Physical Science International Journal

24(9): 8-17, 2020; Article no.PSIJ.61742

ISSN: 2348-0130

Dynamics of the Optical Pulse in a Nonlinear Medium: Approach of Moment Method Coupled with the Fourth Order Runge-Kutta Method

Fessomon Koki[1], Gaston Edah[2], Minadohona Maxime Capo-Chichi[3*], Gaétan Finagnon Djossou[1], Camille Elloh[1] and Marc Ayéla[4]

[1] *International Chair of Mathematical Physics and Applications (ICMPA-Unesco Chair), University of Abomey-Calavi, Benin.*

[2] *Département de Physique, Faculté des Sciences et Techniques, Benin.*

[3] *Laboratory of Radiations Physics, Faculté des Sciences et Techniques, University of Abomey-Calavi, Benin.*

[4] *Institute oh Mathematical and Physical Science, University of Abomey-Calavi, Benin.*

Authors' contributions

This work was carried out in collaboration between all authors. Author FK designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors GE, MMCC and GFD managed the analyses of the study. Authors CE and MA managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/PSIJ/2020/v24i930211

Editor(s):

(1) Dr. Humaira Yasmin, King Faisal University, Saudi Arabia.

(2) Dr. Thomas F. George, University of Missouri, USA.

Reviewers:

(1) Olumuyiwa James Peter, University of Ilorin, Nigeria.

(2) Saad Fawzi Al-Azzawi, University of Mosul, Iraq.

(3) Saad Shakir Mahmood, Universitu of Mustansiriyah, Iraq.

Complete Peer review History: http://www.sdiarticle4.com/review-history/61742

Received 01 August 2020

Accepted 05 October 2020

Published 14 November 2020

Original Research Article

ABSTRACT

In this paper, we considered the nonlinear Schrödinger equation and applied the moment method in order to investigate the evolution of pulse parameters in nonlinear medium. This mathematical model described the effects of cubic nonlinear and the nonlinear dispersion terms on the soliton.

**Corresponding author: E-mail: cachi1550@gmail.com;*

The application of the moment method leads to variational equations that is integrated numerically by the fourth order Runge-Kutta method. The results obtained shows the variations of some important parameters of the pulse namely the energy, the pulse position, the frequency shift, the chirp and the width. It reveals the effects of the nonlinear dispersion and nonlinear cubic terms on each parameter on the pulse. The moment method is appropriate to study the dynamics of the optical pulse in a nonlinear medium modelled by the nonlinear Schrödinger equation.

Keywords: Moment method; nonlinear Schrödinger equation.

1 INTRODUCTION

The generalized nonlinear Schrödinger equation (GNLSE) as a nonlinear model has been studied due to its importance in many fields of physics such a nonlinear optics, plasma physics, superconductivity, quantum mechanics [1]-[12]. It's the fundamental model that governs the transmission of information through optical fibers. This equation plays the role of Newton's laws and conservation of energy in classic mechanics. In order to better understand nonlinear phenomena, it's important to solve this equation. In the general case, it is very difficult to find the analytic solution [13]. With development of soliton theory and computer algebraic system like mathematics, much research papers has been devoted to exact solution of nonlinear evolution equations, especially travelling wave soliton [1]. Various effective method of searching for exact solution to GNLSE have been presented in the literature: the inverse scattering method, the Blacklund transformation, the Adomian method, homothopy perturbation method, the Hirota bilinear method, the Lie group method, the variable separation method, the variational iteration, the Jacobi elliptic function, the expansion method, the auxiliary equation method, the trial function method, the moment method [14]-[26]. The strength of these methods depends on the system that had been studied. For physical systems well defined, the choice of the method could not be uncertain. In this paper, our aim is to study the evolution of the different parameters of the pulse through optical fiber with an appropriate method. We chose the moment method because of the double possibilities it offers: one is the choice of the trial function (anstaz) according to the approximative shape of the solution and the fundamental parameters of the system, then the second is to show the influence of each nonlinearity of the system on the fundamental parameters of the soliton. In addition, since this method does not require a Lagrangian, it can be used for both dissipative and non dissipative systems.

The outline of the present paper is as follows. In section 1, we gave the mathematical model. In section 2 we solved the equation by the moment method. In section 3, we used a Gaussian function as ansatz and we obtained the variational equations of the pulse parameters which are solved by the fourth order of Runge Kutta numerical method. The results and discussions are presented in section 4. Finally, we pointed out the concluding remarks.

2 MATHEMATICAL MODEL

The generalized nonlinear Schrödinger equation in the dimensionless form reads [27]:

$$\frac{i\partial\psi}{\partial z} + a\frac{\partial^2\psi}{\partial t^2} + ib\frac{\partial^3\psi}{\partial t^3} + c|\psi|^2\psi - i\frac{c}{\omega_0}\frac{\partial}{\partial t}(|\psi|^2\psi) = 0 \quad (1)$$

where $\psi = \psi(z,t)$ is the envelop of the pulse, $z \in [0, L], L > 0$ is the length of the fiber and $t \in \mathbb{R}$ is the time, a the second order of dispersion, b the third order of dispersion, c the coefficient of self-modulation; $\frac{c}{\omega_0}$ represents the self-steepening term. This equation holds for pulses that contain just a few optical cycles where higher nonlinear terms are included, with initial conditions $\psi(z = 0, t)$.

The parameters a, b, c are related to β_2, β_3 as:

$a = -\frac{\beta_2}{2}$, $b = -\frac{\beta_3}{6}$, $c = \bar{\gamma}$

3 SOLVING THE PROBLEM BY VARIATIONAL MOMENT METHOD

The basic idea of moment method is to treat the optical pulse like a particle whose energy E, position T, the frequency Ω, the root mean square (RMS) σ and the moment related to the chirp of the pulse are defined as [2, 28, 29]:

$$E = \int_{-\infty}^{+\infty} |\psi|^2 dt \quad ; \tag{2}$$

$$T = \frac{1}{E}\int_{-\infty}^{+\infty} t|\psi|^2 dt \quad ; \tag{3}$$

$$\Omega = \frac{i}{2E}\int_{-\infty}^{+\infty} (\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t})dt \quad ; \tag{4}$$

$$\sigma^2 = \frac{1}{E}\int_{-\infty}^{+\infty} (t-T)^2 |\psi|^2 dt \quad ; \tag{5}$$

$$\tilde{C} = \frac{i}{2E}\int_{-\infty}^{+\infty} (t-T)(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t})dt. \tag{6}$$

Obviously, the evolution of these pulse parameters depend on the evolution on the pulse itself in the fiber which is governed by the GNLSE equation (1).To find the evolution of these pulse parameters, we use the equations (2) to (6) along with equation (1)

3.1 Energy Evolution

Differentiating (2) with respect to z, we have:

$$\frac{dE}{dz} = \int_{-\infty}^{+\infty} (\psi^* \frac{\partial \psi}{\partial z} + \psi \frac{\partial \psi^*}{\partial z})dt. \tag{7}$$

Using (1) we find that:

$$\frac{\partial \psi}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 \psi}{\partial t^3} - \frac{\bar{\gamma}}{\omega_0}\frac{\partial}{\partial t}(|\psi|^2\psi) + i\gamma|\psi|^2\psi \tag{8}$$

After performing calculations, we have:

$$\begin{aligned} \frac{dE}{dz} &= \int_{-\infty}^{+\infty} i\frac{\beta_2}{2}\Big(\psi\frac{\partial^2 \psi^*}{\partial z^2} - \psi^*\frac{\partial^2 \psi}{\partial z^2}\Big)dt + \\ &\quad \int_{-\infty}^{+\infty} \frac{\beta_3}{6}\Big(\psi\frac{\partial^3 \psi^*}{\partial z^3} - \psi^*\frac{\partial^3 \psi}{\partial z^3}\Big)dt - \\ &\quad \frac{\bar{\gamma}}{\omega_0}\int_{-\infty}^{+\infty} \Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt \\ &= 0 \end{aligned} \tag{9}$$

3.2 Evolution of Pulse Position

Differentiating (3) with respect to z we get:

$$\frac{dT}{dz} = \frac{1}{E}\int_{-\infty}^{+\infty} t\Big(\psi^*\frac{\partial \psi}{\partial z} + \psi\frac{\partial \psi^*}{\partial z}\Big)dt \tag{10}$$

we get:

$$\frac{dT}{dz} = i\frac{\beta_2}{2E}\int_{-\infty}^{+\infty} t\Big(\psi\frac{\partial^2\psi^*}{\partial z^2} - \psi^*\frac{\partial^2\psi}{\partial z^2}\Big)dt + \frac{\beta_3}{6}\int_{-\infty}^{+\infty} t\Big(\psi\frac{\partial^3\psi^*}{\partial z^3} - \psi^*\frac{\partial^3\psi}{\partial z^3}\Big)dt - \frac{\bar{\gamma}}{\omega_0 E}\int_{-\infty}^{+\infty} t\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt \tag{11}$$

After integrating by parts and the definition of frequency in (4), we obtain:

$$\frac{dT}{dz} = \beta_2\Omega + \frac{\beta_3}{2E}\int_{-\infty}^{+\infty} t\Big|\frac{\partial\psi}{\partial t}\Big|^2 dt - \frac{3\bar{\gamma}}{2\omega_0 E}\int_{-\infty}^{+\infty}|\psi|^4 dt \tag{12}$$

3.3 Evolution of Frequency Schift

Differentiating (4) with respect to z, we have:

$$\frac{d\Omega}{dz} = \frac{i}{2E}\int_{-\infty}^{+\infty}\Big[\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) - \frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial z}\Big)\Big]dt \tag{13}$$

$$\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) = \psi^*\frac{\partial^2\psi}{\partial z\partial t} + \frac{\partial\psi^*}{\partial z}\frac{\partial\psi}{\partial t} \tag{14}$$

From (8), we can write:

$$\psi^*\frac{\partial^2\psi}{\partial z\partial t} = \frac{i}{2}\psi^*\frac{\partial^2\psi}{\partial t^2} - a\psi^*\frac{\partial^4\psi}{\partial t^4} + ib|\psi|^2\frac{\partial}{\partial t}(|\psi|^2) + ib\psi^*|\psi|^2\frac{\partial\psi}{\partial t} - c|\psi|^2\frac{\partial^2}{\partial t^2}(|\psi|^2) - c\psi^*|\psi|^2\frac{\partial^2\psi}{\partial t^2} \tag{15}$$

and

$$\frac{\partial\psi^*}{\partial z}\frac{\partial\psi}{\partial t} = -\frac{i}{2}\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t} - a\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t} - ib|\psi|^2\psi^*\frac{\partial\psi}{\partial t} - c\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\frac{\partial\psi}{\partial t} \tag{16}$$

Adding (15) and (16) and substituting into (14), we find:

$$\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) = \frac{i}{2}\left[\psi^*\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}\right] - a\left[\psi^*\frac{\partial^4\psi}{\partial t^4} + \frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t}\right] + ib|\psi|^2\left[\frac{\partial}{\partial t}(|\psi|^2) + \psi^*\frac{\partial\psi}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right] - c|\psi|^2\left[\frac{\partial^2}{\partial t^2}(|\psi|^2) + \psi^*\frac{\partial^2\psi}{\partial t^2} + \Big|\frac{\partial\psi}{\partial t}\Big|^2\right] - c\psi^*\frac{\partial}{\partial t}(|\psi|^2)\frac{\partial\psi}{\partial t} \tag{17}$$

Also, we can write

$$\frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial z}\Big) = -\frac{i}{2}\Bigg[\psi\frac{\partial^2\psi^*}{\partial t^2} - \frac{\partial^2\psi}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Bigg] - a\Bigg[\psi\frac{\partial^4\psi^*}{\partial t^4} +$$
$$\frac{\partial^3\psi}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Bigg] + ib|\psi|^2\Bigg[\frac{\partial}{\partial t}(|\psi|^2) + \psi\frac{\partial\psi^*}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\Bigg] -$$
$$c|\psi|^2\Bigg[\frac{\partial^2}{\partial t^2}(|\psi|^2) + \psi\frac{\partial^2\psi^*}{\partial t^2} + \Big|\frac{\partial\psi}{\partial t}\Big|^2\Bigg]$$
$$-c\psi\frac{\partial}{\partial t}(|\psi|^2)\frac{\partial\psi^*}{\partial t} \tag{18}$$

Using (17) and (18) into (13), we can find the evolution of frequency along the fiber to be

$$\frac{d\Omega}{dt} = \frac{i}{2E}\int_{-\infty}^{+\infty}\frac{i\beta_2}{2}\Bigg[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t} + \frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big) - \Big(\psi\frac{\partial^3\psi^*}{\partial t^3}$$
$$+\psi^*\frac{\partial^3\psi}{\partial t^3}\Big)\Bigg]dt + \frac{i}{2E}\int_{-\infty}^{+\infty}\frac{\beta_3}{6}\Bigg[\Big(\psi^*\frac{\partial^4\psi}{\partial t^4} - \psi\frac{\partial^4\psi^*}{\partial t^4}\Big) +$$
$$\Big(\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t} - \frac{\partial\psi^3}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Big)\Bigg]dt -$$
$$\frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2} - \psi\frac{\partial^2\psi}{\partial t^2}\Big)dt +$$
$$-\frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt -$$
$$\frac{\bar{\gamma}}{E}\int_{-\infty}^{+\infty}|\psi|^2\frac{\partial}{\partial t}|\psi|^2 dt \tag{19}$$

In order to calculate $\frac{d\Omega}{dz}$, we evaluate one by one the integrals on right hand side of the (19). After computation, we get:

$$\frac{d\Omega}{dt} = -\frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2} - \psi\frac{\partial^2\psi}{\partial t^2}\Big)dt -$$
$$\frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt -$$
$$\frac{\bar{\gamma}}{E}\int_{-\infty}^{+\infty}|\psi|^2\frac{\partial}{\partial t}|\psi|^2 dt \tag{20}$$

Rearanging (20) and after computation, we obtain

$$\frac{d\Omega}{dt} = -\frac{i\bar{\gamma}}{E\omega_0}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt \tag{21}$$

3.4 Evolution of Chirp Parameter

Let's differentiate (6) with respect to z, we have:

$$\frac{d\tilde{C}}{dz} = \frac{i}{2E}\int_{-\infty}^{+\infty}(t-T)\Big[\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) - \frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial z}\Big)\Big]dt \tag{22}$$

From (17) and (18), we have:

$$\frac{d\tilde{C}}{dz}=\frac{\beta_2}{4E}\int_{-\infty}^{+\infty}(t-T)\bigg[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}+\frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big)-$$
$$\Big(\psi\frac{\partial^3\psi^*}{\partial t^3}+\psi^*\frac{\partial^3\psi}{\partial t^3}+\Big)\bigg]dt$$
$$\frac{i}{2E}\int_{-\infty}^{+\infty}(t-T)\frac{\beta_3}{6}\bigg[\Big(\psi^*\frac{\partial^4\psi}{\partial t^4}-\psi\frac{\partial^4\psi^*}{\partial t^4}\Big)+$$
$$\Big(\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t}-\frac{\partial\psi^3}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Big)\bigg]dt-$$
$$\frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}(t-T)|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2}-\psi\frac{\partial^2\psi}{\partial t^2}\Big)dt-$$
$$\frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi}{\partial t}\Big)dt-$$
$$\frac{\bar{\gamma}}{E}\int_{-\infty}^{+\infty}(t-T)|\psi|^2\frac{\partial}{\partial t}|\psi|^2dt \tag{23}$$

After many integrations by parts, we get:

$$\frac{d\tilde{C}}{dz}=\frac{\beta_2}{E}\int_{-\infty}^{+\infty}\Big|\frac{\partial\psi}{\partial t}\Big|^2dt+\frac{i\beta_3}{4E}\int_{-\infty}^{+\infty}\bigg[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}-$$
$$\frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big)dt-\frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi\frac{\partial\psi^*}{\partial t}-\psi^*\frac{\partial\psi}{\partial t}-\Big)dt$$
$$\frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi}{\partial t}\Big)dt+$$
$$\frac{\bar{\gamma}}{2E}\int_{-\infty}^{+\infty}\Big|\psi\Big|^4dt \tag{24}$$

3.5 Evolution of the RMS Width

We differentiate (5) with respect to z to obtain

$$2\sigma E\frac{d\sigma}{dz}=\int_{-\infty}^{+\infty}(t-T)^2(\psi^*\frac{\partial\psi}{\partial z}+\psi\frac{\partial\psi^*}{\partial z})dt \tag{25}$$

$$2\sigma E\frac{d\sigma}{dz}=i\frac{\beta_2}{2}\int_{-\infty}^{+\infty}(t-T)^2\Big(\psi\frac{\partial^2\psi^*}{\partial t^2}-\psi^*\frac{\partial^2\psi}{\partial t^2}\Big)dt-$$
$$\frac{\bar{\gamma}}{\omega_0}\int_{-\infty}^{+\infty}(t-T)^2\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big)+\psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt+$$
$$\frac{\beta_3}{6}\int_{-\infty}^{+\infty}(t-T)^2\Big(\psi^*\frac{\partial^3\psi}{\partial t^3}+\psi^*\frac{\partial^3\psi^*}{\partial t^3}\Big)dt \tag{26}$$

$$\frac{d\sigma}{dz}=\frac{\beta_2 c}{\sigma}+\frac{\beta_3}{2\Gamma E}\int_{-\infty}^{+\infty}(t-T)\Big|\frac{\partial\psi}{\partial t}\Big|^2dt \tag{27}$$

4 NUMERICAL SIMULATION WITH RUNGE-KUTTA 4

Let's choose the pulse shape on the Gaussian form [2] and [30]:

$$\psi(z,t) = A \exp\left[i\varphi - i\Omega(t-T) - (1+iC)\frac{(t-T)^2}{2\tau^2}\right] \tag{28}$$

with $\tau^2 = K\sigma^2$, $C = 2\tilde{C}$, $K = cte$, $A = cte.$

We obtain a variational equations for each parameter as follows:

$$\begin{aligned}
\frac{dE}{dz} &= 0 \ ; \\
\frac{dT}{dz} &= \beta_2\Omega + \beta_3/2\left(\Omega^2 + \frac{1+C^2}{2\tau^2}\right) + \frac{3\bar{\gamma}E}{\sqrt{8\pi}\omega_0\tau} \ ; \\
\frac{d\Omega}{dz} &= \frac{\bar{\gamma}EC}{\sqrt{2\pi}\omega_0\tau^3} \ ; \\
\frac{dC}{dz} &= 2\beta_2\Omega^2 + \beta_2\frac{1+C^2}{\tau^2} + \beta_3\frac{1+C^2}{2\tau^2} + \frac{4\bar{\gamma}E\Omega}{\sqrt{2\pi}\omega_0\tau} + \frac{\bar{\gamma}E}{\sqrt{2\pi}\tau} \ ; \\
\frac{d\tau}{dz} &= \frac{\beta_2 C}{\tau} + \frac{\beta_3\Omega C}{\tau}.
\end{aligned}$$

We solved numerically the variational equations using the fourth order of Runge Kutta algorithm in Matlab [31]. The results are depicted in Fig. 1. for the following values: $\beta_2 = 0.5$, $\beta_3 = 0.6$, $\bar{\gamma} = 2$, $\omega_0 = 0.1$. The initial conditions are given by $E(0) = 1$, $T(0) = 50$, $\omega(0) = 2$, $C(0) = 0$, $\tau(0) = 1$.

5 DISCUSSION

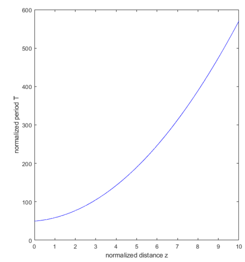
$\frac{dE}{dz} = 0$, therefore the pulse energy remains constant when the pulse propagates along the fiber. Since the width increases Fig. 1(e). then the pulse flattens and according to equation (27), this is due to dispersion effects. The increasing of the chirp and the period, respectively Fig. 1(c). and Fig. 1(a). confirm the flattening of the pulse during his propagation. The equation (12) shows that the pulse position is affected by any frequency shift due to the group velocity dispersion β_2 and the third order dispersion β_3. As the pulse propagates, the frequency increases quickly at the beginning then reach a limit value after a given distance Fig. 1(b).

The equation (22) shows that the chirp is not affected by the group velocity dispersion β_2 nor the self-steepening $\bar{\gamma}$. The equation (27) shows that the evolution of the width depends on the group velocity dispersion β_2 and the third order dispersion β_3; it's not affected by the self-steepening parameter.

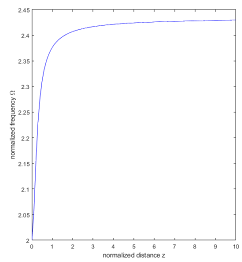
The above equations for the evolution of the pulse parameter reduce the complexity of the problem but they are still not a useful form because they depend on the shape $\psi(z,t)$, which is not known until 1 is solved. If one has some knowledge of the pulse shape and its dependence on the five moments, the problem can be solved approximately. The zoom of parts of the chirp and width curves (respectively Fig. 1(d). and Fig. 1(f).) shows that these fundamental parameters can be modelled by an approximative linear function of z starting from a given distance. The analytic solving would be less difficult.

Let us notice that similar works was carried out by using other variational methods in particular the Lagrangian Variational and Collective Variable methods [32]-[34]. These methods use different formalisms but lead to a set of variational equations. The integration of these differential equations finally make it possible to appreciate the evolution of the parameters defined in the trial function. The major complexity of these methods lies in the choice of the initial conditions as well as the values of coefficients of the nonlinearities in the nonlinear Schrödinger equation in order to obtain physically solutions. They strongly vary from a method to another.

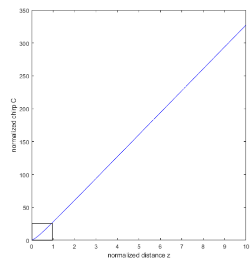
They have nevertheless the common advantage to determine from the variational equations the nonlinear effects which affect each parameter in the anstaz; the appreciation of the intensity of these influences depending on the values coefficients of the nonlinear terms and thus of the intrinsic characteristics of the medium.



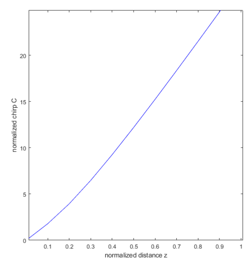
(a) Variations of the period T with respect to the distance z



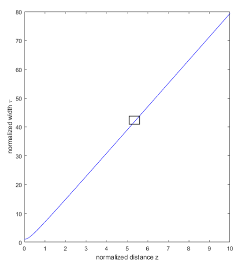
(b) Variation of the frequency Ω with respect to the distance z



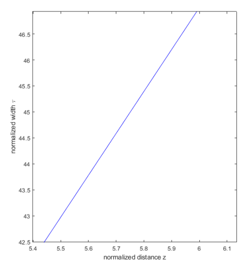
(c) Variation of the chirp C with respect to the distance z



(d) Zoom of the framed part of the chirp C



(e) Variation of the width τ with respect to the distance z



(f) Zoom of the framed part of the width τ

Fig. 1. Variation of some Gaussian pulse parameters: (a) The period; (b) The frequency; (c) The chirp; (d) Zoom of the chirp; (e) The width; (f) zoom of the width

6 CONCLUSION

In this paper, we used the moment method to describe the dynamics of the pulse in nonlinear medium such as optical fiber. Considering the generalized nonlinear equation, we used the moment method formalism to derive a set of ordinary differential equations which we solved with the fourth order of Runge-Kutta algorithm. Indeed, the moment method allowed us to choose the ansatz in accordance with the fundamental parameters of optical. The results obtained reveal the variations of each parameter of systems and show that the pulse flattens. The further work will be interested to the comparison of the variational methods namely the moment method, the collective method and the Langrangian variational approach using the high order nonlinear Schrödinger equation.

ACKNOWLEDGEMENTS

Authors wish to acknowledge Robinson DEGBEGNI, Aubin AIHOULE for useful advice and suggestions.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

[1] Cao Rui, Zhang Jia. Trial function method and exact solutions in the generalized Schrödinger equation with time-dependent coefficient. Chin. Phys. B. 2013;22(10):100507.

[2] Jayanthi Santhanam. Applications of the moment method to optical communications systems: Amplifier noise and timing jitter. University of Rochester; 2004.

[3] Agrawal GP. Applications of nonlinear fiber optics. Academic, San Diego; 2001.

[4] Kodama Y, Hasegawa A. Nonlinear pulse propagation in a monomode dielectric guide. IEEE Journal of Quantum Electronics. 1987;23:510-524.

[5] Mamyshev PV, Chernikov SV. Ultrashort-pulse propagation in optical fibers. Opt. Lett. 1990;15:1076-1078.

[6] Agrawal GP. Fiber-optic communication systems. Wiley; 1997.

[7] Agrawal GP. Fiber-Optic communication systems. 3rd ed. Wiley. New York; 2002.

[8] Agrawal GP. Nonlinear fiber optics. Academic Press; 2007.

[9] Lin Q. Oskar Painter J, Agrawal Govind P. Nonlinear optical phenomena in silicon waveguides: Modelling and applications. Optics Express. 2007;15.

[10] Hasegawa A, Tappert FD. Appl. Phys. Lett. 1973;23:142.

[11] Mollenauer LF, Stolen RH, Gordon JP. Phys. Rev. Lett. 1980;45:1095.

[12] Konar S, Sengupta AJ. Opt. Soc. Am. B. 1994;11:1644.

[13] Cao LV, Bui DT, Piotr G, Nguyen VH, Doan HS. Propagation of ultrashort pulses in a noninear Medium. Computational Methods in Science and Technology; 2010. Special Issue.

[14] Adebisi AF, Peter OJ, Ayoola TA, Oguntolu FA, Ishola CY. Approximate solution of typhoid fever model by variational iteration method. Journal of Science, Technology and Education (JOSTE). 2018;6(3).

[15] Olumuyiwa James Peter. Transmission dynamics of fractional order brucellosis model using caputofabrizio operator. International Journal of Differential Equations; 2020.

[16] Anjan Biswas. Conservation laws for optical solitons with anti-cubic and generalized anti-cubic nonlinearities. Optik, International Journal for Light and Electron Optics 176; 2018.

[17] Biazar J, Ghazvini H. Exact solution for nonlinear Schrödinger equations by He's perturbation method. Phys. Lett. A. 2008;386:79-84.

[18] Belanger N. Etude des moments de l'équation de Schrödinger non linéaire avec perte. Université de Laval; 1996.

[19] Gorji M, Ganji DD, Soleimani S. New application of He's perturbation method. Int. J. Nolinear Sei. 2007;8:319-328.

[20] He JE. New interpretation of homotopy perturbation method. Int. J. Modern Phys. B20.

[21] Sadighi A, Ganji DD. Analytic treatment of linear and nonlinear. Optics Express. 2007;15.

[22] He JH. Wu XH. Variational iteration method: New developpment and applications. Comput. and Math. with Appl. 2007;54:881-894.

[23] Sweilam NH. Variational for solving cubic nonlinear Schrödinger equations. J. of Comput. and Appl. Math. 2007;207:155-163.

[24] Sweilam NH, Al-bar RF. Variational iteration method for coupling nonlinear Schrödinger equations. Comput. and Math. with Appl. 2007;54:993-999.

[25] Turget O, Ahmet Y. Application of He's semi-inverse method to the nonlinear Schrödinger equation with applications. 2007;54:1039-1042.

[26] Edah G, Adanhounmè V, Kanfon A, Guédjé, Hounkonnou N. Pulse propagation in a non linear medium. Open Phys. 2014;13:151-156.

[27] Guanmao Z, Xiaoping Z. Rapid numerical difference recurent formula of nonlinear Schrödinger equation and its applications. Optics Communications. 2007;270:379-383.

[28] Paré C, Belanger P. A1939 Opt. Commun. 168103.

[29] Ablowitz MJ, Clarkson PA. Solitons, Nonlinear evolution equations, and inverse scatterring. Cambridge University Press, New York; 1991.

[30] Fisher RA. Bischel WK. Numerical studies of the interplay between self-phase modulation and dispersion for intense plane-wave laser pulses. J. Appl. Phys. 1975;46.

[31] Butcher JC. Numerical method for ordinary differentials equations. Second Edition. chapter 3: Runge-Kutta Method.

[32] Mir Asma. Othman WAM, Wong BR, Anjan Biswas. Chirped optical Gausson perturbation with quadratic-cubic nonlinearity by collective variables. Optical and Quantum Electronics; 2019.

[33] Marija V, Daniela M, Milivoj B, Qin Zhou, Seithuti PM, Anjan Biswas. Super-sech soliton dynamics in optical metamaterials using collectives variables. FACTA UNIVERSITATIS Series: Electronics and Energetics Vol. 2017;30(1):39-48.

[34] Nakkeeran K, Wai PKA. Generalized projection operator method to derive the pulse parameters equations for the nonlinear Schrödinger equation. Optics Communications; 2004.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle4.com/review-history/61742