International Astronomy and Astrophysics Research Journal

3(3): 27-32, 2021; Article no.IAARJ.72097

Stress: The Forgotten Gravitational Force

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

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Complete Peer review History: https://www.sdiarticle4.com/review-history/72097

Original Research Article

Received 15 June 2021
Accepted 26 August 2021
Published 16 September 2021

ABSTRACT

The fundamental physical concept of shear stress, which governs fluid dynamics, is applied to the classical gravitational results of Newton and of Einstein, and leads naturally to an understanding of the origin of light in the Universe, through the existence of black holes. In this paper, we show how this occurs through an adjunct force of gravity normal to the Newtonian force of gravity, in which the turbulent fluctuations in velocity attain the velocity of light in a ring surrounding the black hole. The existence of this turbulent shear stress has previously been neglected.

Keywords: Gravitation; black hole physics; cosmology; turbulence; methods; analytical; gravitational waves.

1. INTRODUCTION

The concept of shear stress is common place in fluid dynamics, see for example [1], which focusses on the production of waves by the shear stress at the sea surface, however it does not appear to be used in cosmological dynamics. This paper, which addresses this missing physical process in astrophysics, is a companion paper to an earlier paper [2] which addressed

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this missing physical process for the planetary system. The basic model consists of a spinning disk in which ρ(R) and U(R) are respectively the annular mean density and azimuthal velocity at the radius (R), and the shear stress is the local azimuthal stress ($\tau_{R\phi}$(R))where ϕ is the azimuthal co-ordinate.

In Section 2, the expression for the shear stress based on the mixing length hypothesis in fluid dynamics is presented and applied to the Newton and Einstein gravitational models, and in Section 3 it is proposed that these models are fundamentally stress-free, except at their juncture which occurs at the Schwarzschild radius, where an azimuthal shear stress occurs, which gives rise to the adjunct force of gravity which acts normally to the axial Newton force of gravity (Section 4). Section 5 presents the resulting model for black holes, in which addition, collision and structure including the origin of light are considered. and Section 6 is a brief Conclusion.

2. ANALYSIS

2.1 The Prandtl Model for the Azimuthal Shear Stress

On the assumption that the viscous forces may be neglected, the friction velocity in a cylindrical co-ordinate frame is,

$$u_* = \left(\frac{L}{\rho^{1/2}}\frac{d(U\,\rho^{1/2}\,2\pi RW)}{dR}\right)(2\pi RW)^{-1} \qquad (1)$$

where $\tau_{R\phi} = \rho\left|u_*\right|u_*$, and in which $W(R)$ is the thickness of the disk and $L = \kappa R$ is the mixing length due to the interactions of large agglomerations of particles within the disk and κ is von Karman's constant [2]. Eq. (1) is an extension of the mixing length hypothesis of Prandtl [3] for fluctuations in both velocity and density.

Consider a spinning disk in which in the annulus at radius (R) the increase in mass,

$$dM = m\,dR \qquad (2)$$

where the mass density, $m = \rho\,2\pi\,R\,W$, and suppose that

$$\frac{dm}{dR} = 0 \qquad (3)$$

and hence that m is a constant. Then on eliminating ρ in (1) using (3), we obtain,

$$u_* = \frac{\kappa}{W}\left(WR\frac{d(U)}{dR} + ½\ U\frac{d(RW)}{dR}\right) \qquad (4)$$

which is a general relation for the azimuthal shear stress on the disk

2.2 The Newtonian Azimuthal Shear Stress

On assuming Newton's Law of Gravitation,

$$\frac{R^3}{T^2} = \frac{G\,M_o}{4\pi^2}, \qquad R \geq R_o \qquad (5)$$

in which G is the universal gravitational constant, $T(R)$ is the orbital period, and M_o is the principal mass which is contained within the radius (R_o), and no added masses occur beyond R_o, and on taking the positive root, the azimuthal velocity of the disk in the outer region ($R \geq R_o$),

$$U(R) = \left[\frac{GM_o}{R}\right]^{\frac{1}{2}}, \quad R \geq R_o \qquad (6)$$

Thus on substituting for U in (4), we obtain,

$$u_* = ½\,\kappa\,(GM_o)^{1/2}\left[\frac{dW}{dR}\frac{R^{\frac{1}{2}}}{W}\right], \qquad R \geq R_o \qquad (7)$$

which on eliminating W using (3) yields,

$$\rho^{1/2}\,u_* = -\,\kappa\,(GM_o)^{1/2}\frac{d(\rho R)^{1/2}}{dR} \qquad R \geq R_o \qquad (8)$$

and hence $\tau_{R\phi}$

2.3 The Einsteinian Azimuthal Shear Stress

In the inner region, we assume the Einstein relation,

$$U(R) = c, \qquad R \le R_o \qquad (9)$$

where c is the velocity of light. On evaluating (9) at R_o using (6), we obtain,

$$R_o = \frac{GM_o}{c^2} \qquad (10)$$

which is the Schwarzschild relation for the principal mass (M_o), however from (2) we note that the mean mass in the inner region, $M_s = ½M_o$, and hence, the corresponding radius for the mean mass is, $R_o = 2\frac{GM_s}{c^2}$, and on substituting for U(R) from (9) in (4), we have,

$$u_* = ½\frac{c\kappa}{W}\frac{dWR}{dR}, \quad R \le R_o \qquad (11)$$

or alternatively on eliminating W using (3) and substituting for c from (10),

$$\rho^{1/2}\, u_* = -\,\kappa\,(GM_o)^{1/2}\, R^{1/2}\, \frac{d\rho^{1/2}}{dR}, R \le R_o \qquad (12)$$

Within the inner region, from (2),

$$M(R) = m\, R \;, \qquad R \le R_o \qquad (13)$$

and the potential energy, $P(R) = G\, M^2(R)\,/\,R$, is,

$$P\,(R) = G\, m\, M(R)\;, \qquad R \le R_o \qquad (14)$$

where from (10),

$$m = \frac{c^2}{G} \qquad (15)$$

Hence,

$$P(R) = M(R)\, c^2 \quad , \; R \le R_o \qquad (16)$$

In summary, R_o is the matching radius for the Newton dynamics ($R \ge R_o$) and the Einstein dynamics ($R \le R_o$). In the Newton dynamics, the mass ($M(R)$) is a constant and in the Einstein dynamics ($U(R)$) is a constant. On evaluating, we also find that the potential energy (16) has a maximum,

$$E = M_o c^2 \qquad (17)$$

at $R = R_o$ .where $E = P(R_o\,)$. R_o marks a simple division between the Newton and Einstein regions. In this cosmological model there is no mathematical singularity as at the origin, from (2), $M(0) = 0$, R_o marks the radius of maximum energy in the spinning disk.

3. THE STRESS - FREE MODEL

We propose as was verified in [2] that stress-free ($\tau_{R\phi}$ = 0) conditions occur in the Newton model for $R > R_o$ in which W is a constant in (7) and ρR is a constant in (8); and in the Einstein model for $R < R_o$ in which ρ is a constant in (12).

At the juncture of the two regions at R_o , however, there is a discontinuity in the specification of $\tau_{R\phi}$. In the outer region, from (8), as $\rightarrow\; R_o$,

$$\rho^{\frac{1}{2}}\, u_*)_{outer} = -\,\kappa\,(GM_o)^{\frac{1}{2}}\left[\left(\frac{d\rho^{\frac{1}{2}}}{dR}\right)_{R_o} R_o^{\frac{1}{2}} + \rho^{\frac{1}{2}}\left(\frac{dR^{\frac{1}{2}}}{dR}\right)\mathrm{R_o}\right] \qquad (18a)$$

and in the inner region from (12), as $R \rightarrow R_o$,

$$\rho^{\frac{1}{2}}\, u_*)_{inner} = -\,\kappa\,(GM_o)^{\frac{1}{2}}\left[\left(\frac{d\rho^{\frac{1}{2}}}{dR}\right)_{R_o} R_o^{\frac{1}{2}}\right] \qquad (18b)$$

4. THE ADJUNCT FORCE OF GRAVITY

From (18a) and (18b),

$$\rho^{\frac{1}{2}}\, u_*)_{outer} - \rho^{\frac{1}{2}}\, u_*)_{inner} = -\,½\,\rho^{\frac{1}{2}}\,\kappa\left(\frac{GM_o}{R_o}\right)^{\frac{1}{2}} \qquad (19)$$

and hence the azimuthal stress due to this discontinuity,

$$\tau_{R\phi} = -\,¼\;\kappa^2\rho\; c^2 \qquad (20)$$

where from (15), $c^2 = \left(\frac{GM_o}{R_o}\right)$, which over the element ($W\, dR$) gives rise to the differential adjunct force of gravity,

$$dJ = \tau_{R\phi} W \, dR, \tag{21}$$

which is directed circumferentially [2], and on substituting for $\tau_{R\phi}$ from (20) and evaluating at R = R₀ yields,

$$dJ = -\frac{\kappa^2}{8\pi}\frac{GM_o}{{R_o}^2} dM \quad R = R_o \tag{22}$$

The differential adjunct force of gravity (22) is the only non-zero gravitational force at the boundary of the inner region ($R = R_o$), since the Newton differential force of gravity,

$$\begin{aligned} dF &= -\frac{GM_o}{R^2}\, dM, \quad R > R_o \\ &= 0\,, \qquad\qquad\quad R \le R_o \end{aligned} \tag{23}$$

is zero. The division between the two regions ($R = R_o$) occurs at the Schwarzschild radius for the mean mass.

5. BLACK HOLES

5.1 Addition

Consider two black holes of contained masses, M_o and M_1 and suppose that they combine. Then the combined black hole has a mass, $M_c = M_o + M_1$, which on substituting for M_o and M_1 using (10) applied to each black hole yields,

$$M_c = \frac{c^2}{G} R_c \tag{24}$$

where $R_c = R_o + R_1$ and R_o and R_1 are the respective Schwarzschild radii of the two combining black holes. Hence, by virtue of the constancy of the speed of light, the combined black hole has the same structure as its two components. Thus it follows that irrespective of the interactive cosmological physics black holes may populate the Universe on all gravitational scales from the smallest to the greatest. The flow of energy by the addition of the black holes is from small to large scales, i.e. in the conceptual manner of the energy cascade in two-dimensional turbulence in fluids. The flow of energy by subtraction is by division through the formation of the second black hole. Both these processes are quintessentially creative, which is the nature of our Universe.

Eq. (20) shows that the azimuthal stress of a black hole, which occurs at the Schwarzschild radius, is independent of scale. Hence in view of the universality of the black hole population, the azimuthal stress (20) is a universal property of the Universe, which on the assumption of a uniform value for von Karman's constant is an invariant property.

5.2 Collision

From time to time, it would be expected that two black holes may collide. A model for the collision consists of two disks of respective radii, R_1 and R_2, abutting each other, which were originally travelling with a relative velocity (V). This collision occurs during the interval (T), where,

$$V_0 = 2\,(R_1 + R_2)/T \tag{25}$$

where $V_0 = ½\, V$, in which it is assumed that V is the mean relative velocity of the approaching black holes which becomes zero on amalgamation. On substituting for the radii using (10), we obtain,

$$V = 4\frac{(M_1 + M_2)\,G}{Tc^2} \tag{26}$$

where M_1 and M_2 are the principal masses of the two black holes, and T can be identified as the period of the gravitational wave caused by the collision.

An example of a collision is presented in [4] in which $M_1 = 2M_s\,36$ and $M_2 = 2M_s\,29$ and $V = 0.6\,c$. On substituting these values and M_s = 2 10^{30} kg in (25) with c= 2.998 10^{8} ms^{-1} and G = 6.673 10^{-11} $m^3\,kg^{-1}\,s^{-2}$, we find that T = 4 10^{-3} s, which is similar to the period of the gravitational waves at the merger of the two black holes estimated from Figure 1 of [4]. We conclude that the collision model is a basic representation of the physical processes which give rise to the observed gravitational waves.

5.3 Structure

The structure of the black holes is encapsulated in (10) which relates their radius to their mass. Within the black hole, from (2), $M = M_o\, R/R_o$, and hence on substituting for M_o we obtain,

$$R = \frac{GM}{c^2}, \qquad\qquad 0 \le R \le R_o \tag{27}$$

Eq. (27) applies quite generally, and at R_o ,

$$R_o = \frac{GM_o}{c^2} \tag{28}$$

which is the Schwarzschild relation. Thus throughout its structure at all radii a neutral escape velocity occurs, which is a prime property of the black hole as it is for the exterior Newtonian dynamics. In the inner region this is brought about by a radial increase in mass, and in the outer region by a decrease in azimuthal velocity.

We consider next the juncture of the two regions at $R = R_o$.

5.4 The Origin of Light

A comprehensive model of a black hole requires a consideration of its setting in the visible firmament. A key factor is the azimuthal stress which occurs at $R = R_o$., which has the Prandtl form, see Section 2, in which the mean magnitudes of the components of the velocity fluctuations are,

$$< |u'| > = \frac{-L\, d < u >}{dR} \tag{29a}$$

and

$$< |v'| > = \frac{-L\, d < u >}{dR}, \quad \frac{d < u >}{dR} < 0 \tag{29b}$$

where $< >$ denotes a mean, and L is the mixing length [3], and in the notation of fluid dynamics,

$$\rho\, K_v \frac{d < u >}{dR} = \tau_{R\phi} \quad , \tau_{R\phi} < 0 \tag{30}$$

where K_v is the coefficient of eddy viscosity [5], which in the mixing length theory has the form, $K_v = \kappa\,(R - R_o)\, u_*$ in which the friction velocity, $u_* = (-\tau_{R\phi}/\rho\,)^{½}$ On substituting for K_v and for $\tau_{R\phi}$ from (20), (30) yields,

$$\frac{d < u >}{dR} = -½\, \frac{c}{(R - R_o)} \tag{31}$$

which has arisen from the mixing length theory. On comparing (31) with the kinematic relation for the velocity shear,

$$\frac{d < u >}{dR} = \frac{(U - c)}{(R - R_o)} \tag{32}$$

we obtain,

$$U = ½\, c \tag{33}$$

which applies over the constant stress layer ($\tau_{R\phi}$ = const.). Eq. (31) enables the speed of the velocity fluctuations in the constant stress layer, $S = \left[< |u'| >^2 + < |v'| >^2 \right]^{½}$ to be evaluated. We find from (29a) and (29b) that,

$$S = \frac{1}{\sqrt{2}} \frac{L}{(R - R_o)}\; c \tag{34}$$

Hence $S = c$ if,

$$\frac{1}{\sqrt{2}} \frac{L}{(R - R_o)} = 1 \tag{35}$$

Eq. (35) is the condition for the origin of light. On evaluating the mixing length at the mid-point radius, ½ $(R - R_o)$, we obtain $L = ½\, \kappa\,(R + R_o)$ and substituting in (35) we obtain the expression

$$\frac{1}{8}\, \kappa^2\, (\theta + 1)^2 = (\theta - 1)^2 \tag{36}$$

where $\theta = R/\,R_o$, which shows that the annulus (R_o, R) in which light appears as a bright ring, is controlled by von Karman's constant. For κ = 0.4, we find that $\theta = 1.32$. Thus it is predicted that a bright ring would appear around the black hole, of a width to mean radius ratio $2(\theta - 1)/(\theta + 1)$ of 28%.

Recent observations show a bright rim of emission surrounding the black hole in the massive elliptical galaxy M87 [6], which appears to be consistent with this model, and a visual inspection of the imagery of the bright ring indicates a width to mean radius ratio of about 30%, which suggests that the value of von Karman's constant is about 0.4 as is observed in turbulent boundary layer observations on Earth [7]. This important question of the universality of κ as a fundamental constant was left unresolved in [2].

6. CONCLUSIONS

In summary, the adjunct force of gravity controls the dynamics of the inner region, and the Newton force of gravity controls the dynamics of the outer region. Both these regions are stress–free, and the adjunct force of gravity occurs at their juncture. This is the principal conclusion of the analysis. This perspective completely changes theoretical ideas on black holes as the maximum in mass resides at $R = R_o$ and not at $R = 0$ where the mass is zero ($M(0) = 0$). The black hole occupies the inner region.

The existence of the black hole requires the acquisition of a mass density (m) which satisfies (15). In the outer region, the adjunct force of gravity is still present, as has been demonstrated by evaluating (8) for pairs of the discrete massy bodies in the planetary system, on the assumption that the von Karman constant, $\kappa = 0.4$ [2]. The mass densities (m) of all these bodies are however very much less than (15), and their adjunct forces of gravity are much smaller than the Newton forces of gravity.

The centrality of the adjunct force of gravity to our very existence is clear. It is the origin of light.

ACKNOWEDGEMENTS

The comments of five Reviewers are gratefully acknowledged and also the continuing support of Charles James in the final preparation of the paper.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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