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Project Planning Application to Juice Production Using PERT/CPM Technique: A Case Study

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Timely delivery of natural and fresh juice to its' growing customers is the focus of SCUBED 100%. The CPM (Critical Path Method) and PERT (Program Evaluation and Review Technique) techniques were applied in minimizing the expected time duration for juice production at SCUBED 100%. The current expected production time for a batch stood at 656.66 minutes. Modifications were made on the initial model to minimize the expected duration and after two modification processes, an estimated time duration of 458.33 minutes was realized, saving 198.33 minutes (about 3.3 hours) of production time. This study encapsulates the interplay of theoretical insights and practical implementation, confirming the potential of operational research and management techniques in designing real-world outcomes. SCUBED 100%'s journey towards operational excellence demonstrates the transformative potential of time optimization.

Keywords: Optimization; PERT; CPM; time efficiency; production planning; operational excellence.

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1 Introduction

Projects have been a part and parcel of our existence, ranging from mega-projects like construction of roads and building of schools to daily routine of a typical student. For instance, an average student's routine begins with waking up, and ends with going back to bed. Though the number and nature of activities vary per student, the major activities still remain brushing their teeth, taking a shower and getting dressed. A complete project in this case will be waking up and going back to bed while other activities like brushing of teeth, taking a shower, taking breakfast, and walking back home are activities which must be carried out in a specific order for the project to be complete. This is project planning in its' simplest form. Irrespective of whether it's done mentally or using pen and paper, the fact remains that every project is planned in an effort to save time, money, and avoid unpleasant outcomes [1].

Production planning involves all arrangements made and carried out during the course of producing goods and services, from raw materials acquisition through allocation of resources to final distribution to consumers [2]. Project network analysis is a vital tool used to analyze, control and monitor business processes and workflow [3], taking into consideration the chronological order of activities, their durations and dependencies. Over the years, project network analysis has had important contributions from extremely wide variety of fields, including mathematics, biology, computer science, and social sciences. In a broad sense, project network analysis is a field that mostly concerns itself with relational data and research questions. Every project entails certain activities which are interrelated in a logical sequence. A project can be described as successful in terms of its' timely completion, being within the stipulated budget (or at lesser cost) and the satisfaction of end users. The knowledge of project network analysis has helped project managers consider relevant factors such as dependencies and buffer times between activities, earliest and latest start and end dates, duration of activities and critical paths, when creating project plans [4]. Before the emergence of project network, the planning tool was Gantt bar (milestone) chart which specifies start and finish times for each activity on a horizontal time scale, but failed to show the interdependency among these many activities which control the progress of the project. Project network, which is a logical extension of Gantt's chart, incorporates these modifications. Two of the best-known techniques for project network analysis are Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM). Over the years, these methods have gradually merged and are now usually used interchangeably and their application is combined using the name PERT/CPM [5]. The techniques of PERT and CPM prove extremely valuable in assisting managers to handle projects, especially those involving production, thereby effectively discharging their project management responsibilities.

Diverse studies have been carried out in this regard. Chatwal [6] reduced the pessimistic time of the activities on a critical path since it would result in a reduction in the expected project completion time. This led to an increased probability for project completion on or before the scheduled time. Kumar et al. [7], in a comparative study of time-cost optimization studied the CPM & PERT methods as suitable tools to analyze and reduce project duration. They opined that a reduction in project duration leads to increase in direct cost of the project but that the indirect cost of the project, which is as a result of a compromise between the time and cost of the project, reduces. Careful analysis showed that optimum duration at minimum cost will occur at a particular point and this point must be explored by construction managers at planning stage if a reduction in both time and cost of the project is desired. Kholil et al. [8], in a house construction project used the two techniques to determine the optimum time of completion. The duration of the project was set for 173 days. Application of CPM gave a completion time of 131 days while PERT method resulted in a completion time of 136 days. Bahtijar [5] used both PERT and CPM methods to determine the possibility of galvanizing a 240kg-order for an automotive industry in 12 hours and within a shift which lasts for 8 hours. The work concluded that the probability of possible completion in 12 hours was 99.59% while that of 8 hours was 54.04%. Lermen et al. [9], applied PERT/ CPM technique in the production of a horizontal laminator used in a mattress industry, with the aim to optimize time and cost. They concluded that if all critical activities are accelerated, the project can be completed in 186.7 hours less time, although it would lead to an increase in the total cost of the project. However, analysis of the slack activities achieved a reduction in costs which in turn reduced the total cost of the project by 12.56%. Other studies in this field include those of Rautela et al. [10], Karabulut [11], Zareei [12], Badruzzaman et al. [13].

In this study, project network analysis as it pertains to project planning is considered. Time has always been a major factor to be considered in project management, so this piece seeks to use the PERT/CPM technique to show the relationship between the several activities involved in fruit juice production by SCUBED 100%, specifically focusing on the critical path of her production chain to expose the activities which affect the duration of production the most. SCUBED 100% is a brand focused on the production and distribution of 100% natural freshly squeezed juice, which is aimed at encouraging the consumption of healthy drinks as a means of promoting healthy lifestyle in Nigeria. They also cater to events in moderate scale. In over five years of ceaseless production and sensitization on social media platforms, the brand has witnessed some level of success, as more people are going natural with SCUBED 100% fruit drinks. This has resulted in increased bookings and orders, and ultimately a need to adequately meet all customer requests. The aim of this study, therefore, is to model the most effective project network that will increase work efficiency with very minimal time consumption and reduced operation cost at SCUBED 100%. For obvious reasons, not all precise and correct data needed for the study was released to the researchers as some business owners see giving such sensitive data as a risk to their business.

2 Model Formulation

To formulate the model for this work, we employ 4 of the 6 steps for constructing a network planning diagram that apply to both the PERT and the CPM method, Heizer et al., [14]. In PERT, t_{ij} represents the meantime, E and is used to calculate both the earliest times and latest times, E_j and L_j respectively.

The standard operating procedure for production process for SCUBED 100% is divided into two parts as follows:

1. Manufacturing
   - Sort fruits at designated area prior to washing – 45 Minutes
   - Wash and rinse fruits thoroughly with clean treated water – 60 Minutes
   - Temporarily hold the cleaned fruits in clean covered containers, ready for juicing – 20 Minutes
   - Pass the fruits through the juicing machine – 90 Minutes
   - Filter the extracted juice and collect in a clean container – 90 Minutes
   - Transfer the filtered juice immediately into the freezer – 10 Minutes

2. Packaging
   - Sort, wash and rinse PET bottles/screw caps with treated clean water – 45 Minutes
   - Check to ensure that washed PET bottles/screw caps are clean and without defects – 20 Minutes
   - Fill the PET bottles with filtered and chilled juice to specified fill volume and apply the caps immediately. Ensure that the bottles are tightly capped – 120 Minutes
   - Label each bottle with relevant product information and batch number, production/ best-before dates provided – 60 Minutes
   - Store the filled and labeled bottles in the freezer – 45 Minutes

Table 1. Tabular representation of data in CPM

I. D	Activities	Duration of activity (Mins)	Immediate Predecessor
A	Sorting of fruits	45	None
B	Washing of fruits	60	A
C	Temporary holding of fruits before juicing	20	B
D	Juicing of fruits	90	C
E	Filtering the fruit juice	90	C
F	Transfer juice into freezers	10	D, E
G	Washing of the bottles	45	F
H	Checking the bottles thoroughly	20	G
I	Pouring the juice into the bottles	120	H
J	Labeling each bottle properly	60	I
K	Store the bottled juice in freezers	45	J

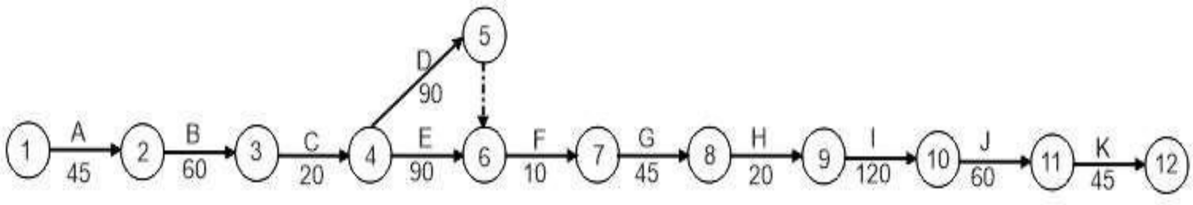

Fig. 1. Model 1

Table 2. Data for model 1 in PERT

Activities	Immediate predecessors	x = most likely time	w = optimistic time	y = pessimistic time	E = mean time	σ = variance
A	None	50	45	60	50.83	6.25
B	A	75	60	90	75	25
C	B	25	20	30	25	2.79
D	C	120	90	150	120	100
E	C	120	90	150	120	100
F	D, E	15	10	30	16.67	11.09
G	F	60	45	75	60	25
H	G	30	20	45	30.83	17.39
I	H	150	120	180	150	100
J	I	65	60	75	65.83	6.25
K	J	60	45	90	62.50	56.25

3 Methodology

A predecessor activity is any activity that must be completed immediately prior to the start of another activity. For instance, if an activity A is predecessor of an activity B, it is denoted by A<B. Thus, activity B can start only if activity A is completed. A successor activity is any activity that immediately begins after the completion of one or more other activities. On the other hand, a dummy activity is one which doesn't consume any kind of resources but merely depicts its' dependence on another activity. It makes activities with same starting and finishing points distinguishable and maintains precedence relationship for activities which are not connected by events. Event (or node) is the beginning and end points of an activity, generally represented by a numbered circle. They could be merge or burst nodes. It is a point in time and does not consume any resources. The head event, called the j^{th} event, has a number higher than the tail event, called the i^{th} event, i.e., $j > i$.

PERT and CPM methods differ in the way they determine the duration of individual activities, as well as time analysis, but the rules for forming a project network diagram and structure analysis remain the same, Bagshaw [15]. Optimistic time, w, is when the estimator thinks that everything will go well without any uncertainties, thereby estimating lowest time possible. Pessimistic time, y, is where the estimator thinks that everything would go wrong and expects all sorts of uncertainties, thus estimating highest possible time. Likely time, x, is in-between optimistic and pessimistic times. Here the estimator expects he may come across some sort of uncertainties and many a time that things will go right.

We shall adopt Fulkerson's rule for proper numbering of events. This rule in summary says, the starting event (the event having no predecessor activity) is numbered J' while other events are numbered in increasing order from event one moving rightwards. In the case of more than one initial event, numbering is done from top to bottom in increasing order [16].

PERT method works with an assumption that the time for an activity is a random variable with a beta distribution since the beta distribution has both minimal and maximum value, and is also capable of assuming a wide variety of shapes, Acuna [1]. In PERT, there are three-time estimates for each activity: optimistic time, w, most likely time, x, and pessimistic time, y. The expected value, denoted by E, which is the mean duration time, is the weighted average of w, x and y, and is given by, $E = ((w + 4x + y) / 6)$. The variance, $\sigma^2 = ((y - w) / 6)^2$.

Thus, provided with the meantime, E, and standard deviation (square root of the variance, σ^2) of the beta distribution, the probability of completing a project at a given time can be calculated using the normal distribution, Bagshaw [15]. PERT also assumes that all job times are independent and equally distributed. Therefore, applying the central limit theorem, T is normally distributed. This implies that the probability of meeting a specific project deadline, D, can be calculated using the standard score Z, where $Z = ((D - \mu (T)) / (\sigma (T)))$.

Unlike the PERT, CPM uses actual time estimate for activities in a project. These three terms play a very important role in CPM: E_j, earliest time of event j, L_j, latest time of event j, and t_{ij} = duration of an activity, where i, j = 1, 2, 3, . . . n. The forward and backward passes, which determines respectively the earliest and latest times of each event, are the two major calculations in CPM. Slack time is the difference between the latest and earliest times of each event. The critical paths of the project are events with zero slack times, Acuna [1].

The forward pass (E_j): Given that events b, d, and k are linked directly to node j where j = 1, 2, 3 by incoming activities (b, j), (d, j) and (k, j). We assume that the earliest times of these events have already been calculated, then the earliest time of event j is calculated with the formula, $E_j = \max \{E_b + t_{bj}, E_d + t_{dj}, E_k + t_{kj}\}$, Acuna [1].

The backward pass (L_j): We start with the last event and move through to the first event. Set $L_n = E_n$ (shows that the earliest and latest times of the last event of the project are the same).

We assume that the latest times of these events have already been calculated, then the latest time of event j is, $L_j = \min \{L_b - t_{bj}, L_d - t_{dj}, L_k - t_{kj}\}$.

Slack time, S_{ij}, helps to determine the amount of time an activity can run late without affecting the duration of the project. A slack time of zero identifies a critical activity (set of activities that affect the duration of the project the most). This is calculated by taking the difference between the latest time and earliest time of each activity, $S_{ij} = L_j - E_j$.

4 Results and Discussion

With regards to model 1 (Fig. 1), the earliest and latest times are first computed, and then the slack times which provides the critical path (those events whose slack times are zero). In PERT, t_{ij} represents the mean time E, which is used to obtain both the earliest times and latest times, E_j and Lj. Determining the earliest and latest time (E_j and L_j) of each event for model 1, gives the following;

$E_1 (T) = 0$
$E_2 (T) = E_1(T)+ t_{12} = 0 + 50.83 = 50.83$
$E_3 (T) = E_2(T)+ t_{23} = 50.83 + 75 = 125.83$
$E_4 (T) = E_3(T)+ t_{34} = 125.83 + 25 = 150.83$
$E_5 (T) = E_4(T)+ t_{45} = 150.83 + 120 = 270.83$
$E_6 (T) = \max\{E_4(T)+ t_{46}, E_5(T)+ t_{56}\} = \max\{150.83 + 120, 270.83 + 0\} = 270.83$
$E_7 (T) = E_6 (T)+ t_{67}= 270.83 + 16.67 = 287.50$
$E_8 (T) = E_7 (T)+ t_{78} = 287.50 + 60 = 347.50$
$E_9 (T) = E_8 (T)+ t_{89} = 347.50 + 30.83 = 378.33$
$E_{10} (T) = E_9 (T)+ t_{9,10} = 378.33 + 150 = 528.33$
$E_{11} (T) = E_{10} (T)+ t_{10,11} = 528.33 + 65.83 = 594.16$
$E_{12} (T) = E_{11} (T)+ t_{11,12} = 594.16 + 62.50 = 656.66$

$L_{12}(T) = E_{12}(T) = 656.66$
$L_{11}(T) = L_{12}(T) - t_{12,11} = 656.66 - 62.50 = 594.16$
$L_{10}(T) = L_{11}(T) - t_{11,10} = 594.16 - 65.83 = 528.33$
$L_9(T) = L_{10}(T) - t_{10,9} = 528.33 - 150 = 378.33$
$L_8(T) = L_9(T) - t_{98} = 378.33 - 30.83 = 347.50$
$L_7(T) = L_8(T) - t_{87} = 347.50 - 60 = 287.50$
$L_6(T) = L_7(T) - t_{76} = 287.50 - 16.67 = 270.83$
$L_5(T) = L_6(T) - t_{65}= 270.83 - 0 = 270.83$
$L_4(T) = \min\{L_6(T) - t_{64}, L_5(T) - t_{54}\} = \min\{270.83 - 120, 270.83 - 120\} = 150.83$

$L_3(T) = L_4(T) - t_{43} = 150.83 - 25 = 125.83$
$L_2(T) = L_3(T) + t_{32} = 125.83 - 75 = 50.83$
$L_1(T) = L_2(T) - t_{21} = 50.83 - 50.83 = 0$

Table 3. Tabular representation of slack times for model 1

Events	$E_j(T)$	$L_j(T)$	$S_{ij} = L_j(T) - E_j(T)$	Remark
1	0	0	0	critical
2	50.83	50.83	0	critical
3	125.83	125.83	0	critical
4	150.83	150.83	0	critical
5	270.83	270.83	0	critical
6	270.83	270.83	0	critical
7	287.50	287.50	0	critical
8	347.50	347.50	0	critical
9	378.33	378.33	0	critical
10	528.33	528.33	0	critical
11	594.16	594.16	0	critical
12	656.66	656.66	0	critical

The critical path is illustrated in Fig. 2 below:

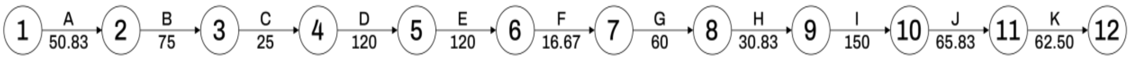

Fig. 2. The critical path for model 1

The length of time by which an event in a project may be delayed without affecting the overall project timeline is referred to as the slack time. An event with a zero slack time implies that any delay in its' completion will automatically result in the extension of total time required for the entire project completion.

Analysing the slack times computed from the Table 3, it can be clearly observed that all events are critical. This implies that any delay in the timely completion of any one event would lead to an ultimate delay in the overall completion of the process. The sequential nature of these events and their interdependency is accentuated by this criticality. It is therefore very important that each event be diligently planned, monitored and promptly executed to ensure that the entire process is finished on schedule.

Considering therefore the model 1 results, the expected duration of production is calculated using the estimated duration of each event. The expected duration of the production process is a sum of the estimated individual events duration, which in this case is calculated as,

$$E(T) = 50.83 + 75 + 25 + 120 + 16.67 + 60 + 30.83 + 150 + 65.83 + 62.50 = 656.66 \text{ minutes.}$$

Emphasis is placed on these critical events because they underscore the overall project timeline and so are pivotal to forestall any inefficiencies and delays.

Insights from the above analysis highlights the importance of every single event in the production process, such that the removal of any event is disputable and unusable. In order to address this issue and heighten the efficacy of the production process, a model that seeks to improve on the existing model by saving on time is formulated. This modified model possesses the potential to optimize time while retaining all the details of the original production procedure.

Model 1 is modified by making necessary adjustments to event structures and sequences with full knowledge of what each activity entails and ensuring that none is compromised. An in-depth assessment of the activities involved in the process exposes the possibility of concurrent or simultaneous execution of certain activity pairs to fast-track the process. Specifically, allowing activities (B and C), (D and E), (G, H and I), (J and K) to run concurrently, a refined model 2 was achieved, and this is illustrated in Table 4 and Fig. 3.

Table 4. Tabular representation of model 2 data in CPM (modification 1)

I. D	Activities	Duration of activity (Mins)	Immediate predecessor
A	Sorting of fruits	45	None
B	Washing of fruits	60	A
C	Temporary holding of fruits before juicing	20	A
D	Juicing of fruits	90	B
E	Filtering the fruit juice	90	C
F	Transfer juice into freezers	10	D, E
G	Washing of the bottles	45	F
H	Checking the bottles thoroughly	20	F
I	Pouring the juice into the bottles	120	F
J	Labeling each bottle properly	60	G, H
K	Store the bottled juice in freezers	45	I

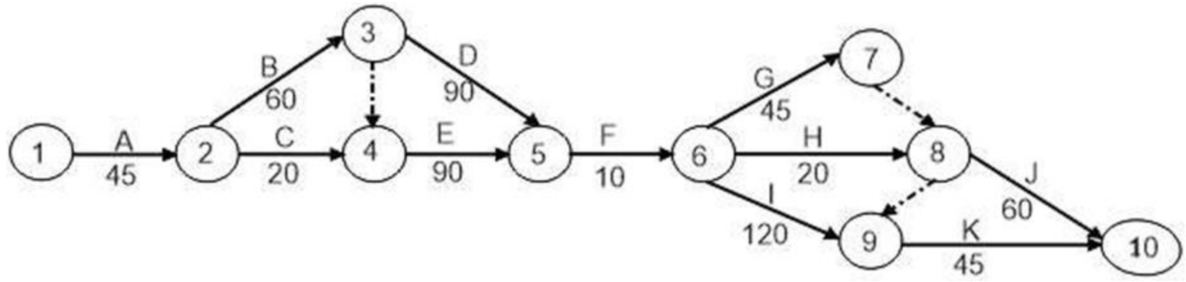

Fig. 3. Model 2

Table 5. Tabular representation of model 2 data in PERT (modification 1)

Activities	Immediate predecessors	x = most likely time	w = optimistic time	y = pessimistic time	E = mean time	σ = variance
A	None	50	45	60	50.83	6.25
B	A	75	60	90	75	25
C	A	25	20	30	25	2.79
D	B	120	90	150	120	100
E	C	120	90	150	120	100
F	D, E	15	10	30	16.67	11.09
G	F	60	45	75	60	25
H	F	30	20	45	30.83	17.39
I	F	150	120	180	150	100
J	G, H	65	60	75	65.83	6.25
K	I	60	45	90	62.50	56.25

To investigate the potential impact of each event in the overall completion of the production process, we compute the earliest possible and latest allowable times that each event can occur. This will lead to the determination of the slack times and hence the development of a critical path for the refined model 2.

$E_1(T) = 0$
$E_2(T) = E_1(T) + t_{12} = 0 + 50.83 = 50.83$
$E_3(T) = E_2(T) + t_{23} = 50.83 + 75 = 125.83$
$E_4(T) = \max\{E_2(T) + t_{24}, E_3(T) + t_{34}\} = \max\{50.83 + 25, 125.83 + 0\} = 125.83$
$E_5(T) = \max\{E_3(T) + t_{35}, E_4(T) + t_{45}\} = \max\{125.83 + 120, 125.83 + 120\} = 245.83$
$E_6(T) = E_5(T) + t_{56} = 245.83 + 16.67 = 262.50$
$E_7(T) = E_6(T) + t_{67} = 262.50 + 60 = 322.50$
$E_8(T) = \max\{E_6(T) + t_{68}, E_7(T) + t_{78}\} = \max\{262.50 + 30.83, 322.50 + 0\} = 322.50$

$E_9(T) = \max\{E_6(T)+ t_{69}, E_8(T)+ t_{89}\} = \max \{262.50 + 150, 322.50 + 0\} = 412.50$
$E_{10}(T) = \max\{E_8(T)+ t_{8,10}, E_9(T)+ t_{9,10}\} = \max \{322.50 + 65.83, 412.50 + 62.50\} = 475$

$L_{10}(T) = E_{10}(T) = 475$
$L_9(T) = L_{10}(T) - t_{10,9} = 475 - 62.50 = 412.50$
$L_8(T) = \min\{L_{10}(T) - t_{10,8}, L_9(T) - t_{98}\} = \min \{481 - 65.83, 412.50 - 0\} = 412.50$
$L_7(T) = L_8(T) - t_{87} = 412.59 - 0 = 412.50$
$L_6(T) = \min\{L_9(T) - t_{96}, L_8(T) - t_{86}, L_7(T) - t_{76}\} = \min \{412.50 - 150, 412.50 - 30.83, 412.50 - 60\} = 262.50$
$L_5(T) = L_6(T) - t_{65} = 262.50 - 16.67 = 245.83$
$L_4(T) = L_5(T) - t_{54} = 245.83 - 120 = 125.83$
$L_3(T) = \min\{L_5(T) - t_{53}, L_4(T) - t_{43}\} = \min \{245.83 - 120, 125.83 - 0\} = 125.83$
$L_2(T) = \min\{L_4(T) - t_{42}, L_3(T) - t_{32}\} = \min \{125.83 - 25, 125.83 - 75\} = 50.83$
$L_1(T) = L_2(T) - t_{21} = 50.83 - 50.83 = 0$

Table 6. Tabular representation of slack times for model 2

Events	$E_j(T)$	$L_j(T)$	$S_{ij} = L_j(T) - E_j(T)$	Remark
1	0	0	0	critical
2	50.83	50.83	0	critical
3	125.83	125.83	0	critical
4	125.83	125.83	0	critical
5	245.83	245.83	0	critical
6	262.50	262.50	0	critical
7	322.50	412.50	90	non-critical
8	322.50	412.50	90	non-critical
9	412.50	412.50	0	critical
10	475	475	0	critical

The critical path is:

Fig. 4. The critical path for model 2

Examining the table of slack times thoroughly, events 1, 2, 3, 4, 5, 6, 9 and10 are observed to be critical to the production framework. This underscores their pivotal role to the seamless advancement of the process flow, as any delay in any of the events will trigger an overall delay in the juice production. Summing up the estimated duration of these critical events provides the anticipated duration for model 2. This expected duration is mathematically computed as follows;

$E(T) = 50.83 + 75 + 120 + 16.67 + 150 + 62.5 = 475$ minutes.

Comparing this estimated duration of 475 minutes to the prevailing practices of SCUBED 100%, a sharp contrast and an improved efficiency is glaring. This refined model 2 proves to be a better improvement of model 1, optimizing time and enhancing gains by strategically adjusting the sequence of events and allowing for parallel execution of certain pairs. A very impressive reduction in production time by 181.66 minutes was achieved by model 2. This intentional refinement of the prevailing model and the resultant optimized efficiency of the new system (model 2) is an affirmation of the benefits of process optimization. This is an immediate gain to the operations management of SCUBED 100%.

Striving for continued improvement to further integrate better practices in the operations management, further modification is made to model 2, seeking possible enhancements. Building on the refined model 2, advanced innovation is sought by further relying on the power of simultaneous execution of events. Activities (A and F), (B and C), (D and E), (G and H), (I and J) are scheduled to run concurrently. Again, being poised to streamline activities and eliminate any seeming inefficiencies, it becomes pertinent to remove the initial activity ‘F’ which represents “Transfer juice into freezers” (see Table 2). The concurrent running of activities ‘A’ and the new ‘F’ makes the initial activity 'F' (transfer juice into freezers) redundant, thereby reinforcing the rationale behind this deliberate removal. A structural rearrangement and modification of model 2, factoring in all the amendments described, led to the emergence of a new model 3, as depicted in Table 7 and Fig. 5.

Table 7. Tabular representation of model 3 data in CPM (modification 2)

I. D	Activities	Duration of activity (Mins)	Immediate predecessor
A	Sorting of fruits	45	None
B	Washing of fruits	60	A
C	Temporary holding of fruits before juicing	20	A
D	Juicing of fruits	90	B
E	Filtering the fruit juice	90	C
F	Washing of the bottles	45	None
G	Checking the bottles thoroughly	20	D, E, F
H	Pouring the juice into the bottles	120	D, E, F
I	Labeling each bottle properly	60	G
J	Store the bottled juice in freezers	45	H

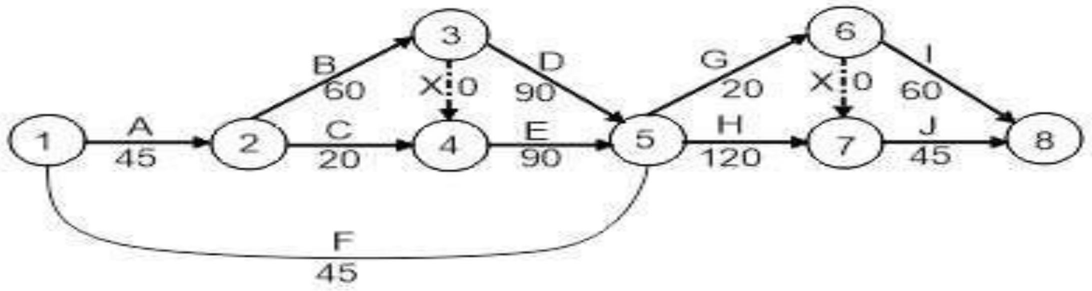

Fig. 5. Model 3

Table 8. Tabular representation of model 3 data in PERT (modification 2)

Activities	Immediate predecessors	x = most likely time	w = optimistic time	y = pessimistic time	E = mean time	σ = variance
A	None	50	45	60	50.83	6.25
B	A	75	60	90	75	25
C	A	25	20	30	25	2.79
D	B	120	90	150	120	100
E	C	120	90	150	120	100
F	None	60	45	75	60	25
G	D, E, F	30	20	45	30.83	17.39
H	D, E, F	150	120	180	150	100
I	G	65	60	75	65.83	6.25
J	H	60	45	90	62.50	56.25

Once again, a key metric to the identification of the critical path, the slack times, is identified from the calculation of the earliest and latest time (E_j and L_j) of each event in model 3. The results will allow for the unravelling of valuable insights into the process dynamics. Computation of these quantities yields the following;

$E_1(T) = 0$
$E_2(T) = E_1(T) + t_{12} = 0 + 50.83 = 50.83$
$E_3(T) = E_2(T) + t_{23} = 50.83 + 75 = 125.83$

$E_4(T) = \max\{E_2(T) + t_{24}, E_3(T) + t_{34}\} = \max\{50.83 + 25, 125.83 + 0\} = 125.83$
$E_5(T) = \max\{E_1(T) + t_{15}, E_3(T) + t_{35}, E_4(T) + t_{45}\} = \max\{0 + 60, 125.83 + 120,$
$125.83 + 120\} = 245.83$
$E_6(T) = E_5(T) + t_{56} = 245.83 + 30.83 = 276.66$
$E_7(T) = \max\{E_5(T) + t_{57}, E_6(T) + t_{67}\} = \max\{245.83 + 150, 276.66 + 0\} = 395.83$
$E_8(T) = \max\{E_6(T) + t_{68}, E_7(T) + t_{78}\} = \max\{276.66 + 65.83, 395.83 + 62.50\} = 458.33$

$L_8(T) = L_8(T) = 458.33$
$L_7(T) = L_8(T) - t_{87} = 458.33 - 62.50 = 395.83$
$L_6(T) = \min\{L_7(T) - t_{76}, L_8(T) - t_{86}\} = \min\{395.83 - 0, 458.33 - 65.83\} = 392.50$
$L_5(T) = \min\{L_6(T) - t_{65}, L_7(T) - t_{75}\} = \min\{392.50 - 30.83, 395.83 - 150\} = 245.83$
$L_4(T) = L_5(T) - t_{54} = 245.83 – 120 = 125.83$
$L_3(T) = \min\{L_4(T) - t_{43}, L_5(T) - t_{53}\} = \min\{125.83 - 0, 245.83 - 120\} = 125.83$
$L_2(T) = \min\{L_3(T) - t_{32}, L_4(T) - t_{42}\} = \min\{125.83 - 75, 125.83 - 25\} = 50.83$
$L_1(T) = \min\{L_2(T) - t_{21}, L_5(T) - t_{51}\} = \min\{50.83 - 50.83, 245.83 - 60\} = 0$

Table 9. Tabular representation of slack times for model 3

Events	Ej (T)	Lj (T)	Sij = Lj (T) - Ej (T)	Remark
1	0	0	0	critical
2	50.83	50.83	0	critical
3	125.83	125.83	0	critical
4	125.83	125.83	0	critical
5	245.83	245.83	0	critical
6	276.66	392.50	115.84	non-critical
7	395.83	395.83	0	critical
8	458.33	458.33	0	critical

The critical path is:

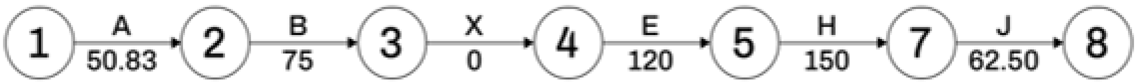

Fig. 6. The critical path for model 3

A careful consideration of the slack times computed above (see table 9) reveals that only events 1, 2, 3, 4, 5, 7, and 8 of model 3 are critical to the successful and timely completion of the juice production process. Fig. 6 visually illustrates this with the critical path diagram. Quantifying the expected duration of the production in the context of model 3, the mathematical expression is as follows;

$E(T) = 50.83 + 75 + 120 + 150 + 62.50 = 458.33$ minutes.

The results from the expected duration for model 3 reveals a further reduction in time from that of the model 2 by 16.67 minutes. This observed decrease in completion time alludes to the fact that model 3 is indeed an optimization of model 2. The modifications made to model 2 that saw to the emergence of model 3 were indeed worthwhile and effective.

Further comparison with the prevailing practices at SCUBED 100% (model 1) portrays a huge advancement in time efficiency, a stupendous gain of 198.33 minutes. This realization is profound and well worthy of note. Painstaking arrangement of sequence of events, leverage on concurrent event execution and streamlining of processes led to an improvement of model 1 to achieve model 3. This typifies the certainty of continuous improvement and innovation in operational management, where each iteration builds on its' predecessor to achieve greater heights of excellence.

5 Conclusion

In this work, a comprehensive study of the operational procedures of SCUBED 100% in their juice production was undertaken.

- The goal of the study was to develop a model that returned the least production time, thus enhancing operational efficiency.
- A model was formulated with data collected from the company, and thereafter modified to seek an optimized version that would provide the needed solution.
- The CPM and PERT techniques were employed in the data analysis and computation to achieve this merit.

The study began with a careful understanding of the processes and the sequence of events leading up to the final product. Necessary data was collected (Table 1 and 2) and a corresponding model (Fig. 1) was developed. Armed with a comprehensive understanding of the process flow and ensuring that no detail was overlooked, an in-depth analysis of the operational management was investigated. The critical path using the CPM was determined, and then an expected duration of 656.66 minutes using the PERT technique was obtained. This value served as a reference point for subsequent modifications and improvements to the prevailing practices.

Thereafter, meaningful and allowable modifications to the sequence and structure of events was made to the existing model 1, leveraging on the possibility of parallel operational flows, to get a modified model 2 (as displayed in Fig. 3). This reconfiguration led to the determination of a new critical path with a new expected duration of 475 minutes, using the PERT technique. This modified model yielded 181.66 minutes decrease in operational time, from the initial expected duration, denoting an enormous stride towards the achievement of operational efficiency.

Probing further, and streamlining activities by doing away with an activity, “Transfer juice into freezer”, the preceding model 2 was re-adjusted to formulate model 3 (Fig. 5). A new reshaped critical path was obtained for model 3, and using the PERT technique, a better expected duration of 458.33 minutes was obtained. This is 16.67 minutes less than the expected duration of the predecessor model 2, and 198.33 minutes less than the base model 1. A diligent deployment of PERT and CPM techniques gave rise to an optimized model that ensured production time gain at SCUBED 100%. This research thus reaffirms the transformational potentials of operations research and management when applied in real-world scenarios. Table 10 summarizes the outcomes for the different models.

Table 10. Summary of outcomes for all three models

Model	**1**	**2**	**3**
Total number of activities	11	11	10
Number of critical activities	11	6	5
Critical activities	ALL	A, B, E, F, I, K	A, B, E, H, J
Total expected duration (Mins)	656.66	475	458.33
Time gained / optimized from preceding model	-	181.66	16.67

As much as carrying out projects in general entails various costs, labour, resources and so much more variables, proper management of time remains a very paramount factor that determines success. As a popular saying goes, "time is money". Time inefficiency could lead to delayed outcomes, financial stress, and resources wastage. A myriad of techniques have been proposed and employed to address the challenges of operational inefficiencies. However, the focus in this study is not just on mere efficiency but on time efficiency, such that the job is started and completed in the least possible time without compromising any critical event. The CPM & PERT techniques have proven very effective in the actualization of this goal.

With the quest by SCUBED 100% to increase its operational efficiency, the optimal model 3 (Fig. 5) was prescribed for its’ juice production, as it guarantees reduced production time which translates to a plethora of advantages. By adopting model 3, the company ensures timely delivery of consumables to its’ clients, heightened customer satisfaction and ultimately increased profit for the brand. This surplus time recovered by

SCUBED 100% also allows the company to have more time to carry out any other processes that will lead to its' business expansion such as product diversification. This time optimization through the deployment of model 3 will set SCUBED 100% on a path of greatness and enhanced operational competitiveness and excellence. The time harnessed thus cascades to increased profit margins and sustainable business growth in the long term.

Competing Interests

Authors have declared that no competing interests exist.

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