gamma\varnothing m_0$ of the particle. These happen as a consequence of the intrinsic motion of the the intrinsic rest mass $\varnothing m_0$ (and the relativistic intrinsic mass $\varnothing\gamma\varnothing m_0$) at intrinsic speed $\varnothing v$ relative to the 'stationary' 3-observer in the proper Euclidean 3-space Σ' in our universe.

As done in [1], we shall be considering the flat four-dimensional proper metric spacetime in the Newtonian gravitational field, usually denoted by $(x^{0\prime}, x^{1\prime}, x^{2\prime}, x^{3\prime})$, but uniformly being denoted by (Σ', ct') for brevity in this monograph. The Σ' in our notation is the three-dimensional proper Euclidean space with dimensions, $x^{1\prime}, x^{2\prime}$ and $x^{3\prime}$.

The rotations of $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ relative to their projections, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{x}$, with respect to the 'stationary' 3-observers in Σ' in our universe and the identical simultaneous rotations of the symmetry-partner intrinsic affine spacetime coordinates, $-\varnothing\tilde{x}'^{*}$ and $-\varnothing c\varnothing\tilde{t}'^{*}$, relative to their projections, $-\varnothing\tilde{x}^{*}$ and $-\varnothing c\varnothing\tilde{t}^{*}$, with respect to the symmetry-partner 'stationary' 3-observers* in $-\Sigma'^{*}$ in the negative universe, are illustrated in Fig. 1 in the two-world picture.

Let us, as done in section one of this paper, consider the primed intrinsic affine spacetime coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, of the primed particle's intrinsic affine frame attached to the intrinsic rest mass $\varnothing m_0$ of the particle in our universe, represented by $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0)$ in sub-section 1.2, to be rotated by continuously increasing intrinsic angle $\varnothing\psi$ relative to their projective intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, of the unprimed particle's intrinsic affine frame, containing the relativistic intrinsic mass $\varnothing\gamma\varnothing m_0$ of the particle $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}\ ; \varnothing\gamma\varnothing m_0)$, due to the continuously increasing intrinsic speed $\varnothing v$ of $\varnothing m_0$ (and $\varnothing\gamma\varnothing m_0$) relative to the 'stationary' 3-observer in Σ'. Let us consider this alongside the simultaneous identical symmetry-partner intrinsic event in the negative universe, as illustrated in Fig. 1.

We shall as done in section 4 of [1], consider the intrinsic angle $\varnothing\psi$ of the continuously increasing rotations of the primed intrinsic affine coordinates relative to their projective unprimed intrinsic affine coordinates, with respect to the 'stationary' 3-observers in the Euclidean 3-spaces, Σ' and $-\Sigma'^*$, of our universe and the negative universe, mentioned in the preceding paragraph, to increase from $\varnothing\psi = 0$ to $\varnothing\psi = \varnothing\pi$, with the proviso that attainment of rotation by, $\varnothing\psi = \varnothing\pi/2$, be avoided (or 'dodged') in the process. Rotation by $\varnothing\psi = \varnothing\pi/2$ must be avoided, because it makes the generalized form of the intrinsic Lorentz transformation ($\varnothing$LT) and its inverse in the two-world picture, presented as systems (42) and (43) of [1] and reproduced as systems (1) and (2) of this paper singular.

The explanation of how rotation by intrinsic angle $\varnothing\psi$ in the range $[0, \varnothing\pi]$ can be achieved without attaining (or passing through) rotation by $\varnothing\psi = \varnothing\pi/2$, shall not be of concern in this article. It shall be regarded as an outstanding problem in this article and explored elsewhere with further development of the two-world picture.

As explained in sub-section 4.4 of [1], with the aid of Figs. 10a and 10b of that article, reproduced as Figs. 3a and 3b of this article, along with the generalized forms of the intrinsic Lorentz transformation ($\varnothing$LT) and its inverse in the two-world picture (Eqs. (42) and (43) of that article), or Eqs. (1) and (2) of this article, along with the generalized intrinsic mass expression in the context of $\varnothing$SR in the two-world picture, given by Eq. (6) of this article, continuous rotation by intrinsic angle $\varnothing\psi$ from $\varnothing\psi = 0$ through $\varnothing\psi = \varnothing\pi$, with the proviso that attainment of rotation by $\varnothing\psi = \varnothing\pi/2$ be avoided, will carry the primed particle's intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0)$ from our universe into the negative universe to become primed particle's intrinsic affine frame $(-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*; -\varnothing m_0^*)$. This is expressed by the transformation scheme (10a).

Now rotation by intrinsic angle, $\varnothing\psi = \varnothing\pi$, corresponds to zero intrinsic speed $(\varnothing v = 0)$ of the particle's intrinsic rest mass $\varnothing m_0$ (and also the particle's relativistic intrinsic mass $\varnothing\gamma\varnothing m_0$) relative to the 'stationary' 3-observer in Σ' in our universe. This follows from the derived relation, $\sin\varnothing\psi = \varnothing v/\varnothing c$, in the two-world picture in [1], presented as Eq. (16) of that article. Thus the primed particle's intrinsic affine frame $(-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*; -\varnothing m_0^*)$ formed on the flat two-dimensional proper intrinsic metric spacetime $(-\varnothing\rho'^*, \varnothing c\varnothing t'^*)$ in the negative universe in the transformation scheme (10a) is at rest relative to the symmetry-partner 'stationary' 3-observer* in $-\Sigma'^*$ in the negative universe.

Once the primed particle's intrinsic affine frame $(-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*; -\varnothing m_0^*)$ has been formed on the flat proper intrinsic metric spacetime $(-\varnothing\rho'^*, \varnothing c\varnothing t'^*)$ in the negative universe, at rest relative to the symmetry-partner 3-observer* in $-\Sigma'^*$, in the transformation scheme (10a), it is made manifested outwardly in the primed particle's affine frame $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*; -m_0^*)$ on the flat four-dimensional proper metric spacetime $(-\Sigma'^*, -ct'^*)$ of the negative universe, at rest relative to the symmetry-partner 3-observer* in $-\Sigma'^*$.

In effect, the outward manifestation of the transformation scheme (10a), presented as Eq. (10b), and re-written more instructively as follows obtains

$$\underset{\text{(our universe)}}{(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0)} \xrightarrow{\text{intr. rot. by } \psi=\pi} \underset{\text{(negative universe)}}{(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*; -m_0^*)} \quad (71)$$

Although the rotations of the affine coordinates, $\tilde{x}', \tilde{y}', \tilde{z}'$ and $c\tilde{t}'$, relative to their projections, $\tilde{x}, \tilde{y}, \tilde{z}$ and $c\tilde{t}$ (which would be the rotations of $\tilde{x}'$ and $c\tilde{t}'$ relative to $\tilde{x}$ and $c\tilde{t}$), by angle ψ, do not exist, as supported by argument in sub-section 4.4 (in the paragraph leading to Eq. (28)) of [1], the transformation scheme (10a) nevertheless implies the transformations scheme (10b) re-written as Eq. (71) above.

As shown from the generalized mass and generalized intrinsic mass expressions in the context of $\varnothing$SR and SR in the two-world picture in section one of this paper, presented as Eqs. (5) and (6) of this paper, and also from the requirement of perfect (unbroken) symmetry of natural laws between our universe and the negative universe in section 2 of this paper, the sign of the intrinsic rest mass and rest mass (and the other intrinsic mass concepts and mass concepts) is negative in the negative universe. This derived fact is contained in Table I of this article.

Now the identical simultaneous relative rotations of the primed and unprimed intrinsic affine coordinates in our universe and the negative universe, illustrated in Fig. 1, which is mandated by the unbroken symmetry of state between our universe and the negative universe, when considered along with the fact that mass concepts are negative in the negative universe, stated in the preceding paragraph, imply that the transition of particle from our universe into the negative universe, stated by the transformations scheme (10a) and (10b) or (71), occur simultaneously with the transition of the symmetry-partner particle from the negative universe into our universe, stated by the following transformation schemes, re-written from systems (11a) and (11b) of this paper.

$$\underset{\text{(negative universe)}}{(-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*; -\varnothing m_0^*)} \xrightarrow{\text{rot. by } \varnothing\psi = \varnothing\pi} \underset{\text{(our universe)}}{(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0)} \qquad (72)$$

and

$$\underset{\text{(negative universe)}}{(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*; -m_0^*)} \xrightarrow{\text{intr. rot. by } \psi = \pi} \underset{\text{(our universe)}}{(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0)} \qquad (73)$$

The primed particle's intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0)$ formed on the flat 2-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ and the primed particle's affine frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0)$ formed on the flat four-dimensional proper metric spacetime (Σ', ct') in our universe, in the transformation schemes (72) and (73), are at rest relative to the 'stationary' 3-observer in Σ' in our universe.

As follows from the foregoing, a particle with positive intrinsic rest mass $\varnothing m_0$ and positive rest mass m_0 in our universe, makes transition into the negative universe through the second quadrant of the spacetime/intrinsic spacetime hyperplane of the combined spacetimes/intrinsic spacetimes of our universe and the negative universe in Fig. 2 and Fig. 1, with respect to the 'stationary' 3-observer in Σ' in our universe, in the transformation schemes (10a) and (71), and an identical particle with positive intrinsic rest mass $\varnothing m_0$ and positive rest mass m_0, appears simultaneously from the negative universe, through the fourth quadrant, to replace the lost particle, in the transformation schemes (72) and (73), with respect to the 'stationary' 3-observer in Σ' in our universe.

In symmetry, the symmetry-partner particle* with negative intrinsic rest mass $-\varnothing m_0^*$ and negative rest mass $-m_0^*$, in the negative universe, simultaneously makes transition into our universe through the fourth quadrant, with respect to the symmetry-partner 'stationary' 3-observer* in $-\Sigma'^*$ in the negative universe, in the transformation schemes (72) and (73), and an identical particle* with negative intrinsic rest mass $-\varnothing m_0^*$ and negative rest mass $-m_0^*$, appears simultaneously from our universe, through the second quadrant, to replace the lost particle*, with respect to the 'stationary' 3-observer* in $-\Sigma'^*$ in the negative universe, in the transformations schemes (10a) and (71).

As follows from the preceding two paragraphs, the 3-observer in Σ' in our universe cannot distinguish between the particle that made transition into the negative universe and the particle that appeared from the negative universe to replace it. The 3-observer* in $-\Sigma'^*$ in the negative universe can likewise not distinguish between the particle* that made transition into our universe and the particle* that appeared from our universe to replace it.

The preceding paragraph is so, because intrinsic rest mass and rest mass are the only intrinsic parameter and parameter of material particles (and bodies) considered so far. We shall now add the intrinsic electric charges and the electric charges of the particles and investigate any possible difference this may make on the conclusion reached in the preceding paragraph.

The relative signs of the intrinsic electric charges and electric charges of symmetry-partner particles in our universe and the negative universe have been derived as a consequence of the perfect symmetry of natural laws between our universe and the negative universe in section 2 of this article. The sign of the intrinsic electric charge and the electric charge of a particle* in the negative universe is opposite the sign of the intrinsic electric charge and the electric charge of the symmetry-partner particle in our universe, as derived in section 2 and included in Table I of this article.

Now electric charge is Lorentz-invariant (in the context of SR), as known. It follows that intrinsic electric charge is intrinsic-Lorentz-invariant (in the context of ∅SR). These imply that the net proper (or primed) intrinsic electric charge $\varnothing q'$ of the intrinsic rest mass $\varnothing m_0$ in the proper (or primed) particle's intrinsic affine frame, to be represented by $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0; \varnothing q')$, is the same as the net relativistic (or unprimed) intrinsic electric charge $\varnothing q$ of the relativistic intrinsic mass $\varnothing\gamma\varnothing m_0$ in the projective relativistic (or unprimed) particle's intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}; \varnothing\gamma\varnothing m_0; \varnothing q)$, at all intrinsic angles $\varnothing\psi$ of rotations of the the intrinsic affine coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, relative to their projections, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, or at all intrinsic speeds $\varnothing v$ of the intrinsic rest mass the particle relative to the 'stationary' 3-observer in Σ' in our universe.

It likewise follows that the net primed electric charge q' of the rest mass m_0 of the particle in the primed particle's affine frame $(\tilde{x}', \tilde{x}', \tilde{x}', c\tilde{t}'; m_0; q')$, is the same as the net relativistic (or unprimed) electric charge q of the relativistic mass γm_0 in the relativistic (or unprimed) particle's affine frame $(\tilde{x}, \tilde{x}, \tilde{x}, c\tilde{t}; \gamma m_0; q)$, at all angles ψ of 'rotations' of the the affine coordinates, $\tilde{x}'$ and $c\tilde{t}'$, relative to their projections, $\tilde{x}$ and $c\tilde{t}$, or at all speeds v of the rest mass the particle relative to the 'stationary' 3-observer in Σ' in our universe.

The fact stated in the preceding two paragraphs shall be presented explicitly as intrinsic Lorentz invariance of intrinsic electric charge (in the context of ∅SR) and Lorentz invariance of electric charge (in the context of SR) as

$$\varnothing q = \varnothing q' \quad \text{and} \quad q = q' . \tag{74}$$

On the other hand, intrinsic mass is not intrinsic-Lorentz-invariant and mass is not Lorentz-invariant. These follow from the non-trivial mass relation in SR given by Eq. (48) and intrinsic mass relation in ∅SR by Eq. (49) of [1], reproduced as Eqns. (5) and (6) of this article, which shall yet be re-written more fully as

$$\varnothing m = \varnothing m_0 \sec\varnothing\psi = \varnothing m_0 (1 - \varnothing v^2/\varnothing c^2)^{-1/2} ; \tag{75a}$$

$$m = m_0 \sec\psi = m_0(1 - v^2/c^2)^{-1/2} . \tag{75b}$$

Equation (75a) states the explicit dependence of the intrinsic mass of the particle on the intrinsic angle $\varnothing\psi$ of rotations of the intrinsic affine coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, of the proper (or primed) particle's intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0; \varnothing q')$, relative to the projective intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, of the relativistic (or unprimed) particle's intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}; \varnothing\gamma\varnothing m_0; \varnothing q)$, and on the intrinsic speed $\varnothing v$ of the intrinsic rest mass $\varnothing m_0$ (and also the relativistic intrinsic mass $\varnothing\gamma\varnothing m_0$) of the particle relative to the 'stationary' 3-observer in Σ', and Eq. (75b) states the explicit dependence of mass of the particle on the angle ψ and speed v of the rest mass m_0 (and the relativistic mass γm_0) of the particle relative to the 'stationary' 3-observer in Σ'.

On the other hand, system (74) states the independence of intrinsic electric charge on the intrinsic angle $\varnothing\psi$ or the intrinsic speed $\varnothing v$ and the independence of electric charge on the angle ψ or the speed v, with respect to the 'stationary' 3-observer in Σ'.

Let us include the assumed positive intrinsic electric charge $\varnothing q'$ and the implied positive electric charge q' of the particle in our universe in the transformation schemes (10a) and (71) to have

$$\underset{\text{(our universe)}}{(\varnothing\tilde{x}',\ \varnothing c\varnothing\tilde{t}';\varnothing m_0;\varnothing q')} \xrightarrow{\text{rot. by } \varnothing\psi=\varnothing\pi} \underset{\text{(negative universe)}}{(-\varnothing\tilde{x}'^{*}, -\varnothing c\varnothing\tilde{t}'^{*}; -\varnothing m_0^{*}; \varnothing q'^{*})} \quad (76)$$

and

$$\underset{\text{(our universe)}}{(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0; q')} \xrightarrow{\text{`rot'. by } \psi=\pi} \underset{\text{(negative universe)}}{(-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}, -c\tilde{t}'^{*}; -m_0^{*}; q'^{*})} \quad (77)$$

The intrinsic rest mass and rest mass change sign with rotation through $\varnothing\psi = \varnothing\pi$ and $\psi = \pi$ in the transformations schemes (76) and (77), by virtue of relations (75a) and (75b) for $\varnothing\psi = \varnothing\pi$ and $\psi = \pi$. Whereas intrinsic electric charge and electric charge are unchanged in magnitude or sign with rotation by $\varnothing\psi = \varnothing\pi$ and $\psi = \pi$ in the transformations schemes (76) and (77), because $\varnothing q$ is independent of $\varnothing\psi$ and q is independent of ψ in system (74).

Let us also include the implied negative intrinsic electric charge $-\varnothing q'^{*}$ and negative electric charge $-q'^{*}$ of the symmetry-partner particle* in the negative universe (from Table I) in the transformation schemes (72) and (73), knowing that, $-\varnothing q'^{*}$ and $\varnothing q'$ are equal in magnitude and $-q'^{*}$ and q' are equal in magnitude to have

$$\underset{\text{(negative universe)}}{(-\varnothing\tilde{x}'^{*}, -\varnothing c\varnothing\tilde{t}'^{*}; -\varnothing m_0^{*}; -\varnothing q'^{*})} \xrightarrow{\text{rot. by } \varnothing\psi=\varnothing\pi} \underset{\text{(our universe)}}{(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0; -\varnothing q')} \quad (78)$$

and

$$\underset{\text{(negative universe)}}{(-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}, -c\tilde{t}'^{*}; -m_0^{*}; -q'^{*})} \xrightarrow{\text{intr. rot. by } \psi=\pi} \underset{\text{(our universe)}}{(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0; -q')} \quad (79)$$

As follows from transformation schemes (76) – (79), a particle with positive intrinsic rest mass and positive intrinsic electric charge $(\varnothing m_0;\ \varnothing q')$ and with positive rest mass and positive electric charge $(m_0;\ q')$ in our universe, makes transition into the negative universe, through the second quadrant of the spacetime/intrinsic spacetime hyperplane of the combined spacetimes/intrinsic spacetimes of our universe and the negative universe, with respect to the 'stationary' 3-observer in Σ' in our universe, in the transformation schemes (76) and (77), and a particle with positive intrinsic rest mass and negative intrinsic electric charge $(\varnothing m_0; -\varnothing q')$ and with positive rest mass and negative electric charge $(m_0; -q')$, appears simultaneously from the negative universe through the fourth quadrant to replace the lost particle, in the transformation schemes (78) and (79), with respect to the 'stationary' 3-observer in Σ' in our universe.

In symmetry, the symmetry-partner particle* with negative intrinsic rest mass and negative intrinsic electric charge $(-\varnothing m_0^{*};\ -\varnothing q'^{*})$ and with negative rest mass and negative electric charge $(-m_0^{*};\ -q'^{*})$, in the negative universe, simultaneously makes transition into our universe through the fourth quadrant, with respect to the symmetry-partner 'stationary' 3-observer* in $-\Sigma'^{*}$ in the negative universe, in the transformation schemes (78) and (79), and a particle* with negative intrinsic rest mass and positive intrinsic electric charge $(-\varnothing m_0^{*};\ \varnothing q'^{*})$ and with negative rest mass and positive electric charge $(-m_0^{*};\ q'^{*})$, appears simultaneously from our universe, through the second quadrant, to replace the lost particle*, with respect to the 'stationary' 3-observer* in $-\Sigma'^*$ in the negative universe.

As follows from the penultimate paragraph, the particle that made transition into the negative universe from our universe and the particle that appeared from the negative universe to replace it, both have positive rest mass and positive intrinsic rest mass, but opposite signs of electric charge and opposite signs of intrinsic electric charge. Consequently, the particle that appeared from the negative universe (through the fourth quadrant) into our universe is the antiparticle of the particle that made transition from our universe into the negative universe (through the second quadrant).

As also follows from the penultimate paragraph, the particle* that made transition into our universe from from the negative universe and the particle* that appeared from our universe to replace it, both have negative rest mass and negative intrinsic rest mass, but opposite signs of electric charge and opposite signs of intrinsic electric charge. Consequently, the particle* that appeared in the negative universe from our universe (through the second quadrant) is the antiparticle* of the particle* that made transition from the negative universe into our universe (through the fourth quadrant) to become the antiparticle in our universe.

Now let the particle that made transition into the negative universe from our universe in the discussions above be proton with positive rest mass $m_{0\mathrm{p}}$ and positive unit electric charge $+$e, to be represented by $\mathrm{p}:(m_{0\mathrm{p}}, +\mathrm{e})$. Then the particle which appeared from the negative universe to replace the lost proton has positive rest mass $m_{0\mathrm{p}}$ but negative unit electric charge $-$e. Let us represent this particle by $\bar{\mathrm{p}}$:$(m_{0\mathrm{p}}, -\mathrm{e})$. This particle is the antiproton, since it differs from proton p:$(m_{0\mathrm{p}}, +\mathrm{e})$ by opposite sign of its electric charge.

The particle* that made transition into our universe from the negative universe is the symmetry-partner proton* with negative rest mass $-m^*_{0\mathrm{p}}$ and negative unit electric charge $-$e, to be represented by $\mathrm{p}^*:(-m^*_{0\mathrm{p}}, -\mathrm{e})$, while in the negative universe. Then the particle* which appeared from our universe to replace the lost proton* has negative rest mass $-m^*_{0\mathrm{p}}$ but positive unit electric charge $+$e. Let us represent this particle by $\bar{\mathrm{p}}^* : (-m^*_{0\mathrm{p}}, +\mathrm{e})$. This particle is the antiproton*, since it differs from proton* $\mathrm{p}^* : (-m^*_{0\mathrm{p}}, -\mathrm{e})$ by opposite sign of its electric charge.

The events of transition of a particle from our universe into the negative to be replaced by its antiparticle from the negative universe and simultaneous transition of the symmetry-partner particle* from the negative universe into our universe to be replaced by its antiparticle* described above, are summarized in Table 2. for proton and proton* as the particle and particle*.

Table 2. Proton in our universe and symmetry-partner proton* in the negative universe make transitions simultaneously into the negative universe and our universe and are replaced by antiproton from the negative universe and antiproton* from our universe respectively

Negative universe		Positive universe
anti-proton* $\bar{\mathrm{p}}^* : (-m^*_{0\mathrm{p}}, +\mathrm{e})$ (appears)	$\leftarrow$	proton $\mathrm{p} : (m_{0\mathrm{p}}, +\mathrm{e})$ (disappears)
proton* $\mathrm{p}^* : (-m^*_{0\mathrm{p}}, -\mathrm{e})$ (disappears)	$\rightarrow$	anti-proton $\bar{\mathrm{p}} : (m_{0\mathrm{p}}, -\mathrm{e})$ (appears)

The particle p^* is the symmetry-partner proton in the negative universe (referred to as proton* in Table I), while $\bar{\mathrm{p}}^*$ is the symmetry-partner anti-proton in the negative universe (referred to as anti-proton* in Table I). Thus observations are identical in the positive and negative universes: proton disappears and anti-proton appears to replace it in each universe.

Another parameter used to characterize particles and anti-particles is the magnetic moment μ. For an elementary particle of electric charge q and mass m, the magnetic moment is defined as

$$\mu = \frac{q\hbar}{2mc} \ . \tag{80}$$

For the electron, $q = -\text{e}$ and $m = m_\text{e}$, and for the positron, $q = +\text{e}$ and $m = m_\text{e}$, from which it is clear that the electron and the positron have magnetic moments of equal magnitude but opposite signs. This is equally true for proton and anti-proton and for every other particle and anti-particle pair [7].

Now for a proton $\text{p}:(m_{0\text{p}}, +\text{e})$ in the positive universe and its symmetry-partner proton* $\text{p}^*:(-m^*_{0\text{p}}, -\text{e})$ in the negative universe, the magnetic moments are the following respectively

$$\mu_\text{p} = \frac{+\text{e}\hbar}{2m_\text{p}\,c} \quad \text{and} \quad \mu^*_\text{p} = \frac{-\text{e}\hbar}{2(-m^*_\text{p})c} = \frac{\text{e}\hbar}{2m^*_\text{p}\,c} \ .$$

Hence proton in the positive universe and its symmetry-partner proton* in the negative universe have identical magnetic moments (of the same magnitude and the same positive sign), according to the two-world picture.

Upon the proton* making transition into the positive universe, it becomes the anti-proton $\bar{\text{p}}:(m_{0\text{p}}, -\text{e})$ with positive mass m_p and negative electric charge $-\text{e}$. Likewise upon the proton making transition into the negative universe, it becomes the anti-proton* $\bar{p}^* = (-m^*_{0\text{p}}, +\text{e})$ with negative mass $-m^*_\text{p}$ but positive electric charge $+\text{e}$. Hence magnetic moments change signs according to the following scheme with transitions of proton from the positive to the negative universe and simultaneous transition of proton* from the negative to the positive universe.

Negative universe		Positive universe
$+\mu^*_\text{p} = -\text{e}\hbar/2(-m^*_\text{p})c$	$\rightarrow$	$-\mu_\text{p} = -\text{e}\hbar/2m_\text{p}\,c$
(proton* disappears)		(anti-proton appears)
$-\mu^*_\text{p} = \text{e}\hbar/2(-m^*_\text{p})c$	$\leftarrow$	$+\mu_\text{p} = \text{e}\hbar/2m_\text{p}\,c$
(anti-proton* appears)		(proton disappears)

Thus the two-world picture (or symmetry) is consistent with the known fact in particle physics that, a particle and its anti-particle have the same magnitude but opposite signs of magnetic moments.

Finally let us investigate the consequence on conservation of electric charge in the positive and negative universes of the event of the disappearance of a proton and simultaneous appearance of an anti-proton in the positive universe, which occur simultaneously with the disappearance of a proton* and simultaneous appearance of an anti-proton* in the negative universe. In the positive universe, a charge $+\text{e}$ of a proton is lost and a charge $-\text{e}$ of an anti-proton is gained. Thus the change in electric charge of the positive universe, to be denoted by ΔQ is

$$\Delta Q = -\text{e} - (+\text{e}) = -2\text{e} \ . \tag{81}$$

And in the negative universe, a charge $-\text{e}$ of proton* is lost and a charge $+\text{e}$ of an anti-proton* is gained. Thus the change in electric charge of the negative universe, to be denoted by ΔQ^* is

$$\Delta Q^* = +\text{e} - (-\text{e}) = +2\text{e} \ . \tag{82}$$

Electric charge is neither conserved in the positive universe nor in the negative universe according to Eqs. (81) and (82). It is not clear if this will prevent inter-universe transition of particles described in this section.

However by adding Eqs. (81) and (82) we obtain the change in electric charge of the two universes as

$$\Delta Q + \Delta Q^* = 2\text{e} - 2\text{e} = 0 \ . \tag{83}$$

The change in electric charge of the two universes is zero according to Eq. (83). Hence the events described conserve the electric charge of the two universes as a whole. Indeed it is conservation of electric charge within the two universes that we should worry about, since the events occur on the larger spacetime of combined positive and negative universes. The baryon number of each universe and, consequently, the baryon number of the two universes, are conserved.

One crucial fact that emerges from the description of how an antiproton, or any antiparticle for that matter, in our universe can originate from the negative universe in this section is that, it makes transition into our universe (first and fourth quadrants of the spacetime hyperplane in Fig. 2) through the fourth quadrant with negative (or time reversal) dimension $-ct^*$ of our universe in that figure (and not from the past time axis of the past light cone of the existing picture in one-world). For as long as the antiparticle remains in the fourth quadrant during its transition into our universe, it dwells in the four-dimensional spacetime $(\Sigma, -ct^*)$ with natural time reversal without natural parity inversion.

This section should have been entitled as, "Antiparticles in our universe originate from the negative universe and conversely", but for the outstanding problem of explaining how a particle in our universe can make transition into the negative universe and its symmetry-partner in the negative universe can simultaneously make transition into our universe without encountering singularities in the intrinsic Lorentz transformation and the Lorentz transformation at the point of crossing the event horizons along the time dimensions, ct and $-ct^*$, during their transitions.

The requirement for (or explanation of) not encountering singularities in ∅LT and LT mentioned in the preceding paragraph, deduced in the first part of this paper [1] and mentioned earlier in this section, is the rotations of the intrinsic affine coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, of the particle's proper (or primed) intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'; \varnothing m_0; \varnothing q')$ relative to the intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, of the particle's relativistic (or unprimed) intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}; \varnothing\gamma\varnothing m_0; \varnothing q)$, from $\varnothing\psi = 0$ to $\varnothing\psi = \varnothing\pi$ (in Fig. 1), without attaining (or passing through) $\varnothing\psi = \varnothing\pi/2$ exactly, that is, upon attaining $\varnothing\psi = \varnothing\pi/2 - \varnothing\epsilon$ at the point of making transition from our universe, where $\varnothing\epsilon$ is a small positive intrinsic angle, along with the simultaneous symmetry-partner intrinsic event in the negative universe.

The requirement is the same as saying that the intrinsic rest masses $\varnothing m_0$ of a particle in our universe and its symmetry-partner $-\varnothing m_0^*$ in the negative universe, must attain speed $\varnothing v$ relative to the 'stationary' 3-observers in Σ' and the symmetry-partner 'stationary' 3-observer* in $-\Sigma'^*$ respectively (in Fig. 1), which is slightly lower that the intrinsic speed of light, $(\varnothing v \lesssim \varnothing c)$ (or $\varnothing v = \varnothing c - \varnothing\delta$, where $\varnothing\delta$ is a small positive intrinsic speed), in the context of ∅SR, corresponding to, $v = c - \delta$, in the context of SR, at the point of $\varnothing m_0$ making transition into the negative universe through the second quadrant and $-\varnothing m_0^*$ making transition into our universe through the fourth quadrant. Meeting this requirement implies the compactification of SO(3,1) as discussed in the next section.

Now once a particle has attained the speed $v = c - \delta$ and made transition into the negative universe relative to an observer in our universe, it has done so relative to all observers in our universe, lest it will make transition into the negative universe with respect to one observer and yet remain in our universe with respect to other observers in our universe, which is impossible. Thus another outstanding issue associated with inter-universe transition of particles is that, it must be shown that the intrinsic speed, $\varnothing v = \varnothing c - \varnothing\delta$, and the speed, $v = c - \delta$, at which a particle in our universe and its symmetry-partner particle* in the negative universe make transition into the negative universe and our universe respectively, are the same relative to all observers.

Once the outstanding problem discussed in the penultimate paragraph has been resolved, then the explanation of the origin of antiparticles in the two-world picture presented in this section should be another theory of the origin of antiparticles in our universe. It shall not be compared with the existing theories of the origin of antiparticles in this paper, but comparison should merit investigation upon resolving the outstanding problem. The Diracs hole theory of the origin of the positron [9], the Ernst Stueckelberg and Richard Feynman theory of the origin of the positron [10], and the theory of the origin of antiparticles in general in the context of the quantum field theory [11], are the existing theories of the origin of antiparticles in our universe. All the existing theories are based on the existence of our universe only, along with the existence of the past time furnished by the past light-cone, in the case of the Stueckelberg-Feynmann theory and the theory in the context of the quantum field theory.

6 PROSPECT FOR MAKING THE LORENTZ GROUP COMPACT IN THE TWO-WORLD PICTURE

The impossibility of making the Lorentz group SO(3,1) compact in the context of the Minkowski's geometry in the one-world picture is mentioned in section 3 of [1]. It arises from the fact that the unbounded parameter space, $-\infty < \alpha < \infty$, of the Lorentz boost (the matrix $L(\alpha)$ in Eq. (7) of [1]), in the one-world picture, is unavoidable. The matrix $L(\alpha)$ is reproduced here for convenience of propagation in this section.

$$\mathrm{L}(\alpha) = \begin{pmatrix} \cosh\alpha & -\sinh\alpha & 0 & 0 \\ -\sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \quad (84)$$

Now the new intrinsic matrix $\varnothing\mathcal{L}(\varnothing\psi)$ that generates the intrinsic Lorentz boost, $\varnothing\mathbf{x}' = \varnothing\mathcal{L}(\varnothing\psi)\varnothing\mathbf{x}$, on the flat two-dimensional proper intrinsic metric spacetime in Eq. (11) of [1], reproduced as system (1) of this paper, in the positive universe, or Eq. (21) of that paper in the negative universe, in the two-world picture is the following

$$\varnothing\mathcal{L}(\varnothing\psi) = \begin{pmatrix} \sec\varnothing\psi & -\tan\varnothing\psi \\ -\tan\varnothing\psi & \sec\varnothing\psi \end{pmatrix} , \quad (85)$$

where the intrinsic angle $\varnothing\psi$ takes on values in the concurrent intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ in the positive and negative universes, as explained earlier and illustrated in Figs. 3a and 3b.

The corresponding new matrix $\mathcal{L}(\psi)$ that generates the Lorentz boost, $\mathbf{x}' = \mathcal{L}(\psi)\mathbf{x}$, on the flat four-dimensional proper metric spacetime in Eq. (26) of [1] in the positive universe or Eq. (34) of that paper in the negative universe, in the two-world picture, is the following

$$\mathcal{L}(\psi) = \begin{pmatrix} \sec\psi & -\tan\psi & 0 & 0 \\ -\tan\psi & \sec\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \quad (86)$$

where the angle ψ takes on values in the concurrent open intervals, $(-\pi/2, \pi/2)$ and $(\pi/2, 3\pi/2)$, like $\varnothing\psi$, in the positive and negative universes.

The matrix $\mathcal{L}(\psi)$ can be said to be the outward manifestation on the flat four-dimensional proper metric spacetime of SR of the intrinsic matrix $\varnothing\mathcal{L}(\varnothing\psi)$ on the flat two-dimensional proper intrinsic metric spacetime of $\varnothing$SR. It is to be recalled however that, while the intrinsic angle $\varnothing\psi$ in Eq. (85) measures actual rotation of intrinsic affine coordinates $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$ of the particle's primed intrinsic affine frame relative to the intrinsic affine coordinates $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$ of the particle's unprimed intrinsic affine frame in Fig. 1, in the context of $\varnothing$SR, the angle ψ in Eq. (86) represents intrinsic (i.e. non-observable or non-actual) rotation of affine spacetime coordinates $\tilde{x}'$ and $c\tilde{t}\,'$ of the particle's primed affine frame relative to $\tilde{x}$ and $c\tilde{t}$ of the particle's unprimed affine frame.

The concurrent open intervals $(-\varnothing\pi/2\,,\,\varnothing\pi/2)$ and $(\varnothing\pi/2\,,\,3\varnothing\pi)$ in which the intrinsic angle $\varnothing\psi$ takes on values in the positive and negative universes, constitute a bounded parameter space for the intrinsic matrix $\varnothing\mathcal{L}(\varnothing\psi)$ (the intrinsic Lorentz boost) and the Lorentz boost $\mathcal{L}(\psi)$ in the two-world picture. On the other hand, the matrix $L(\alpha)$ of Eq. (84) that generates the Lorentz boost in the Minkowski's one-world picture is unbounded, because the parameter α in that matrix takes on values in the unbounded parameter space $(-\infty, \infty)$.

Also letting $\varnothing\psi \to \varnothing\pi/2$ and $\varnothing\psi \to -\varnothing\pi/2$ or $3\varnothing\pi/2$ in the intrinsic matrix $\varnothing\mathcal{L}(\varnothing\psi)$ we have, $\sec\varnothing\psi = \tan\varnothing\psi \to \infty$ and $\sec\varnothing\psi = \tan\varnothing\psi \to -\infty$ respectively, which shows that $\varnothing\mathcal{L}(\varnothing\psi)$ (the intrinsic Lorentz boost) and, hence, the Lorentz boost $\mathcal{L}(\psi)$, in the two-world picture, are not closed. This is so because the values of some entries of $\varnothing\mathcal{L}(\varnothing\psi)$

and $\mathcal{L}(\psi)$ are outside the concurrent intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ over which intrinsic angle $\varnothing\psi$ takes on values. Whereas as $\alpha \to \infty$, $\cosh\alpha \to \infty$, $\sinh\alpha \to \infty$, and as $\alpha \to -\infty$, $\cosh\alpha \to \infty$, $\sinh\alpha \to -\infty$ in the matrix $L(\alpha)$, which implies that the Lorentz boost in the Minkowski's one-world picture is closed (since no entry of $L(\alpha)$ is outside the range $(-\infty, \infty)$ of the parameter α) [12]. Thus the range of α is unbounded but $L(\alpha)$ is closed, while the ranges of $\varnothing\psi$ and ψ are bounded but $\varnothing\mathcal{L}(\varnothing\psi)$ and $\mathcal{L}(\psi)$ are not closed. The matrices $L(\alpha)$, $\mathcal{L}(\psi)$ and the intrinsic matrix $\varnothing\mathcal{L}(\varnothing\psi)$ are therefore non-compact.

It is required that the ranges of $\varnothing\psi$ and ψ be bounded and the intrinsic matrix and matrix $\varnothing\mathcal{L}(\varnothing\psi)$ and $\mathcal{L}(\psi)$ be closed over the bounded ranges of $\varnothing\psi$ and ψ, for $\varnothing\mathcal{L}(\varnothing\psi)$ and $\mathcal{L}(\psi)$ to be compact (the Heine-Borel theorem)[12]. It follows from this and the foregoing paragraphs that making the the intrinsic Lorentz boost (85) and, consequently the Lorentz boost (86), in the two-world picture compact has not been achieved in this article and its first part [1]. As deduced in sub-section 1.1 of that paper, making the Lorentz boost compact implies making SO(3,1) compact. Thus SO(3,1) has yet not been made compact in the two-world picture, since the Lorentz boost has not been made compact.

There is prospect for making SO(3,1) compact in the two-world picture however. This is so because the intrinsic matrix $\varnothing\mathcal{L}(\varnothing\psi)$ and, consequently, the matrix $\mathcal{L}(\psi)$ (the intrinsic Lorentz boost and the Lorentz boost in the two-world picture), will become compact by justifiably replacing the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$, in which the intrinsic angle $\varnothing\psi$ takes on values in $\varnothing\mathcal{L}(\varnothing\psi)$ by the concurrent closed intervals $[-(\varnothing\pi/2 - \varnothing\epsilon), \varnothing\pi/2 - \varnothing\epsilon]$ and $[\varnothing\pi/2 + \varnothing\epsilon, 3\varnothing\pi/2 - \varnothing\epsilon]$, where $\varnothing\epsilon$ is a small non-zero positive intrinsic angle. This will make $\varnothing\mathcal{L}(\varnothing\psi)$ and, consequently, $\mathcal{L}(\psi)$, to be closed over the bounded intervals $[-(\varnothing\pi/2 - \varnothing\epsilon), \varnothing\pi/2 - \varnothing\epsilon]$ and $[\varnothing\pi/2 + \varnothing\epsilon, 3\varnothing\pi/2 - \varnothing\epsilon]$ and, hence to be compact. It will certainly require further development of the two-world picture than in this article and its first part to make SO(3,1) compact in two-world — if it will be possible.

Let us shine more light on the discussion in the preceding paragraph. Expressing the rotations of intrinsic affine spacetime coordinates $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ relative to their projections $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$ in terms of the trigonometric ratios $\sec$ and $\tan$ of the intrinsic angle $\varnothing\psi$, in the intrinsic Lorentz transformation ($\varnothing$LT) in our universe of Eq. (11) and its inverse of Eq. (14) of [1], reproduced as systems (1) and (2) of this paper, in the context of $\varnothing$SR in the two-world picture, is the first step toward making the intrinsic Lorentz boost $\varnothing\mathcal{L}(\varnothing\psi)$ (85) above in the two-world compact. The intrinsic length contraction and intrinsic time dilation formulae implied by system (40) of [1] with respect to a 3-observer in the proper Euclidean 3-space Σ', show explicitly that the rotations of $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ relative to $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$ respectively, are expressed in terms of the trigonometric ratios of the intrinsic angle $\varnothing\psi$ in the context of $\varnothing$SR in the two-world picture.

The second step toward making the intrinsic Lorentz boost $\varnothing\mathcal{L}(\varnothing\psi)$ compact is to justifiably replace the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ within which the intrinsic angle $\varnothing\psi$ can take on values in both the positive and negative universes, as illustrated in Figs. 3a and 3b, with the concurrent closed intervals, $[-(\varnothing\pi/2 - \varnothing\epsilon), \varnothing\pi/2 - \varnothing\epsilon]$ and $[\varnothing\pi/2+\varnothing\epsilon, 3\varnothing\pi/2-\varnothing\epsilon]$, where $\varnothing\epsilon$ is a small non-zero positive intrinsic angle, which expectedly will depend on the particle in relative motion. In other words, we must justify the rotations of the intrinsic affine coordinates $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ relative to $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$ and the rotations of $-\varnothing\tilde{x}'^{*}$ and $-\varnothing c\varnothing\tilde{t}'^{*}$ relative to $-\varnothing\tilde{x}^{*}$ and $-\varnothing c\varnothing\tilde{t}^{*}$, over the entire range $[0, 2\varnothing\pi]$ of the intrinsic angle $\varnothing\psi$ in the first cycle, while avoiding (or dodging) the intrinsic angles $\varnothing\pi/2$ and $3\varnothing\pi/2$ in the diagram of Fig. 1 (or in Figs. 3a and 3b).

Once the second step toward making $\varnothing\mathcal{L}(\varnothing\psi)$ compact described in the preceding paragraph has been accomplished, then the rotations of the intrinsic affine coordinates $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ relative to $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$ in the positive universe and the simultaneous rotations of $-\varnothing\tilde{x}'^{*}$ and $-\varnothing c\varnothing\tilde{t}'^{*}$ relative to $-\varnothing\tilde{x}^{*}$ and $-\varnothing c\varnothing\tilde{t}^{*}$ in the negative universe, in the context of $\varnothing$SR, can take place over the entire range $[0, 2\varnothing\pi]$ of $\varnothing\psi$, or over the entire intrinsic hyperplane formed by the combined two-dimensional intrinsic metric spacetimes, $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$, of our universe and the negative universes in Fig. 2, without singularity appearing in $\varnothing\mathcal{L}(\varnothing\psi)$ (in Eq. (85)). The intrinsic Lorentz boost $\varnothing\mathcal{L}(\varnothing\psi)$ is then compact. The Lorentz boost $\mathcal{L}(\psi)$ (in Eq. (86)), being the outward manifestation of $\varnothing\mathcal{L}(\varnothing\psi)$, is then compact likewise. There is certainly a prospect for making the Lorentz boost and, hence, the Lorentz group SO(3,1), compact in the two-world picture.

The prospect for making the intrinsic Lorentz boost $\varnothing\mathcal{L}(\varnothing\psi)$ and the Lorentz boost $L(\psi)$ in the two-world picture compact and, consequently, for making SO(3,1) compact, is the same for a particle of our universe to make transition into the negative universe to become the antiparticle* and for its symmetry-partner particle* in the negative universe to simultaneously make transition into our universe to become the antiparticle, discussed in the preceding section. It must be added that a less-developed form of this paper has appeared in [13].

7 CONCLUSION

This article has enhanced the possibility of the two-world picture started in its first part. In particular, the preceding two sections have exposed a prospective experimental test of the picture. The possibility of experimental verification of the two-world picture rests on the possibility of exchange of elementary particles between the two universes at high energy, corresponding to the large speed $c - \epsilon$ of the particle (usually proton) involved. Experimental verification cannot be described until the outstanding problem of the explanation of how particles can make transition across the universes without hitting the event horizons and singularities in LT and ∅LT are resolved however.

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COMPETING INTERESTS

No competing interests are involved in this work.

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