Al Azhar Bulletin of Science Vol. 29, No. 1, (June) 2018, pp. 31-44

INVESTIGATION OF ENERGY STAGGERING, IDENTICAL TRANSITION ENERGIES AND SHAPE BEHAVIORS IN ROTATIONAL BANDS OF ACTINIDE NUCLEI BY USING SOFTNESS MODEL

Asmaa Abdelsalam

Physics Department - Faculty of science (Girls College) - Al-Azhar University - Cairo – Egypt
asmaaabdelsalam@azhar.edu.eg

ABSTRACT

The nuclear two – parameters softness model has been used to calculate the energy levels of the ground state bands in even - even actinide nuclei namely ^{228,230}**Th**, $^{230-238}$ **U**, $^{236-244}$Pu, $^{242-248}$ **Cm**, ^{248,250}**Fm** *and* ^{252,254} **No**. *For each band the optimum values of the softness parameter and the ground state moment of inertia are calculated by the fitting procedure between the calculated and the experimental excitation energies using a computer simulated search program. Very good agreement is found between the calculated and experimental data. The nuclear kinematic and dynamic moments of inertia have been calculated; a smooth gradual increase in both moments of inertia as function of rotational frequency was seen. The* $\Delta I = 2$ *energy staggering index represents the finite difference approximation of fourth order derivative of the transition energies is extracted and examined. The transition energies in the ground state bands of* ^{236}**U** *and* ^{238}**U** *have quite identical energies within* **2 KeV** *up to spin 24* $\hbar$*, which indicate that the phenomenon of identical bands is not restricted to superdeformed bands. The study indicates also that these conjugate pair of nuclei* ^{236}**U** *and* ^{238}**U***have moments of inertia nearly identical. The potential energy surfaces for isotones* ^{234}**Th,**^{236}**U***and* ^{238}**Pu** *are calculated and show rotational behavior mainly prolate deformed.*

1. INTRODUCTION:

Theoretically, a number of models were introduced for correlating the large number of experimental data for energy levels of ground state bands in even- even nuclei. In particular the Bohr- Mottelson model [1], the Holmberg-Lipas model [2] and the variable moment of inertia model [3]. The interacting boson model [4] and the geometric collective model [5] represent two major phenomenological models that successfully describe nuclear collectivity. All the above mentioned models have been very successful in unfolding ground state rotational bands. In the present work, it is possible to describe the ground band of actinide nuclei by using the nuclear softness model [6,7] which was proposed by treating the variation of the moment of inertia with spin in a very simplified and generalized manner.

An interesting feature that happen in rotational bands is the observation of $\Delta \mathbf{I} = \mathbf{2}$ staggering in energies [8-13], the energy levels are consequently separated into two $\Delta I = 4$ sequences with spin values I, I + 4, I + 8, ---, and I + 2, I + 6, I + 10 ,--- respectively, (a zigzage behavior in staggering indices as a function of rotational frequency).

One striking and unexpected feature happen in superdeformed rotational bands is the identical bands (**IB's**) [14] in which nuclei have almost identical energies within ~ **2 KeV** and therefore they requires that the moments of inertia in the two bands be identical. Many theoretical explanations were proposed [15-22] to interpret the existence of **IB's** but a satisfactory explanation is still lacking. Also the IB'S were seen in the ground state bands in normal deformed nuclei [23] and a number of

IB's were observed at both low and high spins in different mass regions [24-26]. The shape transitions is phenomenon which are well known to exist in various regions of nuclear chart [27].In the present work, we resolve the problems of the anomaly Δ **I = 2** energy staggering, the identical bands in normally deformed nuclei and the shape phase transitions. We used the nuclear softness model. Our method is applied to even – even actinide nuclei $_{90}$**Th,**$_{92}$**U**, $_{94}$**Pu**, $_{96}$**Cm**, $_{100}$**Fm**and $_{102}$ **No**.

2. Outline of Nuclear Softness Model

In pure rotor model, the excitation energies of the member of ground state band with angular momentum **I** is given by [1]

$$E(I)=\frac{\hbar^2}{2J}I(I+1) \qquad (1)$$

In nuclear softness model (NSM) [6,7] the variation of moment of inertia **J** with spin **I** is given by

$$J_I = J_0(1+\sigma I) \qquad (2)$$

where, $\mathbf{J_0}$ is the ground state moment of inertia and **σ** is the softness parameter

$$\left(\sigma=\frac{1}{J_0}\left(\frac{\partial J_1}{\partial_1}\right)_{1=0}\right)$$

Substituting the value of moment of inertia **J** in terms of nuclear softness parameter **σ** in equation (1) we get

$$E(I)=\frac{\hbar^2}{2J_0}\left[\frac{I(I+1)}{(1+\sigma I)}\right] \qquad (3)$$

The transition energies take the following formula

$$E\gamma(I) = E(I) - E(1-2)$$

$$= A\left[\frac{I(I+1)}{(1+\sigma I)}-\frac{(I-2)(I-1)}{1+\sigma(I-2)}\right] \qquad (4)$$

With $A = \hbar^2/2J_0$

Now , we define the energy ratio **R (I)** as

$$R(I)=\frac{E(I)}{E(2)}$$

$$=\frac{I(I+1)}{6}\frac{1+2\sigma}{1+I\sigma} \qquad (5)$$

In particular

$$\frac{R(6)}{R(4)}=\frac{21}{10}\frac{I+4\sigma}{I+6\sigma} \qquad (6)$$

As an approximate estimation of the nuclear softness parameter **σ** one can get

$$\sigma=\frac{21R(4)-10R(6)}{60R(6)-84R(4)}\frac{1+4\sigma}{1+6\sigma} \qquad (7)$$

3. The Δ I = 2 Energy Staggering

In the Δ **I = 2** staggering, the rotational band is splitted into two sequences with states separated by Δ **I = 4** shifting up in energy and the intermediate states shifting down in energy. The two sequences have spin values I, I + 4 , I + 8 ,----- and I + 2 , I + 6 , I + 10 , -------- respectively.

In order to explore more clearly the Δ **I = 2**staggering in a band, the deviation of the transition energies from a smooth reference is determined by calculating the finite difference approximation to higher order derivative of the transition energies $\mathbf{E_\gamma}$ **(I)** at a given spin $\mathbf{d^n E_\gamma / d I^n}$. The staggering indices $\mathbf{S^{(n)}}$ **(I)** is given by

$$S^{(n)}(I)=\frac{1}{2^n}\sum_{k=0}^{n}(-1)^{n+k}\binom{n}{k}E_\gamma(x+2k) \qquad (8)$$

where x = I , I – 2 , I – 2 , and I + 4 for first , second, third and fourth derivative and the binomial coefficient is given by

$$\binom{n}{k}=\frac{n!}{k!(n-k)!} \qquad (9)$$

For each band the deviation of the gamma – ray transition energies from a smooth reference has been determined. Therefore

$$S^{(1)}(I) = \frac{1}{2}\left[E_\gamma(I+2) - E_\gamma(I)\right] \quad (10)$$

$$S^{(2)}(I) = \frac{1}{4}\left[E_\gamma(I-2) - 2E_\gamma(I) + E_\gamma(I+2)\right] \quad (11)$$

$$S^{(3)}(I) = \frac{1}{8}\left[-E_\gamma(I-2) + 3E_\gamma(I) - 3E_\gamma(I+2) + E_\gamma(I+4)\right] \quad (12)$$

$$S^{(4)}(I) = \frac{1}{16}\left[E_\gamma(I-4) - 4E_\gamma(I-2) + 6E_\gamma(I) - 4E_\gamma(I+2) + E_\gamma(I+4)\right] \quad (13)$$

Thus last, staggering index include five consecutive transition energies and is denoted by a five point formula. We say that Δ **I** = **2**staggering is observed if the staggering index exhibit alternating signs with increasing spin or angular frequency.

4. Rotational Frequency and Moments of Inertia

The rotational frequency **ħ ω** is defined as a derivative of the energy **E** with respect to the angular momentum as

$$\hbar\omega = \frac{dE}{d\hat{I}} \quad (14)$$

The use of **Î = [I (I + 1)]**$^{1/2}$ rather than angular momentum **I** provides the proper limiting case for an ideal rotor with energy proportional to the I (I + 1) rather **I** ^{2}.

Two possible definitions for nuclear moment of inertia were suggested [28] reflecting two different aspects of nuclear dynamic : the kinematic moment of inertia **J** $^{(1)}$ is equal to the inverse of the slope of the curve of energy **E** versus **Î**^{2}(or I (I + 1)) times **ħ** ^{2} / **2** andthe dynamic moment of inertia **J**$^{(2)}$ which is related to the curvature in the curve of **E** versus **Î** (or [I (I + 1)]$^{1/2}$).

$$\frac{J^{(1)}}{\hbar^2} = \frac{1}{2}\left[\frac{dE}{d(\hat{I}^2)}\right]^{-1} = \frac{\hat{I}}{\hbar\omega} \quad (15)$$

$$\frac{J^{(2)}}{\hbar^2} = \left[\frac{d^2E}{d(\hat{I}^2)}\right]^{-1} = \frac{1}{\hbar}\frac{d\hat{I}}{d\omega} \quad (16)$$

If the rotational excitation energies **E (I)** obey the **I (I + 1)** rule, we can determine the rotational frequency, the kinematic **J**$^{(1)}$ and dynamic **J**$^{(2)}$ moments of inertia as

$$\frac{1}{4}\left[E_\gamma(I) + E_\gamma(I+2)\right] = \frac{\hbar^2}{2J}(2I+1) = \frac{\partial E}{\partial I} = \hbar\omega \quad (17)$$

$$E\gamma(I) = E(I) - E(1-2)$$

$$= \frac{\hbar^2}{2J^{(1)}}(4I-2) \quad (18)$$

2

$$\Delta E_\gamma(I) = \frac{\partial E}{\partial I}dI + \frac{\partial E}{\partial J}dJ = \frac{\hbar^2}{2J}(4)dI - \frac{\hbar^2}{2J^2}(4I-2)dJ \quad (19)$$

$$\frac{\Delta E_\gamma(I)}{\Delta I} = 4\frac{\hbar^2}{2J} - \frac{\hbar^2}{2J^2}(4I-2)\frac{dJ}{dI} = 2\frac{\hbar^2}{J} - \frac{1}{J}E_\gamma\frac{dJ}{dI} = 2\frac{\hbar^2}{J} - E_\gamma\frac{d(\ln J)}{dI} \quad (20)$$

If we use Δ I = 2, then

$$\Delta E_\gamma (I) = 4\frac{\hbar^2}{J} - 2E_\gamma \frac{d(\ln J)}{dI} \quad (21)$$

If the moment of inertia does not change very rapidly with **I**, we obtain

$$\Delta E_\gamma (I) = 4\frac{\hbar^2}{J^{(2)}} \quad (22)$$

That is, the dynamical moment of inertia can be extracted from the energy difference between two consecutive transitions in the band.

5. Identical Bands

Identical **b**ands**(IB's)** are two bands have essentially identical transition energies and thus essentially identical moments of inertia. The initial discovery of IB's was observed in superdeformed nuclei [14].Many theoretical explanation were proposed [15-19] to interpret the existence of IB's but a satisfactory explanation is still lacking. This fascinating phenomenon of IB's was seen also in **n**ormal – **d**eformed **(ND)** nuclei [23]. Since then, a number of IB's were observed at both low and high spins and they span different shapes in several mass regions[24-26].

To determine whether a pair is identical or not one can extract the difference between transition energies ΔE_γ for the identical pair and plotted it versus rotational frequency $\hbar\omega$ or the transition energy E_γ. Also one can compare their dynamical moments of inertia $J^{(2)}$.

6. Potential Energy Surface

According to the geometric collective model [5, 29-31],the **p**otential **e**nergy **s**urface **(PES)** as a function of shape parameters$\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ is given by

$$V(\beta,\gamma) = \frac{1}{\sqrt{5}} C_2 \beta^2 - \sqrt{\frac{2}{35}} C_3 \beta^3 \cos(3\gamma) + \frac{1}{\sqrt{5}} C_4 \beta^4 \quad (23)$$

where $\boldsymbol{\beta} \in [\, 0 , \infty \,]$ and $\boldsymbol{\gamma} \in [\, 0 , 2\pi/3 \,]$

The $\mathbf{C_2}$ and $\mathbf{C_4}$terms describe the γ – independent features while $\mathbf{C_3}$term is responsible for the prolate – oblate energy differences in the PES. Since the parameter $\mathbf{C_3}$ controls the steepness of the potential and there for, the dynamical fluctuations in $\boldsymbol{\gamma}$, it strongly affects the energies of excited intrinsic states. The parameter $\mathbf{C_3 = 0}$ gives a γ – flat potential and an increase of $\mathbf{C_3}$ introduces a γ – dependence in the potential with minimum at $\boldsymbol{\gamma} = \mathbf{0}$. Changing $\mathbf{C_3}$ will indeed induce a γ – unstable to the symmetric rotor transition, it is best to simultaneously vary $\mathbf{C_2}$ and $\mathbf{C_4}$ as well.

7. Numerical Calculations and Discussion

To determine the model parameters $\mathbf{J_0}$ and $\boldsymbol{\sigma}$ a fitting procedure has been applied to all measured values of excitation energies **E (I)** in a given band by using a computer simulated search program to minimize χ^2, with

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{E^{cal}(I_i) - E^{exp}(I_i)}{\Delta E^{exp}(I_i)}\right|^2$$

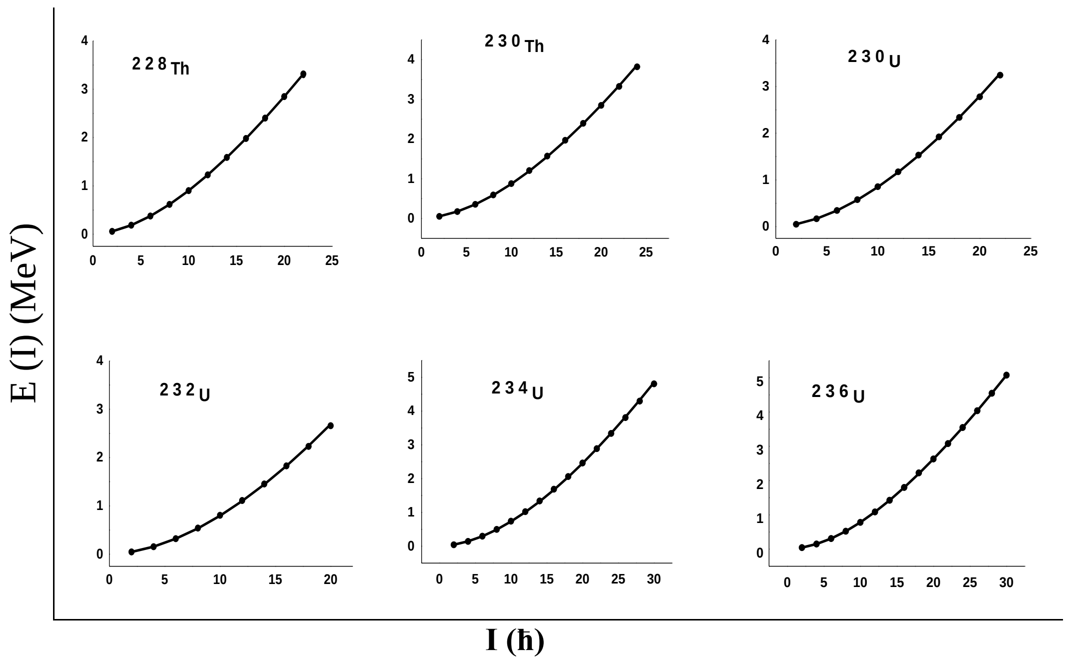
Where **N** is the number of data points entering the fitting and $\Delta\, \mathbf{E^{exp}}$ is the experimental errors in the excitation energies.

The optimized values of the parameters $\mathbf{J_0}$ and $\boldsymbol{\sigma}$ of the softness model results from the fitting procedure for our selected bands are listed in **Table (1)** and have been used to calculate the excitation energies.

To illustrate the quantitative agreement obtained from the excitation energies, we have presentedin Figure (1), a systematic comparison between theoretical and experimental excitation energies. The experimental energies are taken from the National Nuclear Data Center [32].

Table (1) The adopted best parameters **J_0** and **σ** obtained for ground state band in the studied **Th – U – Pu-Cm-Fm-No** actinide nuclei to investigate the **ΔI =2** staggering

Nucleus	A	J_0 ($\hbar^2$ MeV^{-1})	σ (10^{-2})
$_{90}$Th	228	96.291	2.657736
	230	105.815	1.953591
$_{92}$ U	230	109.329	1.873787
	232	119.972	1.504813
	234	129.483	1.603931
	236	123.780	1.513000
	238	123.764	1.585043
$_{90}$Pu	236	130.354	0.985314
	238	130.608	0.986091
	240	134.000	1.074948
	242	128.626	1.099227
	244	118.756	1.684271
$_{96}$ Cm	242	137.490	0.978982
	246	133.416	1.079888
	248	129.351	1.278925
$_{100}$ Fm	248	127.653	0.6676412
	250	130.884	0.8172362
$_{102}$ No	252	125.770	0.7290139
	254	134.179	0.4826813



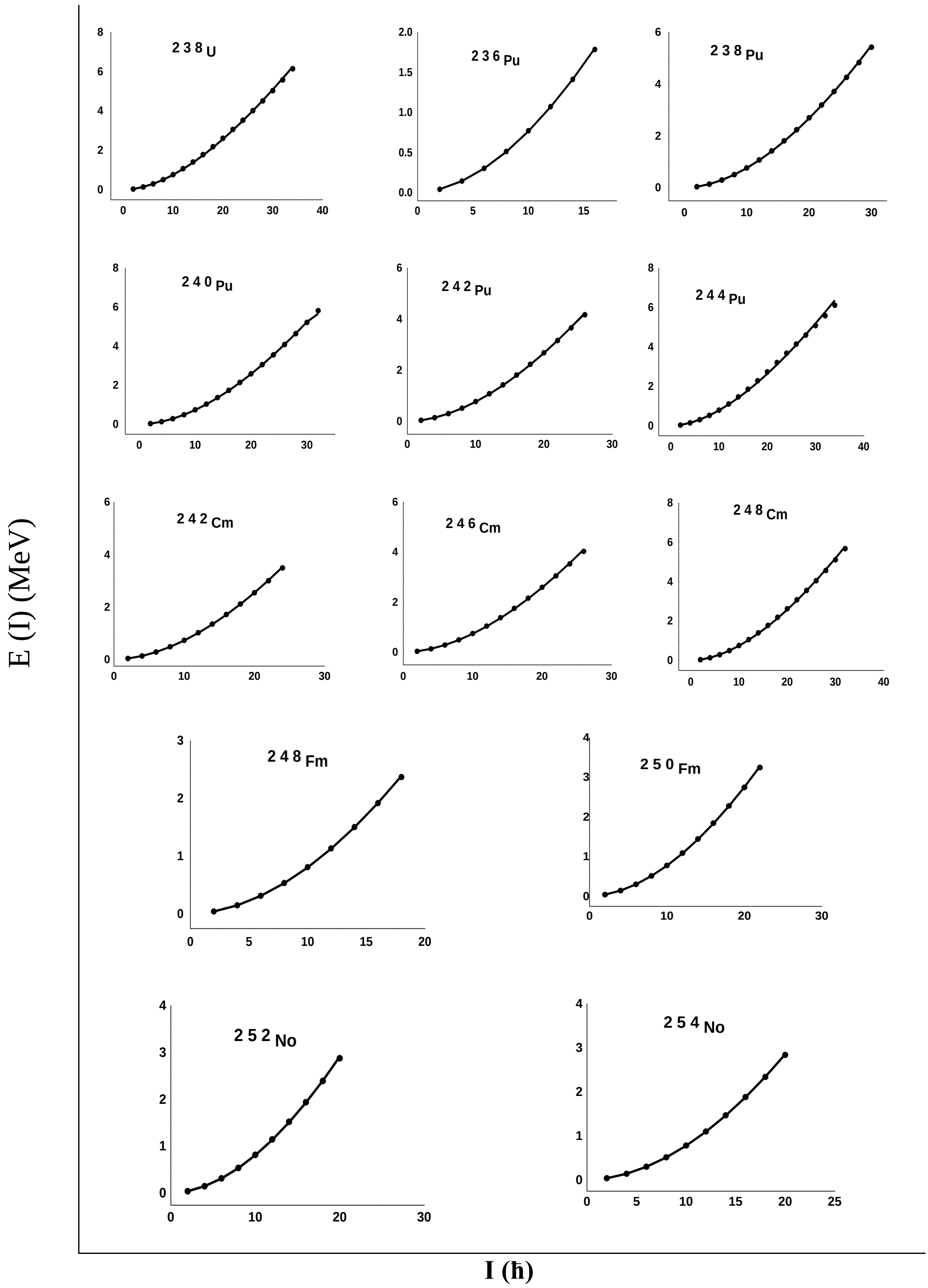
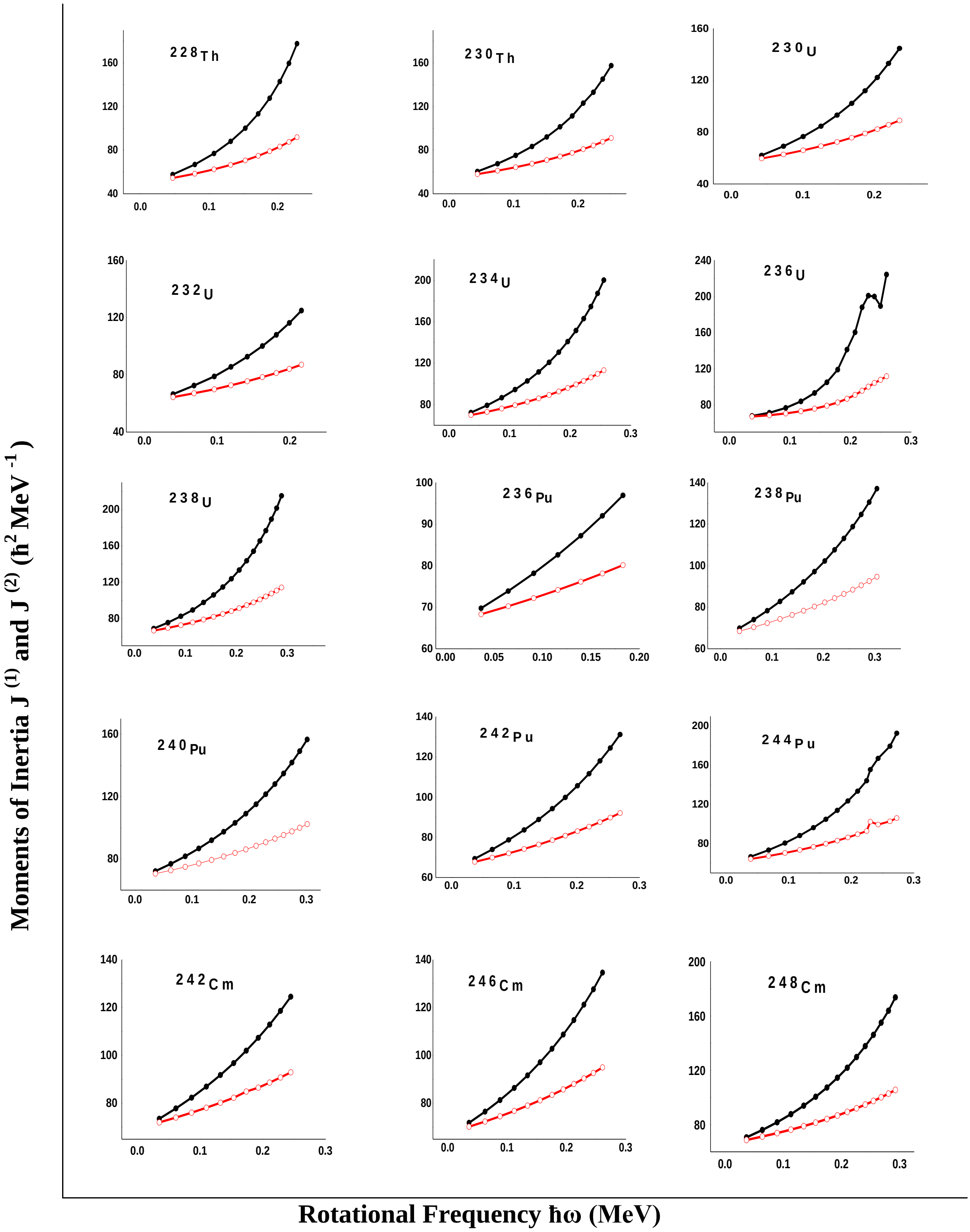


Figure (1) Calculated (solid curves) and experimental (closed circles) excitation energies **E(I)** versus spin **I** for the ground state bands in our selected nuclei

The variation of the deduced nuclear kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia as a function of rotational frequency **ħ ω** are illustrated in **Figure (2)**, a smooth gradual increase in both moments of inertia are seen.



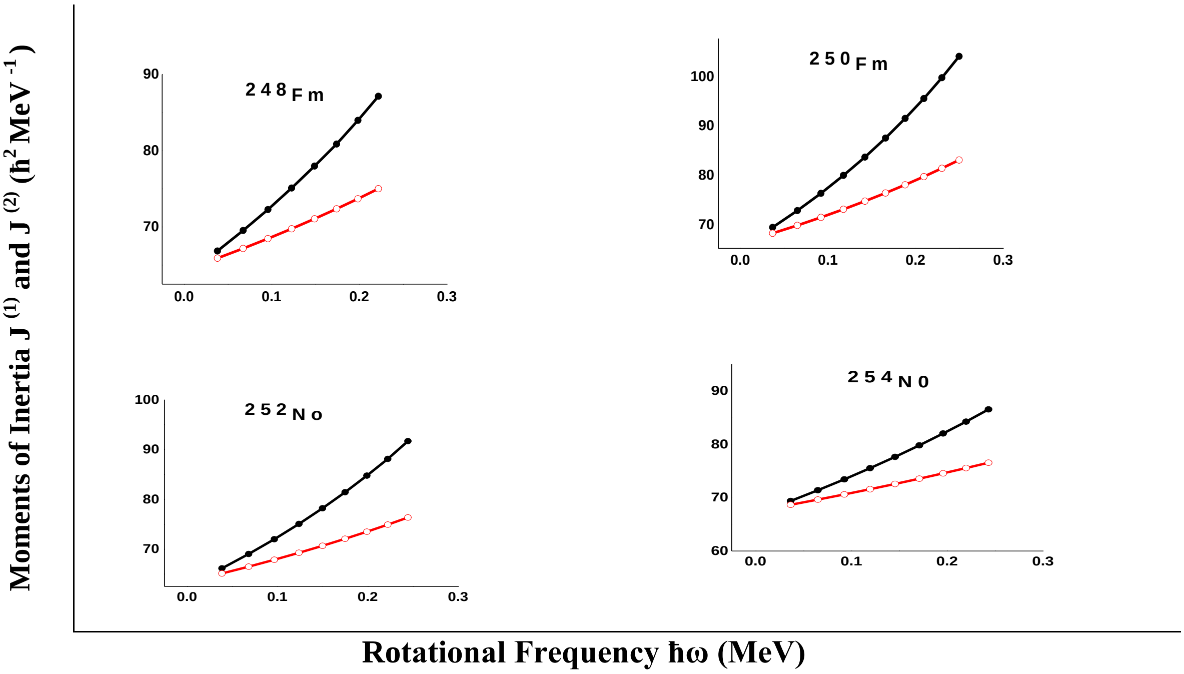


Figure (2) Calculated kinematic $J^{(1)}$(open circles) and dynamic $J^{(2)}$ (closed circles) moments of inertia as a function of rotational frequency **ħω** for the ground state bands in our selected nuclei

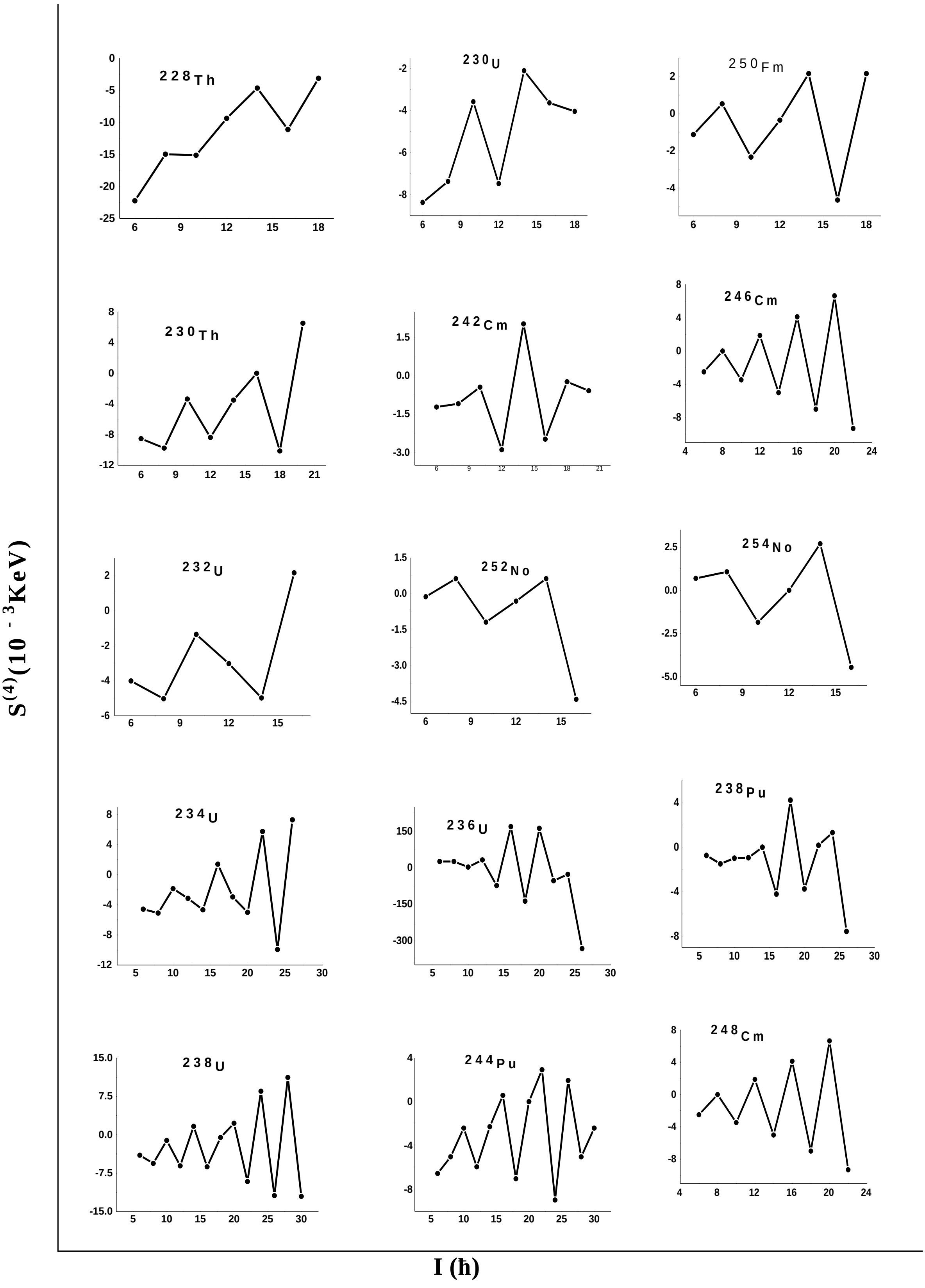
In Table (2) and Figure (3) we present the behavior of Δ **I = 2** staggering index $S^{(4)}$ (I) as a function of nuclear spin I for each rotational band for the studied actinide nuclei. These curves for the five point formula $\mathbf{S^{(4)}}$ show large significant staggering. The levels with spin sequence I , I + 4 , I + 8 , ---- are displaced relative to the sequences I + 2 , I + 6 , I + 10 ,-------- . That is states differing by four units of angular momentum show an energy shift (Δ I = 4 bifurcation).

Table (2) The calculated **ΔI=2** staggering parameter $\mathbf{S^{(4)}}$ obtained by five point formula as a function of spin **I** for the even –even actinide nuclei Th-U-Pu-Cm-Fm-N0. The calculated transition energy $\mathbf{E_\gamma}$ **(I)** are also given.

I (ħ)	$\mathbf{E_\gamma(I)}$ (KeV)	$\mathbf{S^{(4)}}$ (10^{-3}KeV)	**I** (ħ)	$\mathbf{E_\gamma(I)}$ (KeV)	$\mathbf{S^{(4)}}$ (10^{-3}KeV)	**I** (ħ)	$\mathbf{E_\gamma(I)}$ (KeV)	$\mathbf{S^{(4)}}$ (10^{-3}KeV)
	^{228}Th			**^{230}U**			**^{250}Fm**	
2	59.1660		2	52.8974		2	45.10464	
4	128.5777		4	117.2805		4	102.86484	
6	188.4450	-22.25	6	175.1570	-8.369	6	157.92554	-1.124
8	240.4394	-14.98	8	227.3752	-7.373	8	210.45238	0.530
10	285.8764	-15.14	10	274.6493	-3.583	10	260.59299	-2.336
12	325.8319	-9.38	12	317.5757	-7.475	12	308.50351	-0.351
14	361.1394	-4.65	14	357.6934	-2.106	14	354.30268	2.158
16	392.4822	-11.12	16	392.4218	-3.637	16	398.10365	-4.648
18	420.4694	-3.12	18	425.1466	-4.042	18	440.05407	2.157
20	445.5319		20	455.1953		20	480.22723	
22	468.0509		22	482.8307		22	514.73093	
	^{230}Th			**^{242}Cm**			**^{246}Cm**	
2	54.5702		2	42.8015		2	44.021	
4	120.7392		4	97.1898		4	99.678	
6	179.9664	-8.527	6	148.5454	-1.221	6	151.947	-2.500
8	233.1854	-9.760	8	197.1109	-1.092	8	201.102	0.0
10	281.1933	-3.360	10	243.0782	-0.442	10	247.377	-3.465
12	324.6312	-8.370	12	286.6292	-2.893	12	291.006	1.882
14	364.0864	-3.500	14	327.9391	2.033	14	332.168	-5.011
16	400.0122	0.0	16	367.1364	-2.478	16	371.071	4.130
18	432.8060	-10.140	18	404.3826	-0.232	18	407.844	-6.997
20	462.8655	6.527	20	439.7991	-0.584	20	442.681	6.653
22	490.4258		22	473.5040		22	475.666	-9.315
24	515.8267		24	505.6058		24	506.988	
						26	536.686	
	^{232}U			**^{252}No**			**^{254}No**	
2	48.5507		2	47.02		2	44.28	
4	108.6905		4	107.49		4	101.94	
6	163.8497	-4.011	6	165.43	-0.125	6	157.97	0.687
8	214.5634	-5.025	8	220.98	0.627	8	212.44	1.069
10	261.3025	-1.353	10	274.26	-1.189	10	265.40	-1.850
12	304.4578	-3.019	12	325.39	-0.310	12	316.92	0.0
14	344.3981	-4.979	14	374.50	0.628	14	367.04	2.692
16	381.4440	2.161	16	421.68	-4.410	16	415.80	-4.456
18	415.8366		18	467.06		18	463.28	
20	447.8513		20	510.67		20	509.50	
	^{234}U			**^{236}U**			**^{238}Pu**	
2	44.89770		2	45.2454		2	45.05045	
4	100.25067		4	104.2313		4	102.26834	
6	150.74292	-4.580	6	160.3033	25.702	6	156.29292	-0.739
8	196.92855	-5.097	8	212.4508	25.755	8	207.35002	-1.490
10	239.28838	-1.852	10	260.0745	2.830	10	255.65915	-0.982
12	278.22170	-3.139	12	302.9872	32.215	12	301.41231	-0.943
14	314.09815	-4.663	14	341.0468	-73.466	14	344.78609	0.0
16	347.23714	1.403	16	374.6269	169.431	16	385.94437	-4.209
18	377.88347	-2.952	18	402.9255	-137.533	18	425.04834	4.231
20	406.30438	-4.998	20	427.8515	162.435	20	462.19422	-3.747
22	432.71989	5.756	22	449.1134	-53.548	22	497.54532	0.178
24	457.27006	-9.944	24	469.0184	-27.12	24	531.20502	1.313
26	480.18702	7.303	26	489.0172	-333.069	26	563.27951	-7.571
28	501.54381		28	510.1265		28	593.89603	
30	521.53033		30	527.9538		30	623.06067	

^{238}U			^{244}Pu			^{248}Cm		
2	46.98953		2	48.8774		2	45.2287	
4	104.97344		4	108.9060		4	101.8647	
6	157.92236	-4.020	6	163.4231	-6.541	6	154.4661	-2.087
8	206.39913	-5.610	8	213.0886	-5.016	8	203.4025	-2.180
10	250.90225	-1.121	10	258.4577	-2.395	10	249.0101	-3.560
12	291.84046	-6.094	12	300.0056	-5.933	12	291.5903	1.211
14	329.60455	1.643	14	338.1687	-2.268	14	331.3875	-5.580
16	364.48779	-6.286	16	373.2889	0.591	16	368.6652	3.751
18	396.80976	-0.540	18	405.6716	-7.019	18	403.5981	-7.390
20	426.78945	2.242	20	435.6317	0.022	20	436.4205	5.026
22	454.63723	-9.161	22	463.3718	2.939	22	467.2487	-2.970
24	480.59932	8.505	24	489.0941	-8.961	24	496.2794	-5.710
26	504.77537	-11.922	26	513.0479	1.950	26	523.6617	9.447
28	527.40115	11.206	28	535.3391	-5.016	28	549.4534	-13.140
30	548.52163	-12.070	30	556.1049	-2.395	30	573.8635	
32	568.36115		32	575.5610		32	596.8905	
34	586.95089		34	593.7420				

^{240}Pu					
2	43.83369		2	45.6433	
4	99.26671		4	103.2969	
6	151.34301	-1.074	6	157.3831	-1.416
8	200.32371	-1.729	8	208.1901	-2.803
10	246.452762	-1.205	10	255.9834	0.272
12	89.94642	-1.129	12	300.9831	-2.249
14	331.00171	-1.007	14	343.4140	-0.873
16	369.79753	0.089	16	383.4648	0.082
18	406.49671	-4.606	18	421.3103	-4.971
20	441.26350	4.739	20	457.1266	5.096
22	474.18844	-4.140	22	491.0100	-4.465
24	505.43793	0.249	24	523.1388	
26	535.11211	-0.529	26	553.6194	
28	563.31510	-0.478			
30	590.14258				
32	615.68257				
^{236}Pu					
2	45.138		2	46.382	
4	102.471		4	106.217	
6	156.603	-1.314	6	163.744	-1.631
8	207.768	-0.713	8	219.088	1.260
10	256.179	-2.748	10	272.347	-2.950
12	302.039	1.409	12	323.640	3.249
14	345.503		14	373.041	-5.160
16	386.753		16	420.673	
			18	466.577	


^{228}Th
^{230}U
^{250}Fm
^{230}Th
^{242}Cm
^{246}Cm
^{232}U
^{252}No
^{254}No
^{234}U
^{236}U
^{238}Pu
^{238}U
^{244}Pu
^{248}Cm
$S^{(4)}(10^{-3}$KeV)
I (ħ)

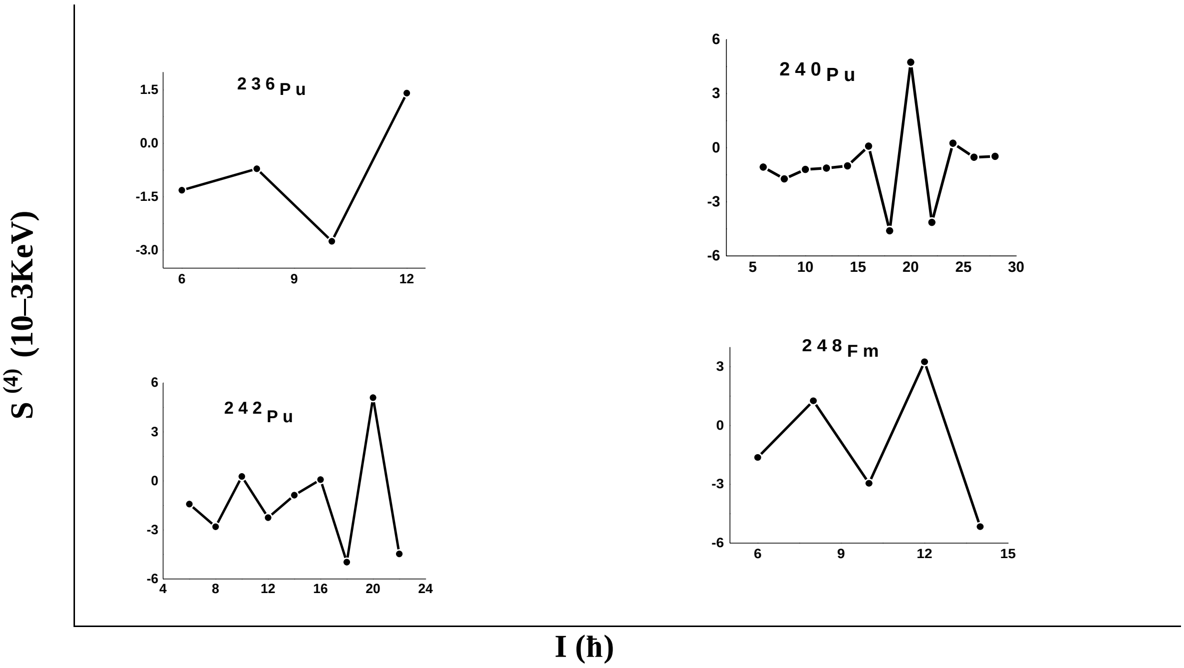


Figure (3) : The calculated **ΔI=2** staggering parameter $\mathbf{S^{(4)}}$as a function of spin **I**for the ground state bands in our selected nuclei

Figure (4)shows the difference in transition energies **δ** $\mathbf{E_\gamma}$ **(I)** between the rotational bands in ^{236,238} **U** versus spin **I** , they are very similar (the average deviation in energy is around **3 KeV**). Therefore, these two bands are considered as identical bands.

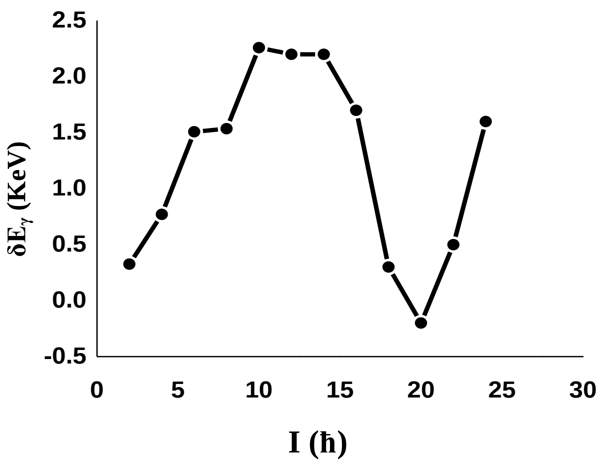


Figure (4) Differences in the γ – ray transition energies between the ground state bands in ^{236} **U** and ^{238} **U**.

The resulting parameters $\mathbf{C_2}$**,** $\mathbf{C_3}$**,** $\mathbf{C_4}$ of the **P**otential **E**nergy **S**urfaces **(PES)** for the isotones ^{234} **Th** – ^{236} **U** –^{238}**Pu** are listed explicitly in **Table (3)**. The corresponding PES's are plotted against the deformation parameter **β** in

Figure (5). The figure shows two wells on the prolate and oblate sides which indicate that these isotones are deformed and have rotational like characters.

Table (3) The geometric collective model parameters in MeV as derived from the fitting procedure used in the calculations

Nucleus	C_2	C_3	C_4
^{234}Th	-2.58700	11.68835	23.07219
^{236} U	-4.89232	16.11576	34.77543
^{238} Pu	-6.22570	18.59511	41.42406

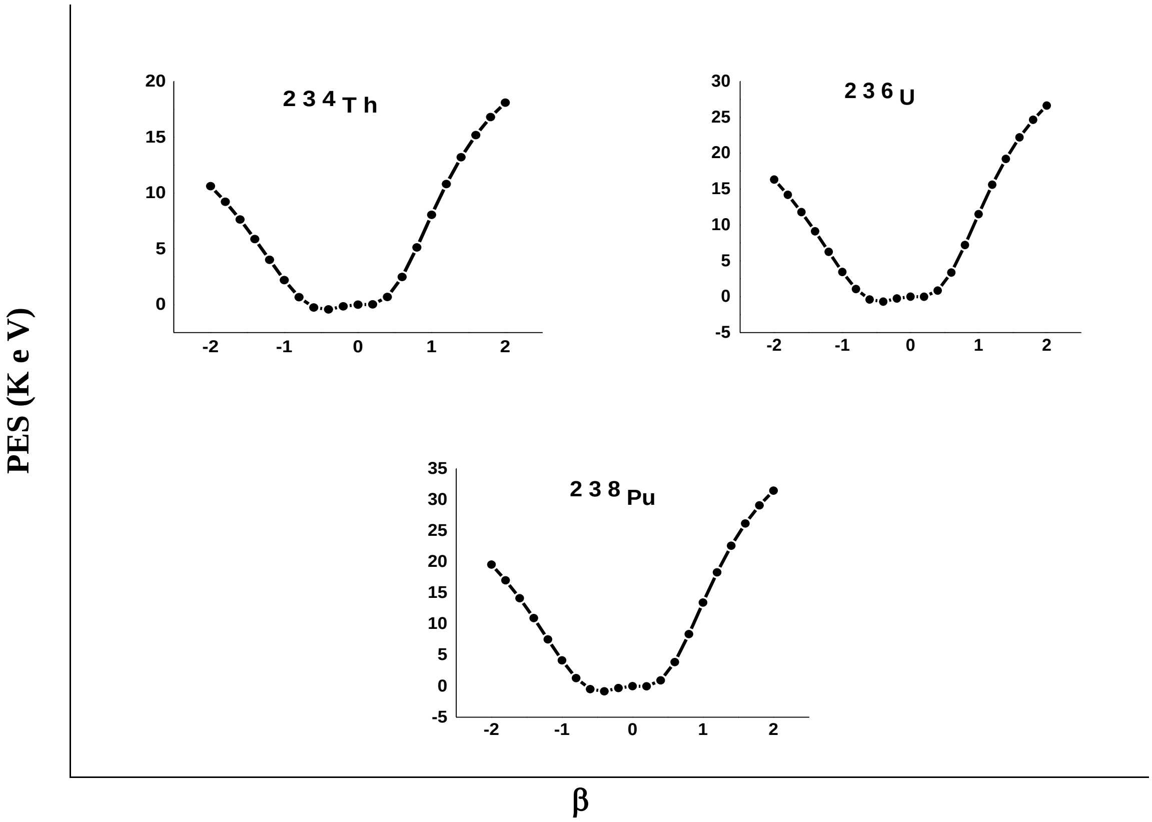


Figure (5) Sketches of the calculated **PES's** as a function of the deformation parameter **β** for the isotones^{234}**Th**, ^{236} **U** and ^{238}**Pu.**

8. Conclusion

The ground state rotational bandsin actinide Th – U – Pu isotopes have been investigated by using the nuclear two parameters softness model. This model is capable to producea systematic comparison between theoretical and experimental excitation energies ,kinematic and dynamic moments of inertia,the Δ **I = 2** staggering, identical bands of **n**ormal – **d**eformed nuclei ^{236} **Uand** ^{238} **U**and shape behavior of ^{234}**Th**, ^{236} **U** and ^{238}**Pu**isotones that are deformed.

References

[1]A. Bohr and B.R. Mottelson, Nuclear Structure (W.A.Benjamin, New York 1975) Volume II

[2] P. Holmberg and P. O. Lipas, Nucl. Phys. A 117 (1968) 552

[3] M. A. J. Mariscotti, G. Scharff- Goldhaber and B. Buck Phys. Rev. 178

(1969) 1864

[4] F.Iachelloand A.Arima,The Interacting Boson Model (Cambridge

University Press, Cambridge 1987)

[5] G. Gneuss, U. Mosel and W. Greiner, Phys. Lett. 30B (1969) 397 and 31B

(1970) 269

[6] Raj. K. Gupta, Phys. Lett. 36B (1971) 173

[7] P. Von Brentano et al, Phys. Rev. C 69 (2004) 044314

[8] S. Flibotte et al, Phys. Rev. Lett. 71 (1993) 4299

[9] B. Caderwall et al, Phys. Rev. Lett. 72 (1994) 3150

[10] I. Hamamoto and B. Mottelson. Phys. Lett. B 333 (1994) 294

[11] A. M. Khalaf, M. Sirag and M. Taha, Turkish J. Phys. 37 (2013) 49

[12] A. M. Khalaf, K. E. Abdelmageed and E. Saber, International Journal of

Theoretical &Applied Sciences 6 (2) (2014) 56

[13] A. M. Khalaf, T. M. Awwad and M. F. Elgabry,International Journal of

Theoretical & Applied Sciences 3 (4) (2016) 47

[14] T. Byrski et al, Phys. Rev. Lett. 64 (1990) 1650

[15] W. Nazarewics al, Phys. Rev. Lett. 64 (1990) 1654

[16] I. Ragnarason, Nucl. Phys. A. 520 (1990) 76

[17] B. Q. Chen. Phys. Rev. C 46 (1992) R 1582

[18] C. Baktash, B. Hass and W. Nazarewicz, Ann.Rev. Nucl. Part. Sci 45

(1995) 487

[19] S. X. Liu, J .Y .Zeng and E. G. Zhao, Phys. Rev. C 66 (2002) 024320

[20] A.M.Khalaf, M. D. Okasha andK. M. Abdelbased , Progress in

Physics 13(1) (2017) 50

[21] A. M. Khalaf and M. D. Okasha, Progress in Physics 10 (2014) 246

[22] A. M. Khalaf, K. E. Abdelmageed and M. Sirag, Turkish J. Phys. 39

(2015) 178

[23] I. Ahmed et al, Phys. Rev. C 44 (1992) 1204

[24] M. Soha and S. Sen, Phys. Rev. C 50 (1994) 2794

[25] J. Y. Zeng et al, Phys. Rev. C 63 (2110) 024305

[26] A. M. Khalaf, M. D. Okasha and K. E. Abdelmageed,Progress in Physics

13 (1) (2017) 50

[27] T. Niksic et al, Phys. Rev. C 65 (2002) 054320

[28] A. Bohr and B. R. Mottelson, Phys. Scr. 24 (1981) 71

[29] J. Eisenberg and W. Greiner , Nuclear Theory Vol. I Nuclear Models:

Collective andSingle – ParticlePhenomena, North – Holland,

Amsterdam , 1987

[30] A. M. Khalaf and A. M. Ismail , Progress in Physics 2 (2013) 51

[31] A. M. Khalaf et al , Progress in Physics 10 (2014) 8,10 (2014) 12

[32] National Nuclear Data Center (NNDC) ,http://www.nndc.bnl.gov/